INTERMEDIATE PHYSICS

Part I.—General Physics, Heat
and Light

Part II.—Magnetism, Electricity
and Sound

Intermediate Practical Physics
by J. Chatterjee

For Calcutta & Dacca Universities
,, Other Indian Universities

(Part I)

BY

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PREFACE TO THE FIRST EDITION

This book is written according to the Intermediate Syllabi of Physics of different Indian Universities. The book has a special recommendation, inasmuch as it is neither too bulky for a student of average merit, nor too meagre in its information for an inquisitive student.

The subject matter herein has been treated in a systematic way and in a simple and clear style with ample illustrations and explanatory diagrams. A sufficient number of Experiments, a Summary at the end of each Chapter and Exercises containing Questions of different Universities with direct reference to the Articles form some special features of the book. Numerous problems of different types put herein will make a student independent of any separate book on Examples. Articles marked with asterisks deal with more advanced portions and may, therefore, be omitted at the first reading.

We take this opportunity to express our gratitude to Prof. Rajani Kanta De, M.A., of the Scottish Church College, Calcutta, who has very kindly given us permission to use some of the blocks prepared for his well-known books on Physics.

Calcutta,
June 25, 1936.

N. BASU
J. CHATTERJEE

PREFACE TO THE THIRD EDITION

In this Edition the book has been thoroughly revised and at some places re-written.

* * * * *

We again cordially thank the colleagues and students for their kind appreciation for the book.

Calcutta,
December 12, 1938.

J. CHATTERJEE

PREFACE TO THE FIFTH EDITION

I have often felt tired to read a book more than once simply because I should have to swallow the same language. To read the 'proofs' of the Intermediate Physics upto the Fourth Edition, I was

even more tired. I have, therefore, thought it worth while to re-write a considerable number of Articles in a simpler language and embody a few additional subjects maintaining the systematic development of ideas as much as possible. Modern practical applications and principles of Sciences have been dealt with in the last Chapter of each Part.

* * * * *

Prof. Basu, although much better in health than last year, has not been able to resume his services and co-operate with me in the modification of the book in this edition. I hope, he would be fit in a short time. One of the principal causes of delay in publishing the book every year is that I am to work for this voluminous book single-handed. In conclusion, I should thank sincerely the patrons of our book.

Calcutta,
August 14, 1941. } J. CHATTERJEE

PREFACE TO THE ELEVENTH EDITION

In this Edition, the book has been thoroughly revised, many old diagrams have been changed and a good lot of new diagrams including line and half-tone blocks have been added. Chapters including Gravitation, Elasticity and Hydrostatic Pressure have been modified. In consideration to the recent developments and fascination for students in Aeronautics, an additional Chapter in General Physics has been incorporated dealing with fundamental ideas in Aerodynamics.

To simplify ideas regarding the formation of images in spherical mirrors and lenses and their combinations, a number of new diagrams have been embodied. Spherical aberrations due to reflection and refraction have been dealt with in a most elementary way. Short sketches of lives of a few Scientists with half-tone blocks have been added at the end of the Chapter. I take this opportunity of expressing my indebtedness to my colleagues of different Colleges, who have always given me suggestions for the modification of the book. I am also thankful to Mr. P. K. Mondal for helping me to make out the new blocks.

Calcutta,
May 25, 1949. } J. CHATTERJEE

PREFACE TO THE FOURTEENTH EDITION

Although quite a lot of additions and alterations was made in its previous Editions, some other subjects of interest such as Utilisation of Water and Air powers, Speedometer, Vacuum Brakes etc., have been embodied in its latest publicity. Questions of many Indian and Pakistan Universities, set in recent years, have also been incorporated,—some as solved problems and some as Exercises. I have every confidence that the modification of the Book, especially during the last two editions, has given it a suitability for its recommendation as a Text Book for Intermediate students of any Indian or Pakistan University, despite the fact that it was a text book of a few universities in its earlier editions.

I would always remain ever grateful to my friends and colleagues both far and near, without whose sympathy and co-operation the Book could not have run through fourteen editions. Their kind and helpful suggestions have made the Book as it appears today. Last but not the least, the students all over the country have esteemed my efforts best in their choice and to them I offer my affectionate thanks.

Calcutta,
June 26, 1953.

J. CHATTERJEE

PREFACE TO THE FIFTEENTH EDITION

In this Edition, the text of th: Book remains almost the same as that of the previous edition except that some modification has been made in the First Chapter of General Physics. There has been a thorough change in worked out Examples and the latest University Questions of many Indian and Pakistan Universities have been embodied. I take this opportunity of offering my heartfelt thanks to my colleagues, distributed throughout India and Pakistan, who never grudged sending me suggestions for all such modifications embodied herein.

Calcutta,
July 27, 1954.

J. CHATTERJEE

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PLATE I

WHERE TWO GREAT MINDS MEET

Galileo helping Milton to have a look through the Telescope

GENERAL PHYSICS

Suppose that you are required to measure the surface area of a rectangular metal sheet. First measure the length of the sheet a number of times by any suitable apparatus and find the mean length. Let it be x cm. Then measure the breadth of the sheet in a similar way. Let the mean breadth be y cm. Then the area of the sheet is $x \times y$ sq. cm. If again you are supplied with a circular sheet, measure the diameter of the circle at several regions and find the mean diameter. Half of the mean diameter is the mean radius r cm., say. Then the area of the circular sheet is $3{\cdot}14 \times r^2$ sq. cm. (as $\pi = 3{\cdot}14$).

Area of Irregular Surfaces.—The area of an irregular surface may be obtained by dividing the surface into suitable areas in the form of triangles, rectangles, or circles and then by adding up the calculated areas of the parts.

By Squared Paper.—The area of a small irregular surface may be determined by placing it on a piece of squared paper, tracing the outline and counting the number of small squares included within its boundary. If half or more than half of a square lies inside the surface, it is taken as one whole and if more than half lies outside, that one is neglected. The area of each square being known, that of the required surface is readily obtained.

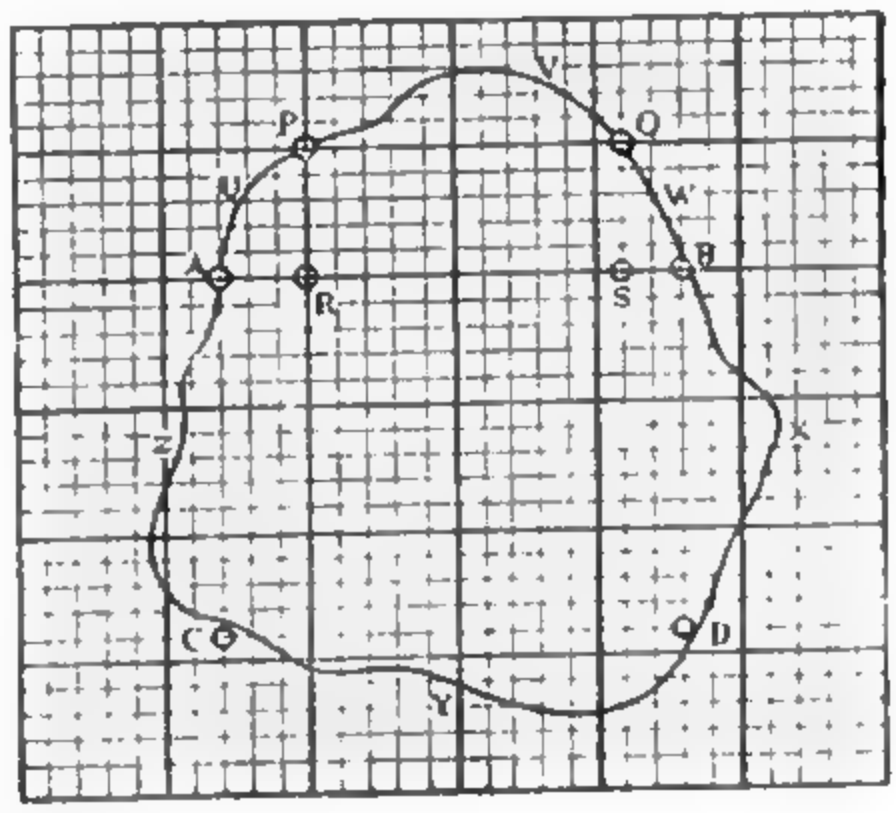


Fig. 14.—Measurement of area

An irregular figure has been traced out on a graph paper (Fig. 14). To find its area, it is divided into as many rectangular figures as possible, *eg.*, PQSR and ABCD. The remaining irregular regions, such as APR, SBQ, BXD, DYC and CZA together with the two rectangular parts and make up irregular figure. Now, the area PQSR = 55 squares. Similarly, area ABCD = 224 small squares; part APR = 12, QSB = 5, BXD = 25, CYD = 31, CZA = 17. Hence the total area is the algebraic sum of the number of squares *i.e.*, 370. If the area of each small square be one sq. mm., then the total area of the figure is 370 sq. mm.

The area of any extended surface, *e.g.*, that of a country, can be determined on this principle by drawing to a scale a map of its boundary on a piece of squared paper. The usual method of measuring the area of a large tract of land is by Trigonometrical survey (vide J. Chatterjee's Practical Physics).

By Weighing = The figure whose area is to be found is drawn accurately on a thin uniform sheet of cardboard or metal plate. The figure is then *very neatly* cut out of the sheet and weighed accurately in a balance. From the same sheet a triangular or rectangular area is cut

out and its weight is also measured. Then since the sheet is uniform,

$$\frac{\text{area of the figure}}{\text{area of rectangle}} = \frac{\text{weight of figure}}{\text{weight of rectangle}}$$

$$\text{Hence, area of figure} = \frac{\text{weight of figure}}{\text{weight of rectangle}} \times \text{area of rectangle.}$$

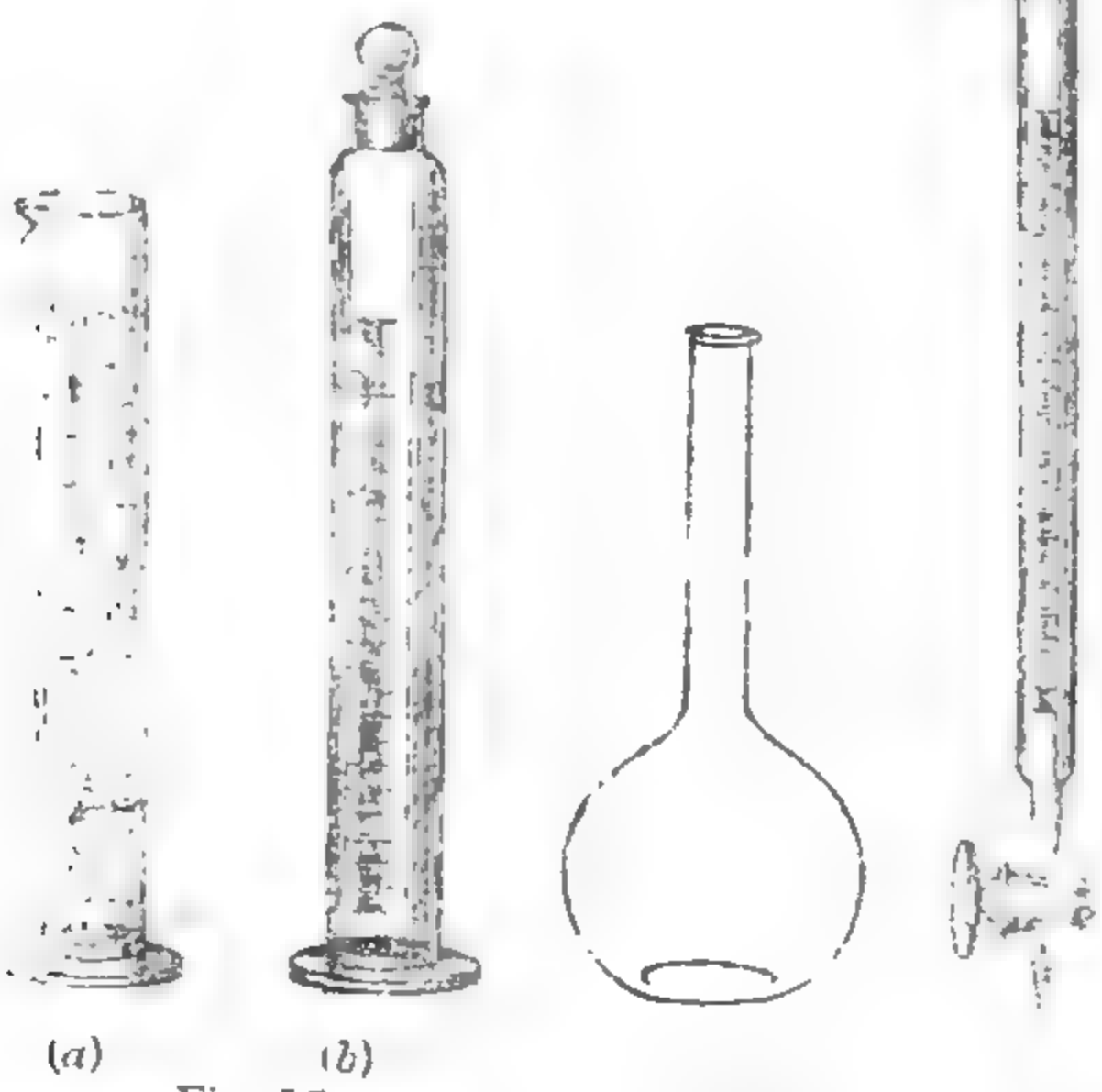
Knowing the area of the rectangle from its dimensions, the area of the figure can be calculated.

10. Measurement of Volume.—The volume of a *solid of a regular form* can be determined from the following relations :—

Volume of a	Parallelopied	= length × breadth × height
,,	,, Cube	= length^3
,,	,, Cylinder or Prism	= area of one end × length
,,	,, Sphere	= $\frac{4}{3}\pi$ × radius^3
,,	,, Pyramid or Cone	= $\frac{1}{3}$ × area of base × perpendicular height.

The volume of liquid can be measured with a graduated cylinder or flask. Fig. 15 (*a*) and (*b*) represent two types of cylinders graduated in cubic centimetres. Fig. 16 is a flask, the capacity of which is marked on its body. The liquid is poured into the cylinder and the upper surface of the liquid is read. This gives the volume of the liquid in units marked on the vessel. Fig. 17 is a burette and is used when a measured quantity of a liquid is to be poured into a vessel.

Volume of Irregular Solid.—The volume of small piece of solid, irregular or regular, can be determined as given below.



(*a*) (*b*)

Fig. 15 Fig. 16 Fig. 17

By Displacement of a Liquid.—A liquid which does not corrode or

dissolve the solid is taken in a graduated cylinder (Fig. 18) and the position of the liquid level is carefully noted. Suppose that original level is at A. The solid is then completely immersed in the liquid when the level of the liquid is found to rise. The position of the meniscus is noted again. Suppose that the new level is at B. The difference of the two readings gives the volume of the solid.

By Weighing—The volume of a solid can be determined by a method based on the principle of Archimedes. (Vide Chap. IX).

11. Measurement of Mass.—The *mass* of a body can be determined by weighing in a balance. Every body has seen a grocer's balance and how he uses it in weighing a substance. It consists of a straight uniform rod, called the *beam*, from the end of which two identical *pans* are suspended. The beam is pivoted at its middle point so that with empty pans, the beam remains horizontal when suspended. The substance to be weighed is placed on one pan and standard weights are placed by trial on the other pan till the beam becomes horizontal. Then the mass of the weights is equal to the mass of the substance. Such a balance is not sensitive to very light objects.

Common Balance.—The *common balance* is one of the contrivances for comparing the masses of different bodies. The masses which are used as standards of reference are called '*weights*'. The essential *parts of* a common balance are (Fig. 19) :—

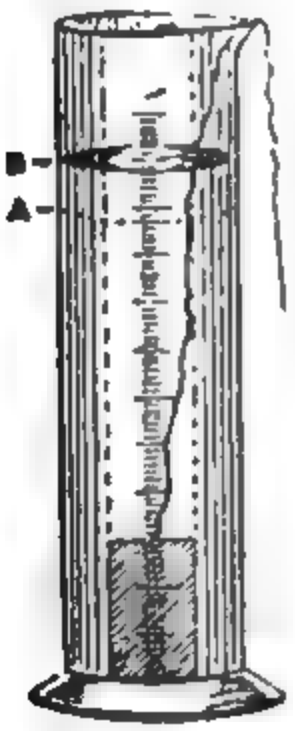


Fig. 18

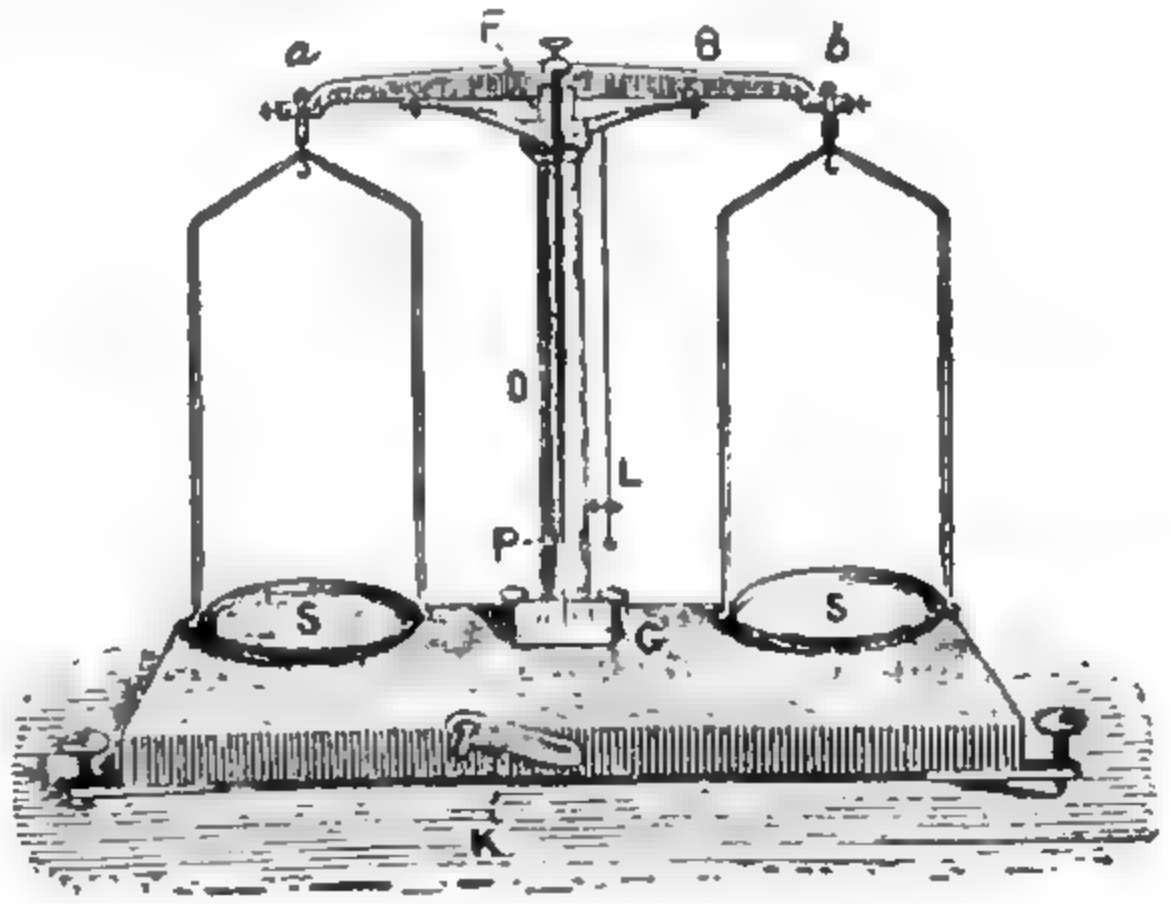


Fig. 19—Physical Balance

(1) The **beam** B.—It consists of a horizontal metal rod, capable of turning freely about a sharp steel or agate **knife-edge** F at the middle, called the *fulcrum*. The sharp end of the knife-edge rests on a small plate of steel or agate to minimise friction. At the ends of the beam two similar agate knife-edges are attached with its sharp edge upwards.

(2) The **stirrups** or pan supporters *a*, *b*.—These rest on the terminal knife-edges. In the better type of balances, these are provided with

agate pieces attached at the lower side of the stirrups. These are furnished with hooks from which the pans are hung. The distances from the fulcrum to the centre of gravity of the stirrups are called the *arms*, which should be equal.

(3) The **scale pans** SS, on which standard 'weights' and bodies to be weighed are placed.

(4) The **pillar** D, to support the beam when at rest.—

This is a vertical rod which can be raised or lowered by a **key** K at the base, when required. The pillar has an agate plate at the top upon which the central knife edge of the beam rests.

(5) The **pointer** P.—This is attached at its upper end to the middle part of the beam and the lower end can move freely over a graduated scale, G attached at the bottom of the pillar. When the beam is horizontal, the pointer should be vertical and its lower end should point the zero mark of the scale.

(6) **The arresting arrangement.**—When the balance is not in use, the pillar supporting the beam is lowered so that the beam now rests on another support, the under-surfaces of the pans just touching the base-board. Thus, the knife-edge at the centre of the beam does not always rest on the agate plate and so its sharpness is preserved.

At each end of the beam there is a screw, by working which the effective weight on each side can be altered through a small range. The base of the instrument is provided with levelling screws whereby the pillar is made vertical and so the beam becomes horizontal. The plumb line, L indicates the correct adjustment. *Weight Boxes* with standard 'weights' are supplied for weighing. A delicate balance for assay weighing has various other parts and is always kept within a glass case to prevent blast of air causing disturbances during an operation of weighing.

A body to be weighed is placed on the left pan and by trial various weights are placed on the right pan to counterpoise the body. When equilibrium is obtained the balance beam is horizontal and the pointer rests vertically downwards. The mass of the 'weights' placed on the pan is then equal to the mass of the body. [For further details regarding a balance and weighing, vide Chap. IV]

12. **Measurement of Time.**—The earliest method of reckoning the hours of the day was done by means of the **Sun dial.** It consists of a horizontal platform containing a dial on which graduations from I to XII are marked as in a clock (Fig. 20). A stick or a plate is fixed at a certain inclination to this dial. When the sun in course of a day moves through the sky, the shadow of the stick is thrown all round the stick in different positions. When the instrument is properly placed, the mark on which the shadow is cast at a particular time reckons the hour of the day. A sun dial can be used only during a sunny day.

The graduations on the sun-dial are in hours and time may be measured with its help to an accuracy of 5 minutes. The edge of the

shadow is just over the graduation 9 on the west side of the plate, as shown in the figure. This means that the time is 9 A.M. at the parti-

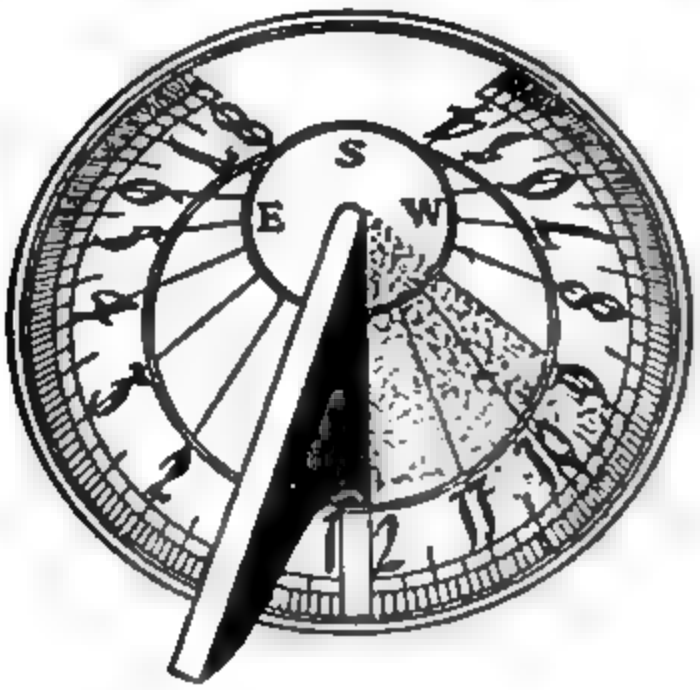


Fig. 20—Sun-dial

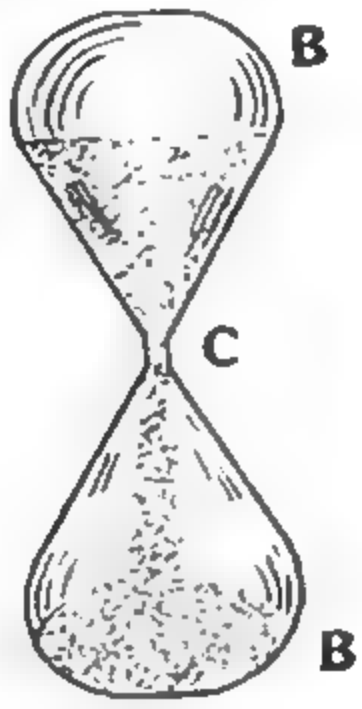


Fig. 21—Sand-glass

Fig. 22—Stop watch

cular instant. To measure smaller intervals clepsydra or *water clock* and *sand glasses* were constructed by ancient people.

Fig. 21 represents a sand-glass which consists of two similar conical glass bulbs B, B connected together by a narrow neck C. Some quantity of sand is put within the vessel so that the sand ordinarily occupies the lower bulb. On now suddenly inverting the vessel, the sand begins to trickle down through the neck and the whole of the sand is collected within the lower bulb in a definite interval. There are different sizes of sand glasses marking intervals of 2 minutes, 1 minute and 30 seconds.

In 1583, Galileo discovered that the time of oscillation of a pendulum of a given length is always constant (Chap. IV) and in 1656, Huygens first used the pendulum to regulate the motion of clocks. The measurement of time is now generally done by watches and clocks. The first use of the coiled hair-spring in watches is credited to Robert Hooke in 1660. A *chronometer* is a watch specially constructed so as to keep time with perfect regularity. **A stop clock** is a type of watch which can measure seconds accurately and which can be started or stopped whenever desired. A *stop watch*, constructed on the same principle, is a more precision instrument (Fig. 22) capable of measuring one-fifth or one-tenth part of a second. The **metronome** is a mechanism, run by clock-work and is used to mark time at regular intervals. (Vide J. Chatterjee's Intermediate Practical Physics).

A reed or thin metal strip can vibrate a large number of times per second and that the period of a single vibration is quite uniform. The vibration of a reed can be utilised to measure an interval as small as one-hundredth or one-thousand part of a second. In recent times scientists have found that even one-thousand part of a second is quite a large interval in comparison to the quickness of action of a few electrical apparatus such as a radio, radar etc. To keep pace with such appara-

tus, electrical instruments have been devised which can measure one-millionth part of a second with a sufficient accuracy. (vide part on Electricity).

13. Units of Angle.—The idea of an angle is obtained when one straight line rotates by some amount about a point on it. Let a straight line AB rotate about the point O on the plane of the paper and occupy the position CD. Then any two points P and X on the straight line AB would trace out two circular arcs PQ and XY about the common centre O. If the lengths of the arcs PQ and XY as well as the distances OP and OX are actually measured, then it would be found that,

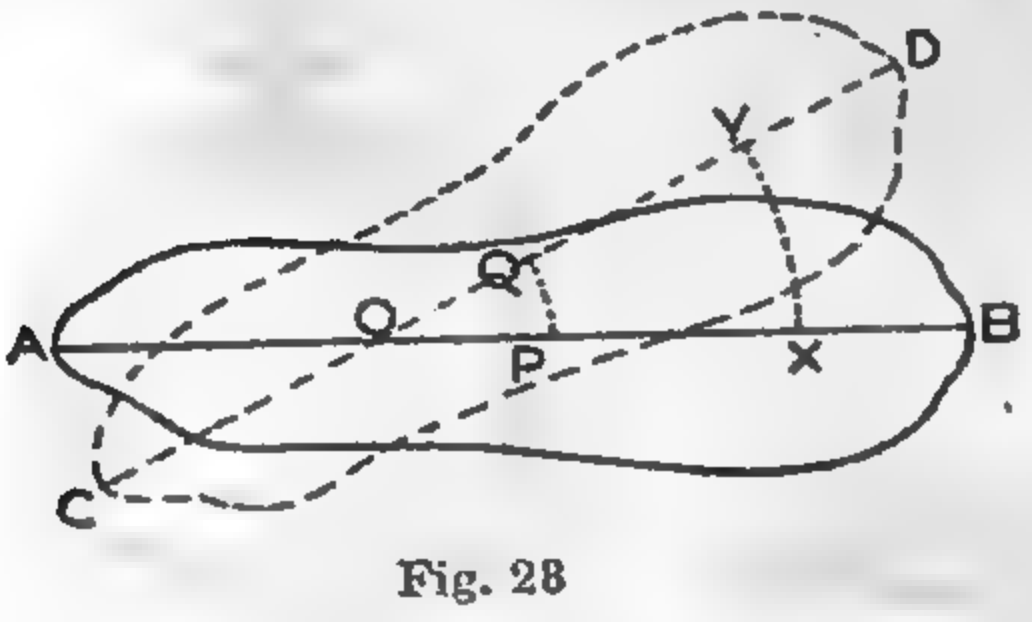


Fig. 28

$$\frac{\text{arc PQ}}{\text{PO}} = \frac{\text{arc XY}}{\text{XO}}$$

This shows that for a given rotation of the straight line, any arc traced by a point on it divided by the distance of that point from the centre of rotation is constant. Therefore the ratio of the arc to the corresponding distance from the centre is a measure of the angle. This system of measuring an angle is called a **circular measure.** According to this system the unit of angle is called a **radian,** which is defined to be the angle subtended at the centre by a circular arc of length equal to the radius. To express an angle in terms of radian, a circular arc is drawn with the apex of the angle as centre and with any radius. The angle *in radian* is given by the quantity,

$$\frac{\text{length of the arc}}{\text{length of the radius}}$$

Lengths of the circumferences and diameters of various circles have been accurately measured and it has been found that in each case the ratio of the length of the circumference to the corresponding diameter of the circle is a constant, having a value of 3·1415 correct to fourth place of decimals. This constant is popularly designated by the Greek letter π pronounced as *pie*,

$$\text{Thus, } \frac{\text{circumference of a circle}}{\text{diameter of the circle}} = \pi = 3{\cdot}1415\ldots$$

$$\text{or, } \frac{\text{circumference}}{\text{radius}} = 2\pi \quad \therefore \text{ circumference} = 2\pi \times \text{radius.}$$

When a straight line makes one *complete rotation* about a point on it on the plane of the paper, any other point on the straight line moves over a complete circle with the fixed point as centre. The number of

radians for one complete rotation of the straight line must then be equal to,

$$\frac{\text{length of the circumference}}{\text{length of the radius}} = 2\pi = 6{\cdot}2830\ldots$$

Another unit of angle is defined by dividing the circumference of a circle into 360 equal arcs ; the angle subtended at the centre by any arc is called a **degree.** Therefore, the angle subtended at the centre by a semi-circle contains 180 degrees and that by ane quarter of a circle, called a *right angle*, contains 90 degree. The multiple and submultiple of a degree are the following.

1 Right Angle = 90 degrees (90°)
1 degree (1°) = 60 minutes (60′)
1 minute (1′) = 60 seconds (60″)

Since the circumference of a circle subtends an angle of 2π radians at its centre and also 360°,

$$2\pi \text{ radians} = 2\pi^c = 360^\circ$$

$$\therefore\ 1 \text{ radian} = 1^c = \frac{360^\circ}{2\pi} = \frac{360^\circ}{6{\cdot}2830} = 57^\circ 17' 44{\cdot}8''$$

14. Measurement of Angle.—The apparatus, most commonly employed to measure an angle is called a protractor. It consists of a semicircular piece of metal, divided on its circumference into 180 equal parts, each called a degree (Fig. 24). The scale is graduated in both directions. The lower straight edge of the protractor forms the diameter of the semicircle having the centre at O. Protractors supplied in ordinary mathematical instrument boxes are graduated with divisions of a degree, but there are more precision types of protractors having $\frac{1}{2}$ or $\frac{1}{3}$ degree as the smallest graduation.

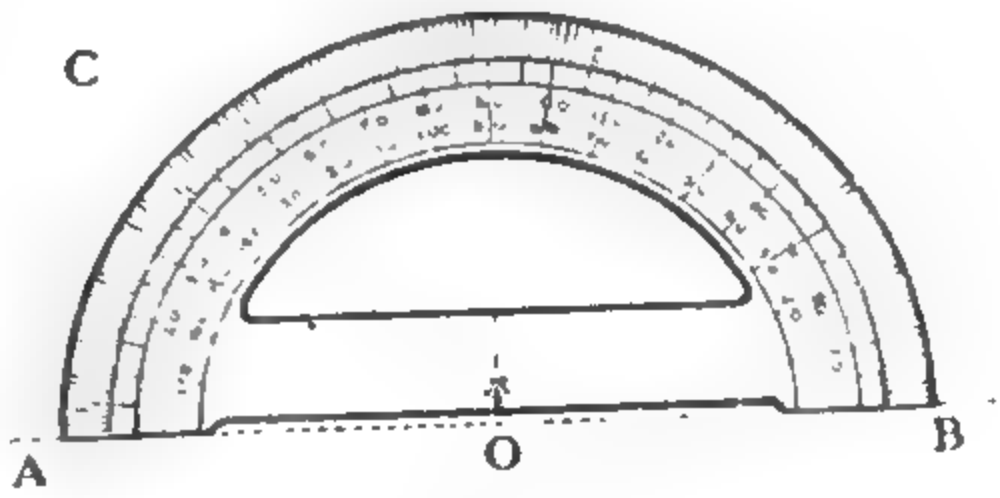


Fig. 24

In order to measure the angle between two straight lines OA and OC, place the protractor with its straight edge just to touch the line OA and the angular point to coincide with O. Now read the circular scale just at the point through which OC emerges out. This gives the angle AOC in degrees. If OA is placed to the right of O, read the inner scale which starts from the right, but if the base line of the angle is to the left of O, read the outer scale. In the particular figure the ∠ AOC is 45°

Where greater accuracy is needed to measure an angle, an angular vernier is used in conjunction with a degree scale. The apparatus consists of a degree scale OO with its centre at B. Just along the side of this scale, another anxiliary vernier scale A slides about the common centre. This vernier scale is graduated in such a way that a certain number n divisions of this scale are equal to $n-1$ divisions of the main scale. The vernier scale is then capable of measuring $1/n$ part of the main scale. If the main scale is graduated in degree, then this angular vernier can read upto $1/n$ of a degree (For details vide J. Chatterjee's Intermediate Practical Physics).

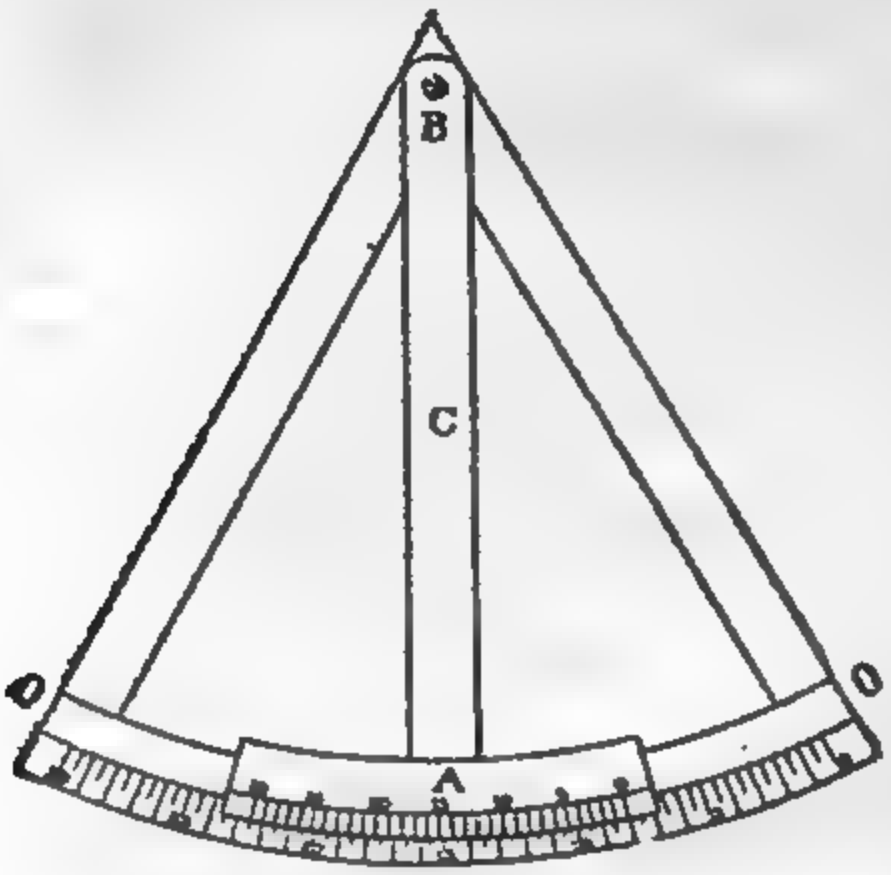


Fig. 25

15. Dimensions of Physical Quantities.—It has been stated (Art. 4), that there are three fundamental units, independent of each other. These are unit of length, unit of mass and the unit of time, conventionally abbreviated by letters L, M and T, Other derived units bear some relations with the fundamental units. Such relations are called the *dimensions* of those derived units. Since all physical quantities are expressed in terms of derived units, they have also certain dimensions.

When we say that a rod is 3 feet long, we refer to the dimension of length and express it in F. P. S. system as a length of 3L (F.P.S.). Again if we say that a body has a mass of 15 grams, we refer to the dimension of mass, which is equivalent to 15 M (C.G.S.). Similarly 20T means 20 seconds on both the systems.

Units of area are in any system obtained by the product of length and breath. Since both length and breath have the dimension of length, area must have a dimension of $L \times L$ or L^2. Similarly, a volume has a dimension of L^3. Thus $35L^3$ (C.G.S.) means 35 cubic centimetres. An angle in circular measure is expressed by the ratio of arc to length and radius. Both being units of length, one radian has the dimension of $L/L = L^0$. Thus an angle has no dimension. As we, in subsequent Chapters, would come across various derived units, we should establish various relations which L, M and T bear with such units. It is to be noted that in an equation between two quantities, the dimensions of the various terms in the equation must be the same. This serves two purposes ; (i) It indicates whether an equation is properly stated or not. (ii) It furnishes a means of finding the dimension of a physical quantity whose relation to the fundamental quantity is not clearly known.

[Students are required to remember the following conversion factors to work out problems dealing with reduction of units.]

British into Metric Conversion Factors.

British		Metric	British		Metric
1 inch	=	2·54 cm.	1 cu. ft.	=	28·3153 litres
1 foot	=	30·479 cm.	1 gallon	=	4·546 litres
1 yard	=	·9144 metre	1 pound (av.)	=	453·593 gm.
1 mile	=	1·6093 km.	1 grain	=	·0648 gm.
1 sq. in.	=	6·451 sq. cm.	1 ton	=	1016 kgm.
1 cu. in	=	16·386 c. c.	1 Indian Tola	=	12 gm.

ARISTOTLE (384—322 B. C.)

There is no record of any man, who made a systematic study of the material world from a scientific stand-point, before the time of Aristotle. He was born in 384 B. C. at Stageira in Macedonia. His father was a physician and a medical adviser to both the grandfather and father of Alexander, the Great. Aristotle, in his eighteenth year, came to Athens as a student and was intimately connected with Plato, the greatest exponent of spiritual philosophy of that time. Socrates the greatest comprehensive thinker of the pre-Christian era, taught the world that the inner life of a man is the thing that matters to him most. Plato interpreted this inner life as the soul that comes from God and is itself divine while Aristotle interpreted it as the reason of a man which might find plenty of knowledge by digging the vast material store-house of the world.

Aristotle was engaged for three years as a private tutor of Alexander the Great in his boyhood. When Alexander set out on his career of conquest, Aristotle returned to Athens and for the next twelve years he spent in establishing a school called the Lyceum. Here he developed his philosophy of arguments and reasoning until practically the whole of the then known knowledge of the world began to systematise itself. After the death of Alexander, there was a conspiracy against the life of Aristotle, who retired to Chalcis in 322 B. C. His health broke down seriously and he died the same year. He set to work out earliest theories of light and gravitation. He also classified animals of the earth and many of the plants and thus laid the foundations of physical sciences for all time to come.

Summary

Fundamental and Derived Units.—There are three *fundamental units* of **length, mass and time.** All other units may be expressed in terms of these units and are therefore termed *derived units.*

System of Units.—(1) The F. P. S. system (2) The C. G. S. system. The units of length and mass in the former system are the **foot** (ft.) and the **pound** (lb.) and in the latter are the **centimetre** (cm.) and the **gram** (gm.) ; the unit of time in both the systems is the **second.**

A length is measured under different circumstances by (*i*) the metre and diagonal scales (*ii*) the measuring tape, (*iii*) the simple callipers, (*iv*) the slide callipers, (*v*) the micrometer screw-gauge and (*vi*) the spherometer.

A mass of a body is measured with a physical *balance.*

Time is measured by *clocks* and *watches* of various types.

Angle is measured in two different units in *radians* and in *degrees.*

Exercises on Chapter I

Reference

Art. 4

1. Name the systems of units commonly used for the measurement of Physical quantities. Which of them is more convenient and why ?

Explain with examples what you mean by fundamental units, derived units and practical units.

Arts. 5 & 6

2. Define mass, volume and density; state the relation that exists between them. [C. U.—1920 ; Dac. U.—1939]

Art. 7

3. What do you mean by an *apparent solar day* and a *mean solar day* ?

Art. 8

4. Give the principle of a vernier and explain its working. Each division of a main scale is 0·5 mm. 9 divisions of the main scale are equal to 10 divisions of the vernier. Length of a cylinder is measured. The readings are : 78 divisions of the main scale ; and the 6th division of the vernier coincides with a division of the main scale. Calculate the length of the cylinder.

A. 3·93 cm. [Del. H. S.—1953]

Art. 8

5. Derive the formula, $R = \frac{a^2}{2h} + \frac{h}{2}$ for a spherometer. The

ets of a spherometer occupy the corners of an equilateral triangle of side 4 cm. When placed on a plane surface, the reading is 4 mm. and on a concave surface, it is 2 mm. Calculate the radius of curvature of the surface. [Anna. U.—1951]

A. 13·4 cm.

Art. 8

6. Describe a spherometer and explain carefully its use for measuring the radius of curvature of a spherical surface

[U. P. B.—1948, '53]

Art. 10

7. How will you find the volume of a solid of irregular shape ?
[C. U.—1917, '29 ; Dac. U.—1932]

Art. 15

8. What is meant by the dimension of a physical quantity ? What is its utility ? What are the dimensions of density and velocity ?

CHAPTER II

MOTION

16. Position and Reference Co-ordinates.—In our every day life we describe the position of any object or a locality by its direction and distance from some other object of reference. For this purpose we choose four standard directions as east, west, north and south. The

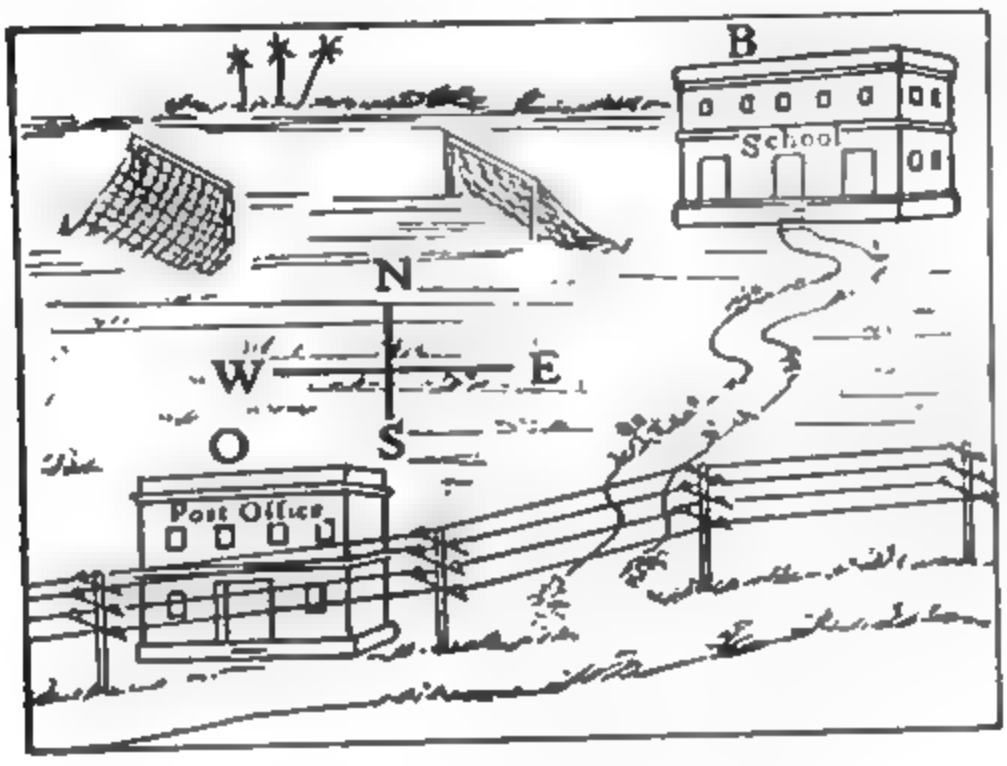


Fig. 26—Cardinal Directions

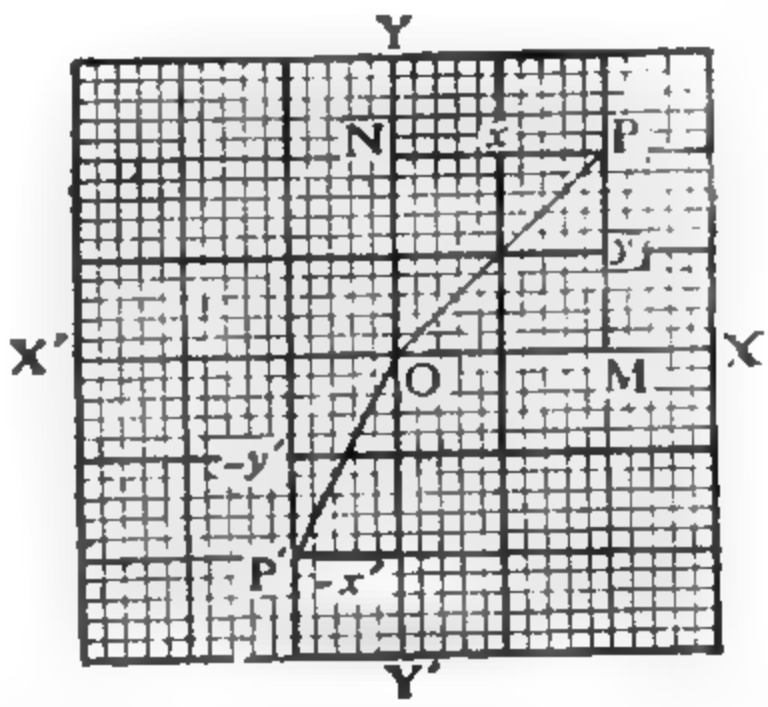


Fig. 27.—Rectangular Co-ordinates

object of reference, may be anything apparently fixed. When we say that the school building B is situated at a distance of half-a-mile to the north-east of the Post-Office O, (Fig. 26), the Post-Office stands for the object of reference, north-east gives the direction and half-a-mile is the distance. Thus we specify the position B of the school building by its distance OB, which is half-a-mile in this case, and also by the north-easterly direction, with reference to the post office. If we know the distance OB and the north-easterly direction, we can easily determine the position of the school building. The position of a point is therefore measured by its distance in a definite direction at any instant from some apparently fixed point in space.

Determination of position of a point on a graph paper is done in a similar manner. Two straight lines are taken at right angles on the graph paper, which we call reference lines XOX′ and YOY′ like the cardinal directions (Fig. 27). The intersection of the two lines is the *origin* O of the co-ordinates. To find the co-ordinates of any point P on the graph paper, the distances of this point from the two lines are measured in terms of the units chosen along abscissa OX and ordinate OY. These co-ordinates, which are, x and y say, determine the distance of the point from the reference origin O. On measuring x and y, the *actual distance* OP *of the point* from the origin is known, which is $\sqrt{x^2 + y^2}$; further we can also measure the angle POX or POY and determine *in which direction the point* P *lies with respect to the axes.*

17. Rest and Motion.—A body is said to be at *rest*, when it does not change its position with respect to some point fixed in space. It is said to be in *motion*. when it changes its position with time. So *motion is a change of position with time* with respect to some reference point. *Absolute rest* is the complete absence of motion. This is beyond our idea, since the earth and all other planets are continually revolving about their respective axes and also each moves round the sun in its own orbit ; while the sun itself with all the planets is moving through space among the stars. The stars are found to move with respect to one another. *Absolute motion* of a body is its motion with respect to a point absolutely fixed in space. But as no such fixed point is known to us, absolute motion is not practically available.

A body may then be in a state of *relative rest* or *relative motion*. A body is said to be at **rest relative to another body, when it does not change its position with respect to that body ; and it is said to be in relative motion with respect to a certain object, when it changes its position with respect to that object Thus, a man seated on the bench of a running train, is at rest *relative* to the bench or the window of the train, although he is in relative motion with respect to a mile post near the rail-road. Again, if two persons are cycling side by side at the same rate and in the same direction, they appear to be at rest with respect to each other, although each is in motion with respect to a man standing near by. We usually refer the motion of all terrestrial bodies with respect to anything fixed on the earth.**

Nature of Motion.—The motion of a body may be either **translatory** or **rotatory** or both. Translatory motion may again be subdivided into *rectilinear* and *curvilinear* motions.

A body is said to possess a *translatory motion* when a line joining any two points on the body remains always parallel for all positions of the body. In Fig. 28, a body is shown to move in a direction shown by the arrow head. Any two points, say *a*, and *c* are taken on the body at its initial position. In the displaced position the corresponding points are *x*, and *z*. If the straight line joining

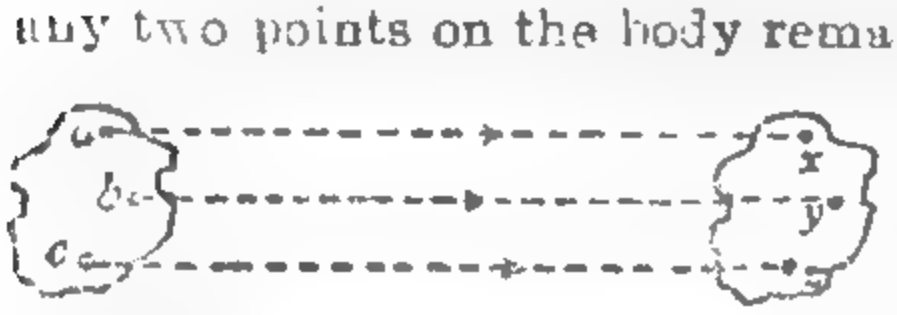


Fig. 28—Rectilinear Motion

point *a* and *c* is parallel to the straight line joining *x* and *z* in the displaced position of the body and if the path of motion of any one point of the body be straight, the type of motion is called a *rectilinear* motion. A stone falling down from a height or a train running on a straight track is an example of such motion.

When the motion of the body takes place over a curved path in such a way that any line drawn on the body remains always parallel to its original direction, it is called a *curvilinear* motion. In Fig. 29, a body is moving in the direction of the arrow head. The path from E to F is rectilinear but the path FGHI is on a curve. The vertical line MP remains always vertical irrespective of its position along the

direction of motion. The motion of the arm chairs of the vertical merry-go-round in fun fairs, is an example of a curvilinear motion.

A body is said to have a *rotatory motion*, when any straight line taken on it moves about a fixed point or axis. Fig. 30 represents a solid wheel rotating about its axis AA. If we consider a point B, on the

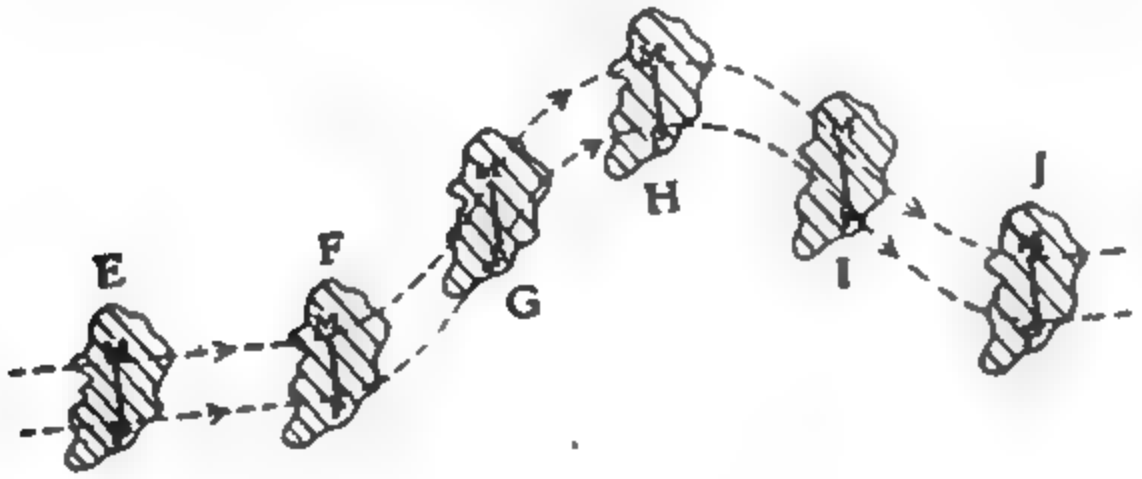


Fig. 29—Curvilinear Motion

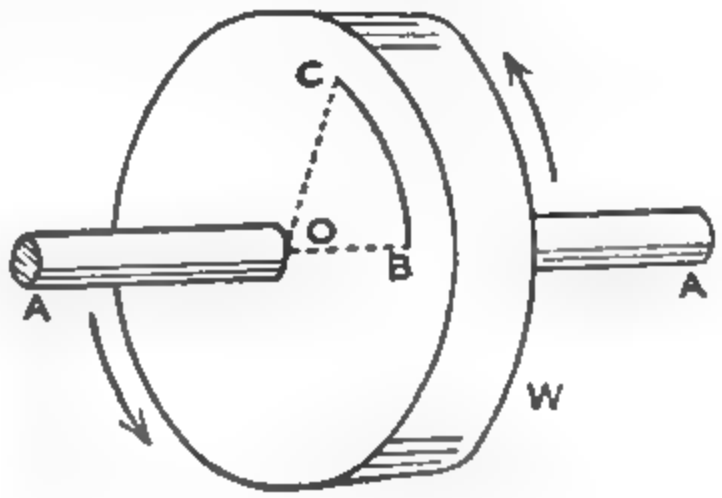


Fig. 30—Rotatory Motion

wheel at any instant, it will be found to occupy a position C at a later instant. By joining the successive positions of the point in between B and C, we get an arc of a circle subtending an angle BOC at the centre of rotation. The same angle is subtended at the centre by an arc made by any point on the radial line OB in a definite interval of time. Therefore in a rotatory motion, a line joining any two points on the body in the displaced state would make an angle to the same line in the initial state of the body.

The motion of a body may often be complex in nature being a combination of a rotation and translation ; thus, the motion of a cart wheel or a ball rolling on the ground or a planet round the sun, comprises both kinds of motion.

18. Terms relating to Motion.—The **displacement** of a moving particle is its change of position in a given time and so it is the distance between its initial and final positions. The displacement of the point P from the origin O (Fig. 27) is the distance OP. Displacement has thus a *direction* as well as a *magnitude*.

The **speed** of a body is the rate of change of its position and is measured by the distance passed over by the body in unit time along a straight or a curved path. The speed does not refer to any direction.

The **velocity** of a moving body is its rate of change of position in a *definite direction*. Since velocity has got some direction and magnitude, it may be represented by a straight line. Evidently speed in a definite direction is velocity. The *velocity* of a moving body *at any instant* is measured by the distance traversed during a small interval containing that instant divided by that interval. Speed and velocity may be *uniform* or *variable*.

Uniform velocity.—The velocity of moving body is said to be *uniform*, when it moves in a definite direction over equal distances in equal intervals of times, however small these intervals may be. When

uniform, the velocity is measured by the distance traversed in a unit of time.

Fig. 31 illustrates a uniform velocity of a body ; *a*, *b*, *c* and *d* represent its various positions at intervals of one minute. Its rate of displacement is found to be 120 ft. for every minute. Therefore its velocity is 2 ft. per second. Thus, is a body, moving with a uniform velocity v, passes over s ft. in t seconds, then $v = s/t$ ft. per sec.

When a body does not move equal distances in equal intervals of time, it possesses a *variable* velocity. In this case velocity may be measured at any given instant or an average velocity may be found

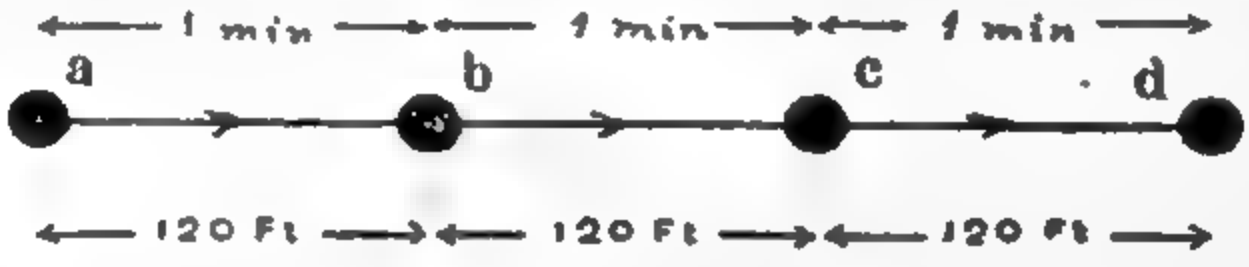


Fig. 31—Uniform velocity

during a given interval. The **average** or **mean velocity** of a moving body during any given interval is measured by the total distance passed over by the body divided by that interval.

The distance may be expressed in any unit of length such as miles, yards, feet, metres, centimetres etc., and the corresponding time may also be expressed in any unit such as hours, minutes, seconds etc. For example, if a body covers l miles in h hours, we may denote its average velocity as l/h miles per hour or even as $\frac{l \times 1760}{h \times 60}$ yards per minute etc.

But since the unit of length is accepted as a foot or centimetre and the unit of time a second, it is customary to express the velocity as so many feet per second or so many centimetres per second. Thus a velocity of 30 miles an hour is equivalent to a velocity of 44 ft. per second.

The *unit of velocity* may be defined as the velocity of a moving body which traverses unit length in a given direction in unit time. Hence in the F. P. S. system, the unit of velocity may be taken as *one foot per second* and in the C. G. S. system, it is *one centimetre per second* sometimes abbreviated as *cm. sec.*$^{-1}$.

Acceleration.— The acceleration of a body is the *rate of change* of its velocity *i.e.*, the change of velocity per unit time. As a velocity has both magnitude and direction, the change may take place either in both or in one only. In the simplest case, the magnitude may change, the direction remaining the same. Acceleration has thus direction as well as magnitude. It can therefore be represented by a straight line.

For example, let a body with continuously increasing *velocity* pass over a distance of 10 ft. in the first second (Fig. 32), a and b representing the positions of the body at the beginning and end of the interval. Then its average velocity during the first interval is 10 ft. per sec. If again it passes over a distance of 15 ft. during the second interval,

its average velocity is 15 ft./sec. Thus the average velocity has been increased by 5 ft. per second. If this increase of velocity is maintained constant, it would pass over distances of 20 ft. in the third second. 25 ft. in the fourth second and so on ; so the rate of change is 5 ft.

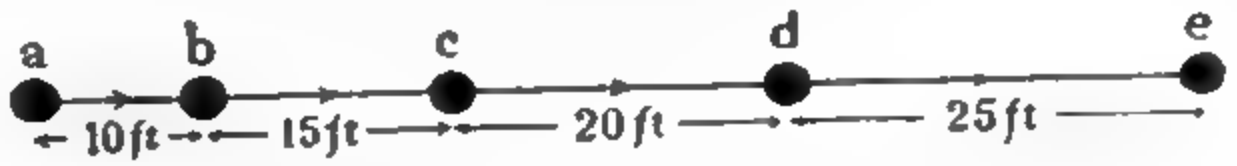


Fig. 32—Accelerated Motion

per sec. of average initial velocity of 10 ft. per sec. Since the term 'per sec.' occurs twice in stating an acceleration, for the sake of brevity it is written as 5 ft. per sec^2. or 5 ft./sec.^{2}. Here the acceleration is *uniform*. It can be proved that the body possesses an initial velocity at the position 'a' of 7·5 ft. per second and at 'b' of 12·5 ft. per sec., so that the average velocity between a and b is 10 ft. sec.

Thus, we find that for a body moving with a uniform acceleration, the velocities at the ends of successive seconds increase and are therefore in arithmetical progression.

In general, if u and v are the velocities at the beginning and at the end of an interval t, the acceleration f is given by

$$f = \frac{v - u}{t} \qquad \ldots \quad (18.1)$$

If the rate of increase or decrease of velocity changes with time, the acceleration is said to be variable. The acceleration may also be expressed in any unit depending upon the units of velocity and time.

A body is said to have a *unit acceleration*, when its velocity changes by unity in unit time. In the F. P. S. system, the unit of acceleration is *one foot per sec. per sec.* and in the C. G. S. system, it is *one cm. per sec. per sec.* sometimes abbreviated as *cm. sec.*$^{-2}$.

Retardation.—When the velocity of a moving body decreases, the motion is said to be *retarded*. Mathematically speaking, *retar lation* is considered as a *negative acceleration* and is expressed by putting a negative sign before the change in velocity taking place per unit of time.

19. Measurement of Velocity.—In order to measure the velocity of a body, we are to find either how far the body travels in a known interval or what time the body requires to pass over a known distance according as the experimental condition favours. Suppose that we are to measure the speed of a railway train in which we are placed and we are provided with a watch capable of measuring seconds. When our compartment just passes by a mile post we record the time in our watch and wait till the compartment passes by the next mile post. The difference of the two readings in the watch measures the interval required for the train to pass over a distance of one mile. The distance divided by time gives to mean speed of the train over a distance of one mile. We can also find the speed by spotting a mark by the side of the railway track at any instant of time by the watch. Then after a definite

interval, say, 10 minutes, we spot another mark. The distance between the two marks is then measured, which gives the distance travelled in the known interval whence the average speed may be found.

Measurement of Acceleration.—It is now evident that when a body moves with an accelerated or retarded speed, the distances passed over by the body in successive intervals of time either continuously increase or decrease. Hence to measure an acceleration, the distances passed over by the body at equal intervals are to be measured. Take the case of a railway train when it starts off from a station. The positions of the train are marked at intervals of say half a minute. It will be found that the distance moved through during the first interval is less than that during the second interval. The difference of these two distances gives the increase of velocity during half a minute. Hence acceleration is calculated. In actual practice observation is continued over a number of intervals and the average value of the acceleration is found.

20. Equations of Motion.—In the case of a body moving in a straight line, the relations between the displacement, time, velocity and acceleration can be represented by some simple equations. These equations were first worked out by Galileo.

Let initial velocity of the body moving in a straight line $= u$
constant acceleration in the direction of motion $= f$
final velocity acquired after a given interval t $= v$
space described during the interval $= s$

(*i*) *Motion with uniform velocity*—If the motion is uniform, which means that the body possesses no acceleration, then the body passes over u units of length in every unit of time and the total space described in an interval t is given by

$$s = u.t \qquad (20,1)$$

(*ii*) *Motion with uiform acceleration*—Let the initial velocity of the body be u. As it is subjected to a uniform acceleration f, the velocity of the body is increased in *each unit of time* by f and the total increase in velocity in time t will be $f \times t$. Hence the final velocity v acquired by the body after the interval t, is given by

$$v = u + ft \qquad (20,2)$$

Whence $$f = \frac{v - u}{t}$$

Acceleration = increase in velocity ÷ time.

If the body possesses a negative acceleration, f will be a negative and the expression would become,

$$v = u - ft$$

(*iii*) *To find the space described with a uniform acceleration*—

Velocity at the end of the 1st second $= u + f$
„ at 1 second earlier than the last second
i.e. at $(t-1)$th second $= v - f$

$\therefore$ Average velocity in each of these two seconds

$$= \frac{(u+f)+(v-f)}{2} = \frac{u+v}{2}$$

Velocity after 2 sec. from the beginning $= u + 2f$

Velocity at 2 sec. earlier than the last sec. $= v - 2f$

Average velocity during each of these two seconds

$$= \frac{(u+2f)+(v-2f)}{2} = \frac{u+v}{2}$$

Thus, taking intervals in pairs equidistant from the beginning and the end, the average velocity in each second is found to be *half* of the sum of u and v. As the acceleration is uniform, the body may be supposed to be moving with this average velocity during the whole period t. The space described by the body is then given by,—

$$s = \frac{u+v}{2} \times t = \frac{u+u+ft}{2} \times t$$

$$s = ut + \tfrac{1}{2}ft^2 \quad \ldots \quad (20,3)$$

or

If the body has a retardation f, eqn. (20,3) takes the form,

$$s = ut - \tfrac{1}{2}ft^2$$

From (20,2), $v^2 = (u+ft)^2 = u^2 + 2uft + f^2t^2$

$$= u^2 + 2f(ut + \tfrac{1}{2}ft^2)$$

$\therefore$ from (19,3), $v^2 = u^2 + 2fs \quad \ldots \quad (20,4)$

For a retardation f, eqn. (20,4) takes the form $v^2 = u^2 + 2fs$.

(*iv*) *Space described in any particular second.*—Let the space described in the t th. second be denoted by s_t.

Then s_t = space passed in t sec. − space passed in $(t-1)$ sec.

$$= (ut + \tfrac{1}{2}ft^2) - \{u(t-1) + \tfrac{1}{2}f(t-1)^2\}$$

whence $$s_t = u + \frac{2t-1}{2} \times f \quad \ldots \quad (20,5)$$

Special cases.—

(*i*) *If the body starts from rest,* $u = 0$.

Then, $u = ft$; $s = \tfrac{1}{2}ft^2$; $v^2 = 2fs$; $s_t = \frac{2t-1}{2}f$

(*ii*) *If the body has a retardation* f (*i.e.* acceleration $= -f$)

then $v = u - ft$; $s = ut - \tfrac{1}{2}ft^2$; $v^2 = u^2 - 2fs$; $s_t = u - \frac{2t-1}{2}f$

Examples :—

1. A body starts with a velocity of 45 miles per hour and moves under a constant retardation of 11 ft. per sec^2. Find (*i*) its velocity after 2 sec. (*ii*) after what time it will stop, and (*iii*) what distance it moves before coming to rest?

A. Here $u = 45$ miles per hour-$= 66$ ft. per second.
$f = -11$ ft. per sec^2.
(*i*) From $v = u + ft$. $v = 66 - 11 \times 2 = 44$ ft. per second.
(*ii*) Let it stop after t seconds. Here $0 = 66 - 11 \times t$ $\therefore$ $t = 6$ seconds.
(*iii*) Let the body move through s ft.; then $s = 66 \times 6 - \frac{1}{2} \times 11 \times 36 = 198$ ft.

2. A body moving with uniform acceleration and with an initial velocity of 14 ft. per sec. passes over 8 ft. and then its velocity is 18 ft. sec. What is the acceleration ? (Anna. U.—1950)

A. Let f = acceleration of the body. Its initial velocity is 14 ft./sec. and after passing over 8 ft. its final velocity is 18 ft. sec. Then according to the eqn. $v^2 = u^2 + 2fs$,

$18^2 = 14^2 + 2f \times 8$ whence $f = 8$ ft./sec^2.

3. A bullet moving at the rate of 200 ft. per second is fired into the trunk of a tree into which it penetrates 9 inches. If the bullet moving with the same velocity were fired into a similar piece of wood 5 inches thick, with what velocity would it emerge, supposing the resistance to be uniform ? (Raj. U.—1953)

A. The striking velocity of the bullet is its intial velocity and its final velocity being zero after moving through a distance of 9 in. $= \frac{3}{4}$ ft. through the trunk. Hence the retardation f of the bullet is obtained from the relation, $v^2 = u^2 - 2fs$.

$$\therefore \quad 0^2 = 200^2 - 2f \times \tfrac{3}{4} \quad \text{whence} \quad f = \frac{8 \times 10^4}{3} \text{ ft.sec}^2.$$

Retardation being same in a similar piece of wood 5 in $= 1\frac{5}{12}$ ft. thick, for its final velocity a similar equation will hold. Thus,

$$v^2 = 200^2 - 2 \times \frac{8 \times 10^4}{3} \times 1\tfrac{5}{12} \quad \text{whence} \quad v = 133 \text{ ft./sec. approximately.}$$

4. A body, starting from rest, travels 150 ft. in 8th second. Calculate the acceleration assuming it to be uniform. (P. U.—1951)

A. Space moved over in 8th second = Space moved over in 8 seconds — that in 7 seconds. Now space covered is 8 seconds, assuming f to be acceleration, initial velocity being zero, is given by,

$$s_1 = 0 + \tfrac{1}{2} f \times 8^2 = 32f$$

Similarly space covered in 7 sec. is given by,

$$s_2 = 0 + \tfrac{1}{2} f \times 7^2 = \tfrac{49}{2} f$$

But $s_1 - s_2 = 150$ ft. $= 32f - \frac{49}{2} f$ whence $f = 20$ ft. per sec^2.

5. A body starts from rest and travels for 6 seconds with a uniform acceleration of 4 ft. per sec^2. It then continues to move for 16 seconds with the unform velocity acquired. A uniform retardation of 2 ft. sec.^{2} is then put to the body which finally comes to rest. Find the total distance the body has travelled before it comes to rest.

A. The space described in the first 6 seconds $= \frac{1}{2} \times 4 \times 36 = 72$ ft.
The velocity acquired at the end of this period $= 4 \times 6 = 24$ ft. per sec.
The space described in the next 16 seconds $= 24 \times 16 = 384$ ft.
Let the body before it comes to rest travel through s ft.
Then from $v^2 = u^2 + 2fs$, $0 = 24^2 - 2 \times 2s$ whence $s = 144$ ft.
$\therefore$ The total space described $= 72 + 384 + 144 = 600$ ft.

21. Vector and Scalar Quantities.—Any physical quantity which has got a *magnitude* only and is subject to ordinary arithmetical laws, is called a **scalar quantity**. Thus, if a man, works for 5 hours in the morning and again for 4 hours in the afternoon, he has worked for 9 hours during the day ; the total period is obtained by the arithme-

tical sum, showing that *time is a scalar quantity*. For a similar reason *mass is a scalar quantity*.

Any physical quantity, which has a directional property in addition to its magnitude, is called a **vector quantity**; so, for the specification of a vector, its direction must be mentioned along with its magnitude. Thus, when we say that a man has moved 20 yards from a certain point, he may be anywhere on the circumference of a circle of radius 20 yds. about the initial position. But if it is stated that the man has moved 20 yds. due *east*, his position can at once be determined. Hence, to state a *displacement* completely, both its *magnitude* and *direction* must be mentioned or in other words *displacement is a vector quantity*. Similarly density, volume, etc. are scalars while velocity, force, etc. are vectors.

Any vector quantity can be represented by a straight line, whose length represents the magnitude and whose direction gives the direction of the vector. A single vector whose effect is the same as the combined effect of two or more vectors is called the **resultant** of these vectors. The process of finding the resultant is called a *vector addition* or the *composition of vectors*.

22. Graphical Representation of Motion.—A motion may be represented graphically by showing the variation of displacement of the body with time. Let OX and OY be two rectangular axes

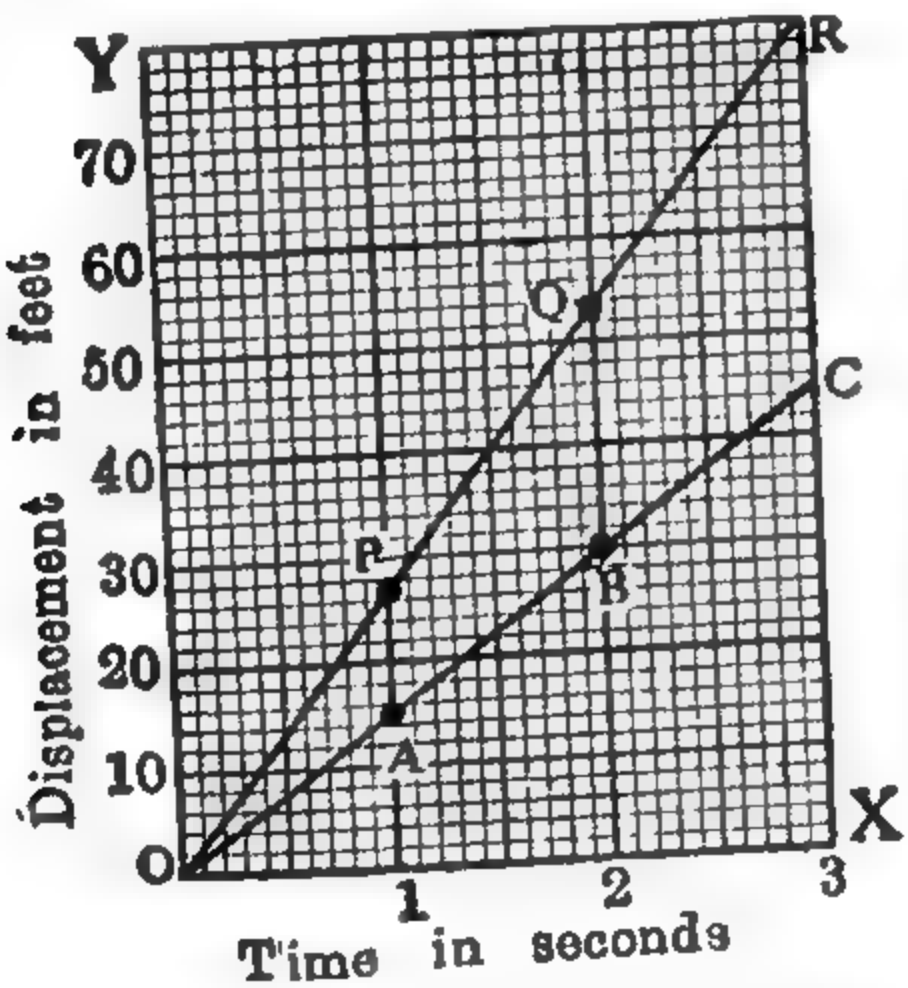


Fig. 33—Graph of Uniform Velocity

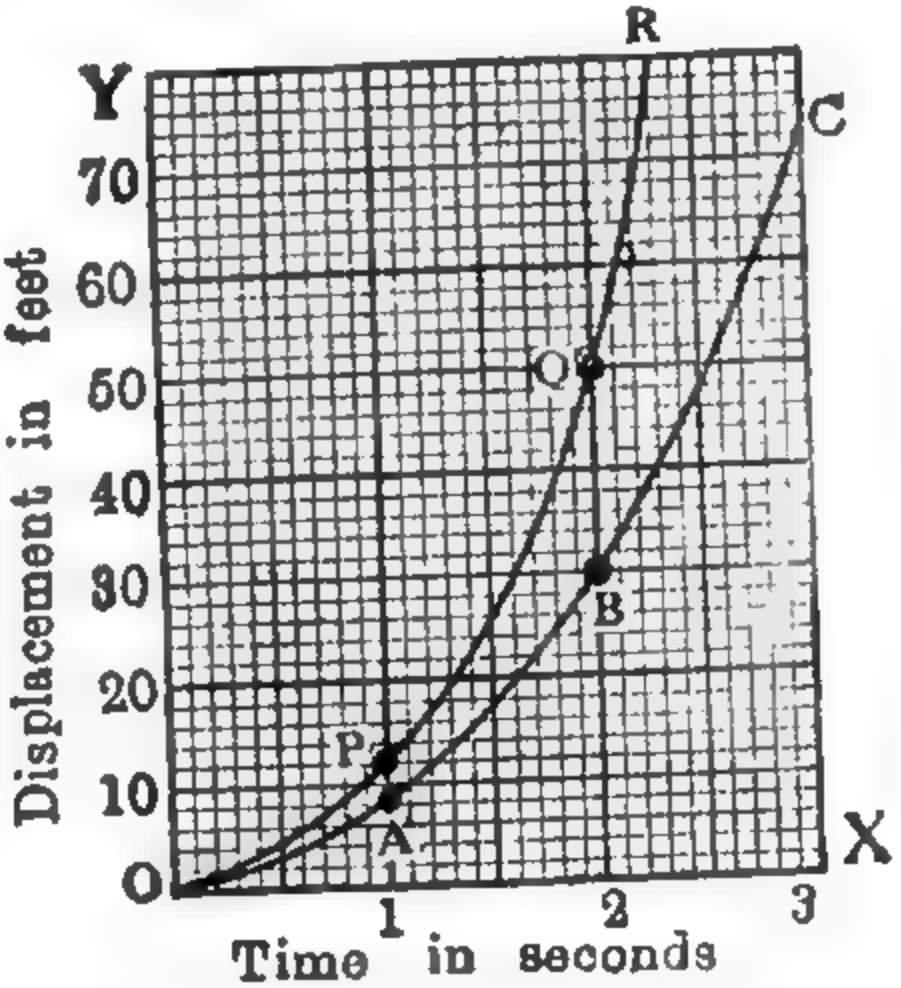


Fig. 34—Uniform acceleration

of co-ordinates with O as the origin (Fig. 33). The time is measured along OX and displacement along OY. The values of the units on the graph paper may be suitably chosen.

Suppose now that a particle has displacements of 15 ft. in the 1st sec., 30 ft. in the first two seconds and 45 ft. in 3 sec. The corresponding points on the graph paper are A, B and C lying on a straight line

passing through the origin. Thus the straight line on the time-displacement graph represents equal displacements in equal times which is a uniform velocity. It is evident that the velocity V representing the line OC is given by

$$V = \frac{CX}{OX} = \tan COX.$$

Hence tangent of the angle made by the straight line with the time axis represents the velocity. The value of the tangent of an angle increases continuously with the angle. Hence the line OPQR which makes a larger angle with the time axis represents a higher velocity than OABC. Any line coinciding with or parallel to OX makes no angle with OX and so this line represents zero velocity, while a line coinciding with or parallel to OY makes an angle of 90° with OX, representing infinite velocity.

A uniform velocity is represented by the equation $v = s/t$ or $s = vt$. Therefore a relation of s with t for a given value of v is similar to the equation $y = mx + c$, where m is a constant and $c = 0$. Hence $s = vt$ represents a straight line passing through the origin.

Again consider that a body is moving with a uniform acceleration. During .st interval, let the body traverse a distance of 9·5 ft. During 1st and 2nd intervals of time, let the displacement be 30 ft. and in first three seconds let the body move a distance of 73 ft. Evidently the body is moving with an accelerated motion Now plot on the graph paper co-ordinates of these points at A, B etc. (Fig. 34). Pass a free-hand smooth curve through these points ; the curve shows a parabolic nature. The more steepy parabola as that given by the curve OPQR signifies a higher acceleration. as it represents larger displacements than given by the curve OABC in any given interval.

We know that $s = ut + \frac{1}{2}ft^2$ represents an equation of acceleration. If $u = 0$, then $s = \frac{1}{2}ft^2$ is similar to the equation of $y = 4ax^2$, which represents a parabola. Therefore the relation of s and t in a uniformly accelerated motion is a parabola, as shown in fig. 34. Even if there is some initial velocity u of the body, the graphical relation between the displacement s and time t is parabolic in nature.

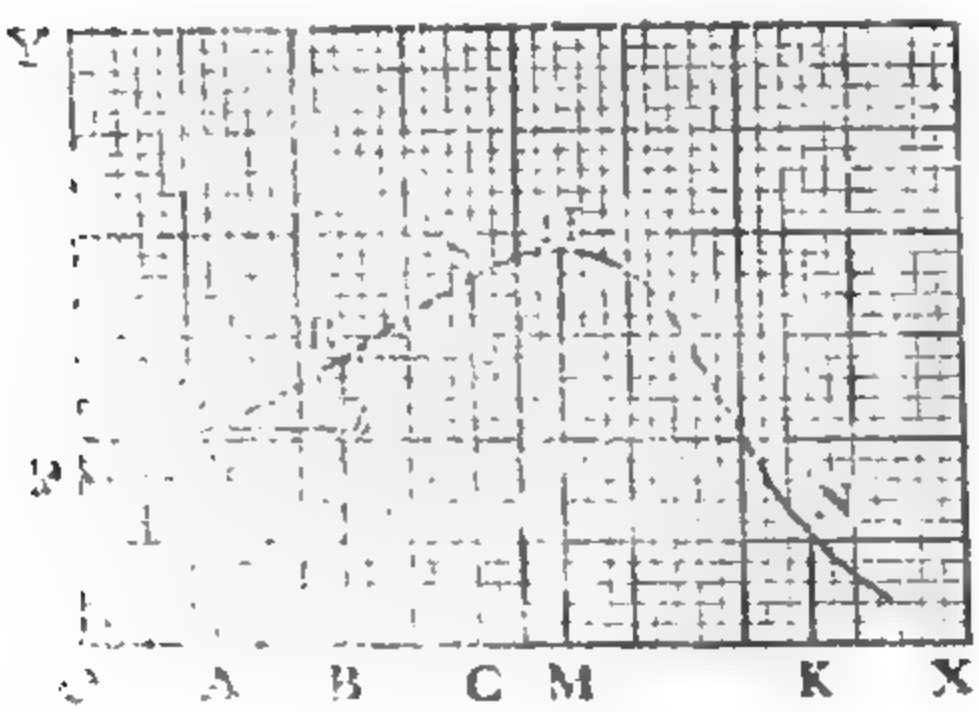


Fig. 35—Speed Time Diagram

Speed-Time Diagram.—Let OX and OY be two axes with O as the origin (Fig. 35). Let equal parts OA, AB, BC, etc. along OX represent equal intervals of time, say, one second each and let the *speed* be represented along OY with any suitable choice of units. Let us count time from O and let the speed of the body at this instant be represented by OP. Similarly AQ, BR, etc. represent the speed at the

corresponding instants. The curve P Q R S.....N obtained by joining P, Q, R, etc. is called the Speed-Time diagram of motion. This curve does not represent the actual path of the body but merely indicates the changes in the speed of the body at successive intervals.

23. Angular Velocity.—We have already seen in Art. 16 that when a rigid body moves *round* any straight line as its *axis of rotation*, all the particles of the body move in circles about the axis as centre. The **angular velocity** of any particle composing the body is measured by the angle swept over in unit time by the line joining the particle to the centre of its path. Let a particle on the body move through an arc MN in one second (Fig. 36). This angle is the same for all particles of the body and is known as its angular velocity. It is usually expressed in radians per second and is denoted by ω. The angular velocity is said to be *uniform* when equal angles are swept over in equal intervals of time.

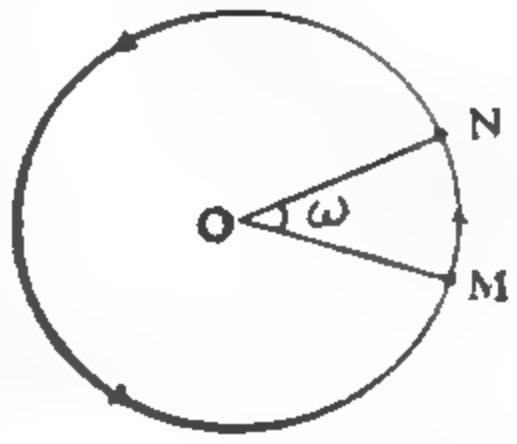


Fig. 36

Let a body, moving with a uniform angular velocity ω, describe an angle θ in time t. Then $\theta = \omega t$ or $\omega = \theta/t$. Again the angle described in one complete revolution is 4 right angles or 2π radians. If the time for one complete revolution be T and n the number of revolutions per second,—

$$\text{Then} \quad \omega T = 2\pi \text{ whence } \omega = \frac{2\pi}{T} = 2\pi n \qquad \ldots \quad (23,1)$$

Relation between Angular and Linear Velocities.—Let the length of the arc described by the particle in unit time be v and if the radius of the circular path be r, then the angle subtended by this arc at the centre is v/r radians.

$$\therefore \quad \text{Angular velocity } \omega = \frac{v}{r} \text{ radians per second}$$

$$\text{whence} \qquad v = r\omega \qquad \ldots \quad (23,2)$$

Hence the *linear velocity* of any particle of a rotating body is proportional to its distance from the axis of rotation and is given by the product of this distance and the angular velocity. Thus, in the case of a rotating wheel, a point on the rim has greater speed than that of any point nearer the centre.

Angular Acceleration.—The speed of revolution of a body may often change for example when the flywheel of an engine is in the process of acquiring speed. In such cases the angular velocity also changes with time. If the angular velocity increases at a constant rate, we call it angular acceleration, and it is measured by the change of angular velocity per second. When the angular velocity decreases at a constant rate, it is called *negative angular* acceleration or angular retardation. If ω_1 be the angular velocity of a rotating body at a certain instant and ω_2 be its angular velocity after a time t, then the total

change in angular velocity during an interval t is $\omega_2 - \omega_1$ and the *average angular acceleration* during the interval is $(\omega_2 - \omega_1)/t$.

Example :—

1. A flywheel rotates about a fixed axis at the rate of 150 revolutions per minute; find the angular velocity of any point on the wheel. What is the linear velocity, if the radius of the wheel is 3½ ft.?

A. The angle described per minute $= 150 \times 2\pi = 300\,\pi$ radians.

$\therefore$ Angular velocity $\omega = \frac{300\pi}{60} = 5\pi$ radians per sec.

Hence linear velocity, $r\omega = 3\frac{1}{2} \times 5\pi = 55$ ft. per sec.

24. Composition of Velocities.—If a body possesses simultaneously more than one velocity, it may due to the combined effect of the velocities, be at rest or in motion with a definite single velocity. In the latter case, effect of the single velocity is the same as the combined effect of the separate velocities. The single velocity is called the **resultant velocity** and the several velocities are called its **components.** The process of finding the resultant when the components are given is known a **composition of velocities.**

To find the resultant of velocities along the same straight line :—

(*i*) If a body has two or more velocities in the same line and in the *same direction*, the *resultant* is a single velocity in that direction and of magnitude equal to the algebraic sum of the components.

(*ii*) If a body has two velocities in exactly *opposite directions*, the resultant is a single velocity in the direction of the greater and its magnitude is equal to the difference of the two velocities.

For example, suppose that a railway train is running uniformly with a velocity of 30 miles an hour. A passenger sitting in a compartment has also the same velocity impressed on him. If now he walks within the compartment at the rate of 4 miles an hour *in the direction of motion of the train*, two velocities are impressed upon him along the same direction and his resultant velocity is 34 miles per hour. But when he moves in opposite direction with the same velocity, his resultant velocity becomes 26 miles per hour along the direction of motion of the train.

To find the resultant velocity not in one straight line :—

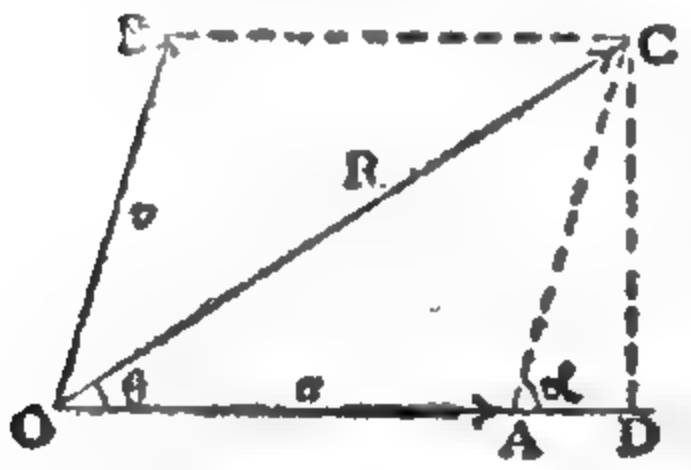


Fig. 87—Parallelogram Law

If a body has velocities not acting in the same straight line but they are inclined at an angle to each other, the magnitude and direction of the *resultant* is obtained by the **parallelogram law.**

Parallelogram of Velocities.—*If a particle be simultaneously acted on by two velocities represented in magnitude and direction by the two adjacent sides of a parallelogram, drawn from a point, their* **resultant** *is represented in magnitude*

and direction by the diagonal of the parallelogram passing through that point.

Let the sides OA and OB of a parallelogram (Fig. 37) represent in magnitudes and directions the velocities u and v respectively acting upon a particle at O and let the directions of the velocities be inclined to each other at an angle α. Complete the parallelogram OACB and join OC. Then the resultant velocity R acting upon the particle is represented in magnitude and direction by the diagonal OC.

Produce OA and from C draw CD perpendicular on it. Let the direction of the resultant OC make an angle θ with that of the component u, so that $\angle AOC = \theta$.

Now
$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$= OA^2 + 2OA.\ AD + AD^2 + CD^2$$
$$= OA^2 + AC^2 + 2OA.AC \cos \alpha$$
since $AD^2 + CD^2 = AC^2$ and $AD = AC \cos \alpha$

or
$$R^2 = u^2 + v^2 + 2uv \cos \alpha \quad \ldots \quad (24,1)$$

Also
$$\tan \theta = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{v \sin \alpha}{u + v \cos \alpha} \quad \ldots \quad (24,2)$$

Thus, knowing u, v and α, the values of R and $\tan \theta$ may be calculated. Further referring to the table of tangents, the value of θ in degrees can be found.

In general, the resultant velocity is inclined at an angle $\tan^{-1} \frac{v \sin \alpha}{u + v \cos \alpha}$ to the direction of the velocity u.

Composition of Displacements.—Since displacement is also a vector quantity having magnitude and direction, the parallelogram law can be applied upon a body under the action of two displacements acting simultaneously or one after another. We can state the *parallelogram of displacements* thus ;—

If two displacements, represented in magnitudes and directions by the adjacent sides of a parallelogram, act upon a body either simultaneously or one after another, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point. The magnitude and direction of resultant displacement may be found exactly in the same way as that followed in velocity.

It is evident from fig. 37 that a vector of length OA, representing a velocity or displacement along with a vector OB of another velocity or displacement produce a resultant vector OC. Since OB is equal and parallel to AC, we may also state that a vector OA and a vector AC produce a resultant vector OC. Thus instead of drawing a parallelogram, the resultant may be found by completing the triangle with the two vectors as its two sides as shown in fig. 38. If OA and AC represent two velocities or displacements acting upon a particle, their resultant is

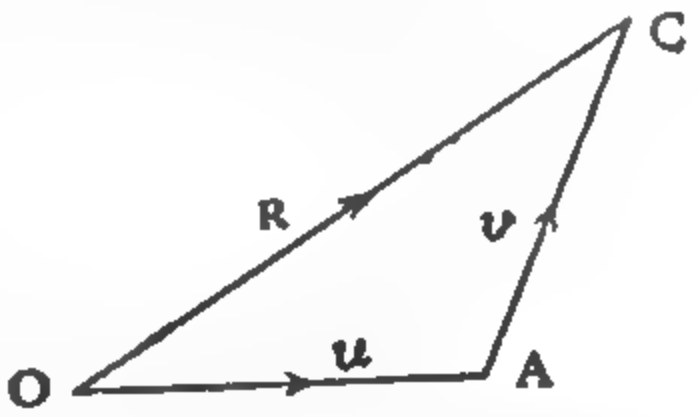


Fig. 38—Vector Sum

represented by the third side of the triangle taken in opposite direction. In terms of vectors we can write the equation in the form,

$$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC} \qquad \ldots (24,3)$$

It is to be remarked that the algebraic sum of the lengths of OA and AC is always greater than that of OC. Such a summation is called a *scalar* summation. But the two vectors OA and AC represented in magnitudes and directions, would combine to make up a vector OC. This is called a *vectorial summation* being represented by the equation (24,3).

The parallelogram law applies also in the case of two or more accelerations imposed on a body. Suppose a body simultaneously possesses two accelerations f_1 and f_2, represented in magnitudes and directions by OA and OB respectively (Fig. 37) acting at an angle α with each other.

The resultant acceleration is given by,

$$f = \sqrt{f_1^2 + f_2^2 + 2f_1 f_2 \cos \alpha} \qquad \text{and } \tan \theta = \frac{f_2 \sin \alpha}{f_1 + f_2 \cos \alpha}$$

Triangle of Velocities.—This is a statement of how three velocities acting upon a particle produce no motion. This is a modification of the parallelogram law. The theorem states that if a body simultaneously possesses *three* velocities, represented in magnitudes and

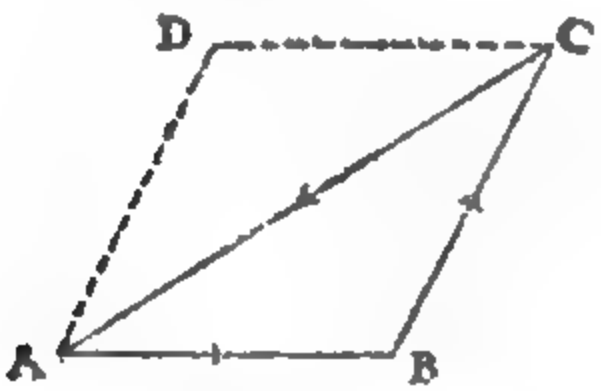


Fig. 39

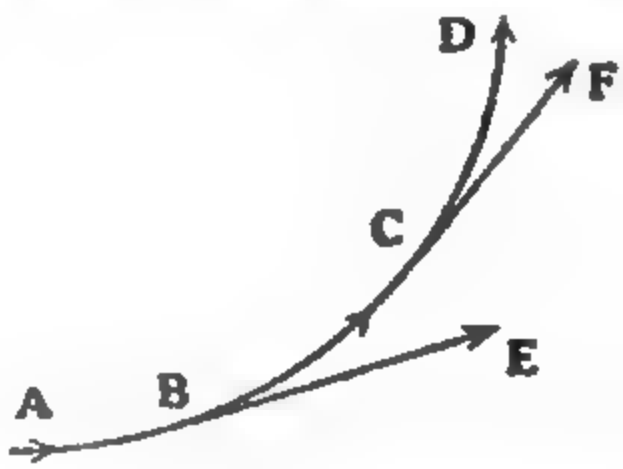


Fig. 40

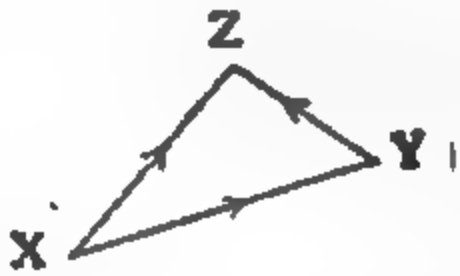


Fig. 41

directions by the sides of a triangle taken in order, the resultant is zero *i.e.* the body remains at rest. In fig. 39, three velocities represented by AB, BC and CA of the triangle ABC act upon a particle. By applying the parallelogram law of velocities, the resultant of any two of these velocities *e.g.*, AB and BC, which is the same as AB and AD, is found to be AC. Consequently, three velocities AB, BC and CA combine to become two velocities AC and CA. Now these two velocities being equal and opposite the *resultant* of the three velocities AB, BC and CA becomes zero.

The principle of the triangle of vectors may be applied to find the change in velocities *during an interval* when a body is moving over a curved path. Imagine a particle to be moving along a path ABCD (Fig. 40). Let its velocity at B be represented in magnitude and direction by the straight line BE and that at C by the line CF. It is evident

that there would be a continuous change of velocity as the particle moves from B to C, the initial velocity being BE and final velocity being CF.

Draw a line XY equal and parallel to BE and another line XZ equal and parallel to CF (Fig. 41). Complete the triangle by joining YZ. Then according to parallelogram law or vector summation,—

$$\vec{XY} + \vec{YZ} = \vec{XZ} \quad \text{or,} \quad \vec{XY} - \vec{XZ} = \vec{YZ}$$

Thus the change of velocities *during the interval* is represented in magnitude and direction by the line YZ.

Examples :—

Two velocities of 12 ft. per sec. and 15 ft. per sec. act upon a particle and are inclined to each other at an angle of 30°. Find the magnitude and direction of the resultant velocity.

A. Let $u = 12$ ft. per sec. and $v = 15$ ft. per sec.

Then $R^2 = 12^2 + 15^2 + 2 \times 12 \times 15 \cos 30°$

$= 369 + 360 \times \cdot 866 = 680 \cdot 76$

When $R = 26 \cdot 08$ ft. per sec.

Again $\tan \theta = \dfrac{15 \sin 30°}{12 + 15 \cos 30°} = \dfrac{7 \cdot 5}{12 + 12 \cdot 99} = \dfrac{7 \cdot 5}{24 \cdot 99} = \cdot 3$

With reference to the table of tangent, $\theta = 16°42'$ with respect to the velocity of 12 ft. per sec.

or generally $\theta = \tan^{-1} \cdot 3$.

2. The wind blows from a point intermediate between north and east. The southerly component of velocity is 5 m.p.h. and the westerly component is 12 m.p.h. What is the velocity with which the wind blows? (Del. U.—1948)

A. Since directions south and west are at right angles to each other, the two component velocities are also at right angles to each other. Hence if R be the velocity of the wind, then

$R^2 = 5^2 + 12^2 + 2 \times 5 \times 12 \times \cos 90° = 169$

$\therefore \quad R = \sqrt{169} = 13$ m.p.h.

3. A particle is found to move with a velocity of 10 ft. per sec. due east at a certain instant. After 2 minutes it is found to move with the same velocity due north. Find the change in velocity during the interval.

A. Although the velocities, at the initial and final stages, remain the same in magnitude, a change in direction calls for a change in velocity. Let BC represent the velocity due east and BA the velocity due north both being equal to 10 ft. per sec (Fig 42). Then from the triangle of velocities, velocities BC and CA combine to produce BA. Hence the change in velocities is CA. Now if D be the middle point of AC.

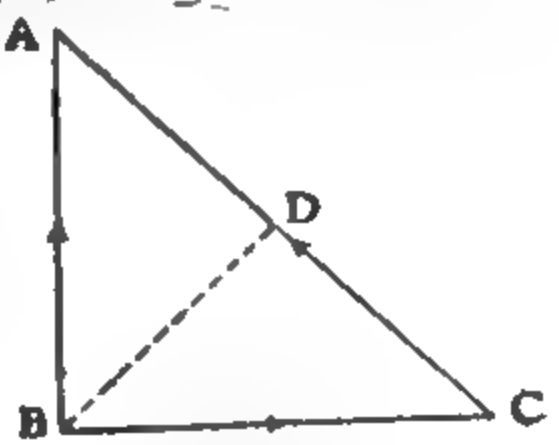


Fig. 42

$DB = CD = BC \sin 45° = 10 \times \cdot 707$

$\therefore \quad CA = 2DB = 14 \cdot 14$

Thus the change is velocity is 14·14 ft./sec. during the interval due north-west.

25. Resolution of a Velocity.—Let AC represent in magnitude and direction a velocity which is to be resolved into two components (Fig. 43). Draw a parallelogram ABCD with AC as diagonal. Then the adjacent sides of the parallelogram AB and AD would represent in magnitudes and directions, the two required components. But since any number of parallelograms can be drawn with a given line as diagonal, a velocity such as AC can be resolved into an *infinite number* of pairs of components. The adjacent sides of any parallelogram AGCH and

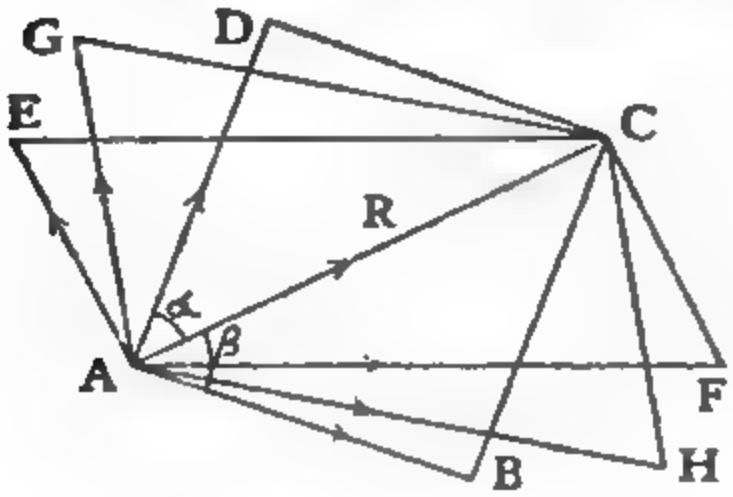


Fig. 43

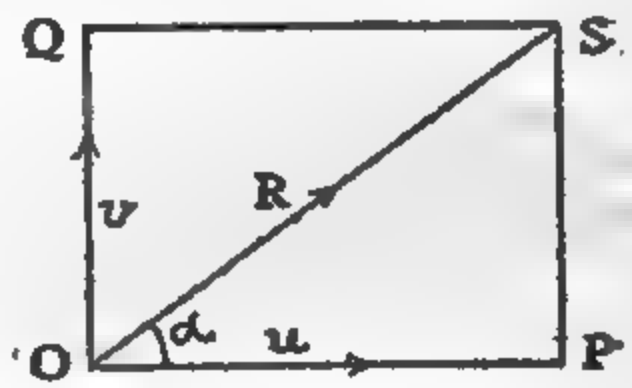


Fig. 44

AECF, for example drawn with AC as the diagonal, may also represent the components.

Let ABCD be a parallelogram of which the angles made by AD and AB with the diagonal AC are α and β. Since $\angle DAC = \angle ACB$, and $\angle DAB = \alpha + \beta$ and consequently $\angle ABC = 180^\circ - (\alpha + \beta)$. Hence from the property of a triangle, if $AC = R$

$$\frac{AB}{\sin \alpha} = \frac{BC}{\sin \beta} = \frac{AC}{\sin [180^\circ - (\alpha + \beta)]}$$

$$\therefore \quad AB = R\frac{\sin \alpha}{\sin (\alpha + \beta)} \quad \text{and} \quad BC = R\frac{\sin \beta}{\sin (\alpha + \beta)} \qquad \ldots (25,1)$$

The components are very often resolved at right angles to each other. In this case each component is said to be a **resolute** or the *resolved part* in the particular direction.

Let OS (Fig. 44) represent in magnitude and direction a given velocity R, which is to be resolved into two mutually perpendicular directions OP and OQ. Let the angle POS be denoted by α. From O draw OQ, perpendicular to OP. From S draw perpendicular SP and SQ. Then by the parallelogram law, OP and OQ represent the required components u and v respectively.

Since $OP = OS \cos \alpha$ and $OQ = OS \sin \alpha$

$$\text{or} \quad u = R \cos \alpha \quad \text{and} \quad v = R \sin \alpha \qquad \ldots (25,2)$$

Evidently, if α be given, *i.e.*, if the direction of one of the components be specified, then both the components become definite in magnitude and direction.

Resolution of an Acceleration.—In a similar way it can be proved that if OS represents a given acceleration in magnitude and

direction, the *resolved parts* of the acceleration along the directions OP and OQ will be represented by R cos α and R sin α respectively.

26. Relative Velocity.—The motion of a body on or near the surface of the earth is usually referred to with respect to some object fixed on the earth. But in some cases it becomes necessary to refer the motion of one body *relative* to a second body, which is also in motion. In this case, if the distance between the two is found to change from time to time, each is said to have a velocity *relative* to the other. Thus, **the relative velocity** of a particle A with respect to a second particle B is the rate at which A changes its position relative to B.

When both the bodies move in the same direction with the same velocity, the relative velocity of each with respect to the other is zero. Thus, if two trains are running along the same direction with the same velocity, then to a passenger in either train it would appear that the other train is at rest.

If the two trains, A and B, run on parallel lines in the same direction but A has a great velocity than B, then to a passenger in A, B will appear to move *away* in the opposite direction, while to passenger in B, A will appear to move *forward*. If the two trains move in opposite directions, to a passenger in either, the other would appear to

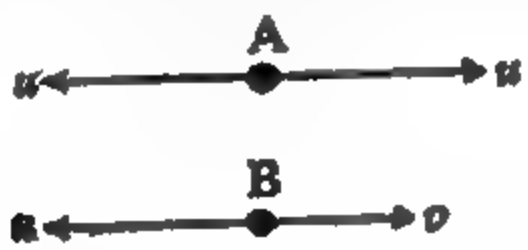


Fig. 45

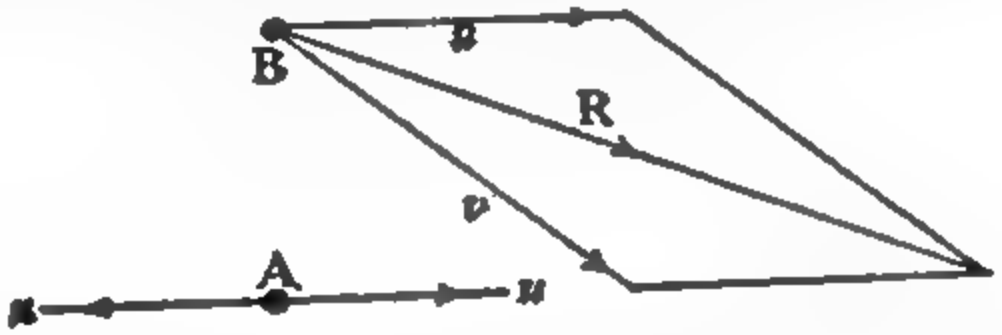


Fig. 46

be moving away faster than its own speed. Similarly when a man is running in a shower of rain, the rain-drops though falling vertically appears to strike his face in a slanting direction.

In general, the *relative velocity* of any body B with respect to another body A, is obtained by taking A to be at rest and compounding with the velocity of B a velocity equal and opposite to that of A. The resultant thus obtained gives the required *relative velocity* in magnitude and direction. For example, imagine that two bodies A and B are moving parallel to each other in the same direction with velocities u and v respectively (Fig. 45). To find the relative velocity of B with respect to A, impose an equal and opposite velocity u on A and B. Thereby A is brought to rest and B moves with a velocity $u \sim v$ The relative velocity of two bodies moving with different velocities in different directions is shown in fig. 46. Here A is moving backwards with a velocity u, and B is moving with velocity v. To find the relative velocity of B with respect to A, the latter is brought to rest by imposing an opposite velocity in the forward direction. An equal and parallel velocity u is imposed on B and their resultant R is obtained. B is then found to be moving along this resultant direction when observed from A.

Examples :—

1. To a man walking at the rate of 2 miles an hour, the rain appears to fall vertically ; when he increases his speed to 4 miles an hour, it appears to meet him at angle of 45° ; find the real direction and speed of the rain.

[Pat. U.—1951 ; Utkal. U.—1951]

A. Let $u =$ velocity of the rain in miles per hour and $\theta =$ the direction of the rain with vertical in degrees. Suppose that the man is walking towards east. To bring him to rest, apply a velocity on him of 2 miles/hour towards west and also apply the same velocity on the following rain drops. The resultant velocity of the rain drops, according to the problem has a vertical direction. Thus $u \sin \theta = 2$. Again when he walks 4 miles/hour towards east, bring him to rest by applying an opposite velocity and apply the same velocity on the rain drops and the resultant makes an angle of 45° with the vertical. By drawing figure, it can be very easily proved that in this case $u \sin \theta + u \cos \theta = 4$. Hence from these two equations, $\theta = 45°$ and $u = 2\sqrt{2}$ miles/hour.

27. Dimensional Representation of Quantities.—It has already been stated in Art. 13, that the dimensions of length, mass and time are represented respectively by the letters L, M and T. Since velocity is the distance covered by a body in unit time, it can be obtained by the quotient of total length covered and total time. Hence the dimension of velocity is L/T or LT^{-1}. The same is the dimension of variable or uniform speed. If a body possesses a velocity of 10 cm/sec. we may as well express it as $10LT^{-1}$ C.G.S. units. Acceleration is the rate of change of velocity and hence it has the dimension of velocity/time or LT^{-1}/T *i.e.*, LT^{-2}. For example, an acceleration of 3 ft. sec^{-2} may be expressed as $3\ LT^{-2}$ F. P. S. units. A retardation has a negative sign before it.

An angle expressed in radians is the quotient of arc and radius. The dimension of an angle is therefore L/L or unity. Hence an angle has no dimension. An angular velocity is expressed as an angle divided by time. Hence its dimension is L^0T^{-1}.

It is interesting and instructive to note that in an equation involving physical quantities, the dimension on both the sides of the equation ought to be identical. In fact, the identity of dimension is a true test of the validity of an equation. Thus, taking the equation, $s = ut + \frac{1}{2}ft^2$, the left hand side term has the dimension of L ; ut has the dimension of $LT^{-1} \times T = L$ and ft^2 has $LT^{-2} \times T^2 = L$. Thus the equation is true.

[Students are required to remember the following equivalences to facilitate quick calculations of mathematical problems.]

30 miles per hour = 44 ft. per sec.
1 mile per hour = 44·703 cm. per sec.
π = 3·1415 ; 3·1415 radians = 180°
1 cm. per sec. = ·0328 ft. per sec.

RENE DESCARTES (1596—1650)

Descartes was born in Touraine in France on March, 31, 1596. From his early age, he got the opportunity of receiving school educa-

tion. At the age of eighteen, he left the college. Teaching of Natural Sciences in those days consisted in imparting to students certain popular beliefs and notions handed down from generations. People would never think it necessary to apply the reasoning to investigate whether the so called notions were correct. Making any experiment to verify the truth of those beliefs was considered to be unlawful and prejudicial to the mandate of the Church.

Descartes was one who thought that a conclusion regarding any problem could be arrived at by a course of systematic and deductive procedure. After leaving the college, he became deeply absorbed in the study of some problems. At the age of thirty-three, he went to Holand and spent eight years in seclusion. Then came out his two world famous books—'Discourse on Method' and 'Meditations'. At the publication of these books, the Dutch theologians conspired against him and he was compelled to go to Sweden in 1649.

He was in fact a mathematical genius and may be said to have founded analytical Geometry. Geometry, after its compilation by Euclid in 300 B. C. did not improve much in about two thousand years, but Descartes gave it an improved outlook. The rectangular co-ordinate system was first inaugurated by him. His health was delicate and he died on February, 11, 1650 in Stockholm. In 1666 his coffin was carried to Paris and was lain there.

JOHAAN KEPLER (1571—1630)

Kepler was born at Wiel, in Wurtemburg on December 21, 1571. His father was reduced to an extreme stage of poverty and he engaged his son to earn money as a pot boy in an inn. After two or three years of service, the boy got an opportunity of being admitted in a monastery school After passing the school final examination in 1589, he entered the University of Tubingen. In 1594 he graduated from the University. Two years later he published his first paper in support of the Copernican Theory that all planets revolve round the sun.

Fig. 47 - Johaan Kepler

At that time the Danish astronomer, Tycho Brahe had certain astronominal apparatus and Kepler requested his permission to use those apparatus for his work. In 1601 Kepler became his assistant. The same year Tycho died and he became

the Director of the laboratory. It took him four years to determine accurately the orbit of Mars, and thus he formulated the laws of planetory motion. But he did not know the nature of the force which holds the sun and the planets.

Of the books compiled by Kepler, the most attractive was "On Celestial Harmonies". His labours and insight revealed the solar system in all its simplicity. He died on November 15, 1630.

Summary

Rest and Motion.—A body is said to be at *rest* when it does not change its position for any period. It is said to be in *motion*, when it changes its position with time.

The *motion* of a body may be (*i*) **translatory** and (*ii*) **rotatory.**

Displacement.—The *displacement* of a body is its change of position.

Velocity.—The *velocity* of a moving body is the rate of change of its position in a definite direction.

Acceleration. is the rate of change of velocity.

Equation of Motion.—(1) $v = u + ft$ (2) $s = ut + \frac{1}{2}ft^2$ (3) $v^2 = u^2 + 2fs$.

If v be the uniform speed of a particle moving in a circle then $v = r\omega$.

Parallelogram of Velocities.—*If a particle possesses simultaneously two velocities, represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

Resolution of a Velocity.—A single velocity R can be resolved along two directions at right angles to each other. The resolved parts u and v are given by $u = R \cos \alpha$, and $v = R \sin \alpha$, where α is the angle between R and u.

Exercises on Chapter I

Reference Arts.

18 & 23 1. Distinguish between acceleration and velocity. What is angular velocity? (Dac. U.—1939; *Cf.* Del. U.—1952; Utkal. U.—1951; Del. H. S.—1949)

2. Derive the relation $s = ut + \frac{1}{2}ft^2$.
(Del. H. S.—1949; Anna. U.—1950)

Art 20 3. A train starting from rest travels with uniform acceleration and acquires a velocity of 30 miles per hour in 2 minutes. Find the acceleration and the distance passed over in the time.

A. $f = \frac{1}{80}$ *ft.* per sec.^{2}; 2640 ft.

Art. 20 4. A train travelling at the rate of 30 miles per hour is stopped in 1 minute by putting a brake; what is the retardation and for what distance was the brake applied?

A. $\frac{11}{15}$ ft/sec.^{2}; $\frac{1}{4}$ of a mile.

Art. 20 5. A body moving with uniform acceleration describes 76 ft. in the 5th second and 116 ft. in the 10th second. Find the distance it passes over in the 15th second.

A. 156 ft.

Reference

6. A train starts from rest with an acceleration of 0·5 ft/sec² and at the same time a dog starts off and runs parallel to it with a constant velocity of 16 miles an hour. How far will the dog have gone before the train overtakes him and what time will have elapsed? Art. 20

A. 2202·7 ft.; 93·9 seconds.

7. Two trains leave the same station on parallel lines, the first starting from rest with a uniform acceleration of ·5 ft/sec² and attaining a maximum velocity of 15 miles per hour which is kept constant, the other staring from rest 40 seconds after the first with a uniform acceleration of 1 ft/sec² and attaining a maximum and constant velocity of 30 miles per hour. At what distance from the station and how long after leaving, will the second pass the first? Art. 20

A. 1760 ft. 62 secs.

8. The distance between the sun and earth is 93×10^6 miles. Assuming that the earth moves round the sun in a circular orbit once in 365 days, find its angular velocity in radian per sec. and linear velocity in miles per sec. Art. 28

A. $1{\cdot}99 \times 10^{-7}$ radian/sec.; 18·5 miles/sec.

9. State and prove the law of parallelogram of velocities. Art. 24

[Del. U.—1948]

10. A river has got a current of 5 miles an hour and a boat is rowed across it with a speed of 4 ft./sec. at an angle of 60° with the bank down the current. Find the resultant speed. Calculate the time to cross the river if its width is half a mile. Art. 24

A. 9·9 ft./sec.; 12 min. 41 secs.

11. A body moving with a velocity of 8 cm. per sec. has an acceleration of 6 cm. per sec.² applied to it in a direction making an angle of 90° with its original direction of motion. Calculate its final velocity and its displacement after 1 sec. Art. 24

A. 10 cm. per sec.; 8·5 cm.

12. Rain drops are falling to the earth vertically with a speed of 4 ft. per second. What should be the speed of a cyclist on a level road at which the rain drops appear to strike him at an angle of 30° to the vertical? Art. 26

A. 2·3 ft./sec.

13. To a motorist going north at 30 miles per hour, the wind appears to blow from north west. Actually it is blowing from west. What is its true velocity and what is its apparent velocity? Art. 26

From what direction and with what velocity would the wind appear to blow to a motorist going south at 15 m.p.h. Illustrate your explanation by diagrams.

A. 30 p.m.h.; 42·4 m.p.h.; 20°34′ S. of W; 33·5 m.p.h.

14. Explain what you mean by 'Relative velocity' and how it can be measured. Art. 26

(Utkal. U.—1951)

CHAPTER III

FORCE : NEWTON'S LAWS OF MOTION

28. **Inertia and Force.**—It is an inherent property of a material body that entirely left to itself, it can not change its state of rest or of motion. If the body is found to be at rest, it continues to be at rest for all time to come. If again it is found to be moving, it keeps on moving for all time to come. This property of a material body is called its **inertia.** Whatever a change is effected, it must be due to some external cause, which is termed a **force.** Hence a ***force is that which acting on a body, changes or tends to change the state of rest or of uniform motion of a body.*** A force must have its (*i*) *point of application* that is, the point at which the force acts (*ii*) *direction* and (*iii*) *magnitude.* A force is thus a vector quantity.

Newton's Laws of Motion.—The three laws of motion, first enunciated by Sir Isaac Newton, are based on general observations of motion of material bodies and form the basis of Dynamics. The three laws are stated thus ;

First Law. - *Every body continues in the state of rest or of uniform motion in a straight line, except in so far as it is compelled by some external impressed force to change that state.*

Second Law.—*The rate of change of momentum of a body is proportional to the impressed force and takes place in the direction of the straight line in which the force acts.*

Third Law.—*To every action there is an equal and opposite reaction.*

29. **Explanation of the First Law.**—This law consists of two parts. The first of these implies that a material body cannot of itself change its state of rest or of uniform motion. It also states that a moving body, when not acted upon by an external force would have free motion *i.e.*, *uniform motion in a straight line.* It is not possible to verify this experimentally, for we cannot make a body perfectly free from external forces. Thus, a very smooth ball rolled on the ground soon comes to rest owing to the resistance of the ground and of air. If the surface were more smooth, the ball would roll for a longer while. The nearest approach to free motion is the motion of a glass ball over smooth ice in the absence of the wind.

The second part of the law furnishes us with an idea regarding the function of a force. Thus (*i*) a force can change or tends to change the state of rest of a body ; (*ii*) a force can change the state of motion of a body, by increasing the speed or decreasing it. The law also implies that the force, which is to produce a change in the state of rest or of motion of a body, must be externally *impressed i.e.*, acting *from outside* on the body and not *stresses* or internal forces between

the parts of the body. It is then evident that inertia may show itself in two forms, namely,

(*i*) *Inertia of rest*, which means that a body at rest has a tendency to remain at rest.

(*ii*) *Inertia of motion*, which indicates that a body in motion tends to continue its motion.

Illustrations of Inertia of Rest.—To show inertia of rest of a body, take a ball and place it on a rectangular card just above a hollow cup fixed on a vertical stand (Fig. 48). A metal spring is swung aside and is fixed to a clamp on one side. On releasing the clamp, the spring jumps back and strikes the card, which is thrown away while the ball owing to its inertia of rest does not move forward but falls down into the cup.

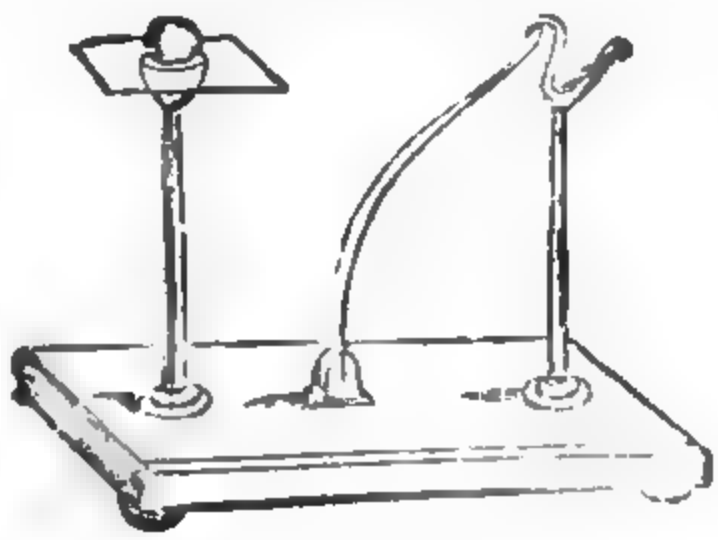


Fig. 48

A bullet fired against a window pane would make a clean hole and not cracks. The glass surface near about the place where the bullet strikes cannot share the quick motion of the bullet and remains as it is. When a train or a tram car suddenly starts, a passenger standing or sitting loosely in the car falls backwards. The lower part of the body, being in contact with the car, takes up the motion but the upper part tends to remain in the former state of rest. In beating the dust off a coat by a stick, the coat is suddenly set into motion, while the loose dust particles, tending to remain at rest, fall off.

Illustrations of Inertia of Motion.—A person, in getting down carelessly from a moving tram car, falls forward. His feet, on touching the ground, come to rest but the upper part of the body due to inertia tends to move forwards and so he stumbles towards the direction in which the car is moving. Again when a car in motion suddenly stops, a passenger sitting loosely in the car falls forwards. If a horse at full gallop suddenly stops, the rider unless sitting firmly on the horse is likely to be thrown forward over the head of the horse.

Before taking a long jump a person runs from a little distance behind, so that the inertia of motion would help his muscular exertion to jump. A ball thrown vertically upwards by a person in a running train comes back to hand. Due to inertia the ball maintains its horizontal motion which it has acquired form the car and so comes back to hand, if the motion of the train or the position of the hands does not change in the meantime. Similarly in the circus show, a performer jumps up from the back of a running horse, and passing through a hoop again comes back on the horse.

30. Explanation of the Second Law.—The second law states that so long as an external force is acting upon a material body, it would continuously *gain* in motion. The acceleration of the body under

the action of the force is a measure of the force acting upon it. This law gives a *quantitative definition of a force.*

Momentum.—The *momentum* of a body is measured by the product of the mass and the velocity of the body. Hence, if the mass of the body be m and its velocity at any instant be v, then *momentum* of the body at the instant is $m \times v$.

The *unit* of momentum is the momentum possessed by a body of unit mass moving with a unit velocity. The F. P. S. unit of momentum is the momentum possessed by a mass of 1lb moving with a velocity 1 ft. per second. It is often termed a *poundem.* The C. G. S. *unit of momentum* is the momentum of a mass of 1 gm. moving with a velocity of 1 cm. per second. For example, a ball of mass 150 gm. moving with a velocity of 1 metre per sec. has a momentum of $150 \times 100 = 15 \times 10^3$ gm. cm. per sec.

Measurement of Force.—Let a body of mass m having an initial velocity u, be acted upon by a constant force P for a time t. Since it is under the action of a force, its velocity would continuously increase. Let its final velocity be v at the end of this period. Then the initial momentum of the body $= mu$ and its final momentum $= mv$,

Hence, the change of momentum in time $t = mv - mu = m(v - u)$

Thus the rate of change of momentum $= \frac{m(v-u)}{t} = mf$.

According to this law, the force P is proportional to the rate of change of momentum, *i.e.*, $P \propto mf$

or $P = kmf$, where k is a constant.

If now the unit of force be so chosen, that it produces unit acceleration when acting on a unit mass, then if $m = 1$ and $f = 1$, P becomes equal to 1 ; hence accordingly k must be equal to 1. Thus with units so chosen, $P = mf$ ... (30,1)

or Force = mass × acceleration.

Unit of Force.—The unit of force may be defined as *that force which acting on a unit mass produces unit acceleration.*

The *F. P. S. unit of force* is that force which acting on a mass of 1 lb. produces an acceleration of 1 ft. per sec. per sec. This unit is termed a **poundal.** The *C. G. S. unit of force* is that force which acting on a mass of 1 gm. produces an acceleration of 1 cm. per sec. per sec. This unit is called a **dyne.**

These two units are called *dynamical* or **absolute units of force,** for they remain constant throughout the universe. There is another system of unit of force, called the **gravitational unit.** (Vide Chapter IV).

Again $P = \frac{mv - mu}{t}$ or $P.t = mv - mu$ ... (30,2)

When a large force acts for a short period, the product **(force × time)** is called the **impulse** of the force during the time. Thus **the impulse**

of a force is equal to the change of momentum produced, and if there be no external force there will be no change in momentu n.

Eqn (30,2) provides another form of definition for a unit of force ; for if $mv - mu$ be unity and t be 1 second, then P becomes unity. Thus a unit of force is that force which *acting on any mass for unit time produces a unit change of momentum in the direction of the force.* Let the mass taken be unity, then a unit force acting upon a unit mass initially at rest would generate unit velocity on it.

Relation between a Poundal and a Dyne.—A poundal is much larger than a dyne. A force of one poundal acting on a mass of 1 lb. produces an acceleration of *one foot* per sec. per sec.

Now 1 lb = 453·59 gm. and 1 ft. = 30·479 cm.

Hence a force of 1 poundal acting on a mass of 453·59 gm produces an acceleration of 30·479 cm. per sec. per sec.

$\therefore$ 1 poundal = 1 lb × 1 ft. sec.$^{-2}$ = 453·59 × 30·479 gm. cm. sec.$^{-2}$
= 13824·969 dynes = 13825 dynes approximately.

Thus momentum is a *vector* quantity, its direction at any instant being the same as that of the velocity at that instant. It can therefore be represented by a straight line. The momentum has the dimension of mass × velocity, which is $M \times LT^{-1} = MLT^{-1}$. The force has the dimension of mass × acceleration, which is MLT^{-2}.

Examples :—

1. A train having a mass of 100 tons running on a level ground is kept going by the locomotive at a uniform speed of 30 miles per hour, the steam is suddenly shut off and the train comes to rest after going 2 miles further. Calculate the average resistance to the motion of the train. (Utkal. U.—1952)

A. The train running with a speed of 30 miles per hour is brought to rest within 2 miles. Thus to calculate the retardation, the formula $v^2 = u^2 - 2fs$ is to be used were $v = 0$ and $u = 30$ miles/hr $= \frac{220}{3}$ ft./sec. and $s = 2$ miles $= 10560$ ft.

Thus $0 = \left(\frac{220}{3}\right)^2 - 2f \times 10560$ whence $f = \frac{605}{2376}$ ft. per sec. per sec

The resisting force acting on the train = mass × retardation

$= 100 \times 20 \times 4 \times 28 \times \frac{605}{2376}$ poundals = 57037 poundals.

31. Verification of the Second Law of Motion.—It can be studied very conveniently with an apparatus known as Flotcher's trolley, which consists of a stout metal bench B, B, about one and a half metres in length having two parallel rails *rr* fixed upon it (Fig. 49). The bench is provided with levelling arrangements. A trolley T provided with wheels can run upon these rails almost without any friction. A pair of springs SS, known as *friction brakes* are fixed at the end of the bench for arresting the motion of the trolley

without a direct collision. A frictionless pulley is also fixed at one end

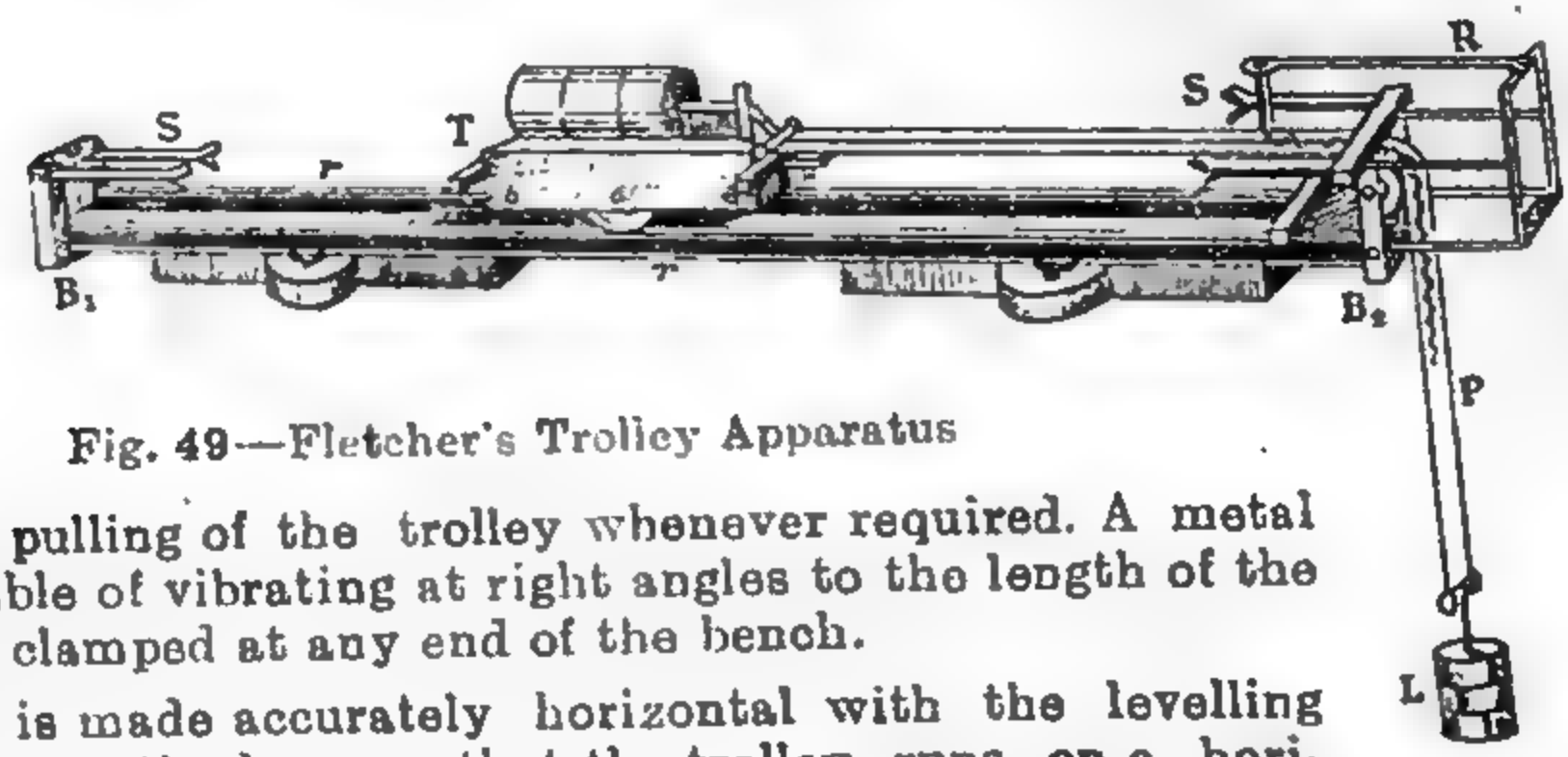


Fig. 49—Fletcher's Trolley Apparatus

to facilitate pulling of the trolley whenever required. A metal reed R capable of vibrating at right angles to the length of the bed may be clamped at any end of the bench.

The bed is made accurately horizontal with the levelling arrangements at the base, so that the trolley runs on a horizontal surface. Then the trolley is tied with a piece of thread to the end B_1 of the bench. One end of a specially made paper tape P is tied to the trolley. The tape is carried over a pulley and scale pan is suspended at its other end. A suitable load is put upon the pan which serves as the pulling force on the trolley. An inked brush is fixed at the end of the reed so as to touch the paper tape.

Fig. 50

The reed is swung a bit at right angles to its length, so that it keeps on vibrating to-and-fro marking a small arc on the paper with the inked brush. The period of one vibration of the reed is determined with a stop-watch or a metronome. Then the trolley is released by cutting down the thread, when it begins to move over the rails under the action of the load. The inked brush is found to trace a wavy curve on the moving strip of paper (Fig. 50). Since time for each vibration of the reed is constant, such a curve on the paper offers a convenient method of accurately measuring short intervals of time. The paper is then placed on the table and a straight line drawn evenly through the wavy curve which serves as the line of reference. The points of intersection of the wavy line with this straight line are marked. The distances *ab*, *bc*, *cd*, etc. are accurately measured. If period of vibration of the reed be T, then average velocity in successive intervals of time is *ab* T, *bc* T etc. Hence change of velocity in successive intervals is (*bc* − *ab*) T, whence acceleration is (*bc* − *ab*) T^2. This acceleration is found to be constant over any region of the paper so long as the active force and the moving mass remain constant. This law can also be verified with an Atwood's machine. (*Vide* next Chapter).

32. Explanation of the Third Law.—When a body A exerts a force on another body B, it is called the action of

the force of A on B. The third law states that if there is any action of the force of A on B, B also exerts an equal and opposite force on A, which is called the reaction of B on A. This law holds good whether the bodies are at rest or in motion and whether they are in contact or acting on each other from a distance. This law is therefore called the **law of reaction.**

Take, for example, a person standing on the ground. The weight of the person shows itself as the thrust exerted by his feet on the ground, which is called the action of his weight. So long as he stands firm, the ground presses his feet *equally* in an upward direction, which is called the ground reaction. These two forces would be exactly equal and opposite. This is an example of static action and reaction. But when he stands upon sandy or marshy soil, the ground resistance may not be sufficient to balance his weight. In that case his feet might be gradually submerged within the soil. Here also at any instant the ground reaction is equal and opposite to *a part of the weight of the person*, which is called the action. The remaining part of the weight serves as the moving force and has nothing to do with the third law ; it is to be treated with the second law. Since the ground reaction is much less on a sandy soil, it is rather difficult to walk fast.

The law also gives the principle of **conservation of momentum.** Thus, if a bat is moved towards an approaching ball, then on collision, the force exerted by the bat on the ball is equal and opposite to that exerted by the ball on the bat. Hence the momentum given to the ball by the bat is equal to that given to bat by the ball in the opposite direction. In other words, the momentum gained by one is equal to that lost by the other, so that *the total momentum of the system remains the same in any particular direction*. The principle of the conservation of momentum may be verified with an apparatus known as a Ballistic Pendulum. It can also be verified with a slight modification of Fletcher's Trolley apparatus. (*vide* Chatterjee's Intermediate Practical Physics).

Different Kinds of Action and Reaction.—The action and reaction between any two bodies may be of the following kinds.—

(*1*) *Thrust* or *Pressure*.—This occurs when one body presses against the other. Thus, when a book rests on a table, it presses the table vertically *downwards* by its weight, while the table exerts a pressure on the book vertically *upwards* called the thrust or the reaction on the book. These two forces are always equal and opposite.

(*2*) *Pull or Tension*.—This occurs when two bodies pull each other through a material medium. Thus, when a body is kept suspended by a string, the weight of the body pulls the string *downwards*, while the string pulls the body *upwards* with an equal force called the **tension** of the string. This upward force is thus equal to the weight of the body suspended.

Horse pulling a Carriage.—Let a carriage be pulled by a horse (Fig. 51). We shall now consider how the motion of the system is

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possible in spite of the fact that the tension in the tie-rod D is always equal and opposite.

To start motion, the horse presses against the ground with its hind leg in a direction with a force, say R along AE. Then the reaction which is also R acts in the opposite direction AF. The reaction R can be resolved into two components—(*i*) a vertical component, AB supporting the weight of the horse ; (*ii*) the horizontal component BF, which tends to move the system forward. Call this horizontal component *p*. There is a resistance *r* acting on the carriage due to friction of the wheels against the ground. So long as *p* is less than *r*, there is no motion. But as the horse exerts a greater and greater force, the component *p* increases and ultimately when *p* exceeds *r* the system is under the action of a resultant force $p - r$, which makes the system move in the forward direction.

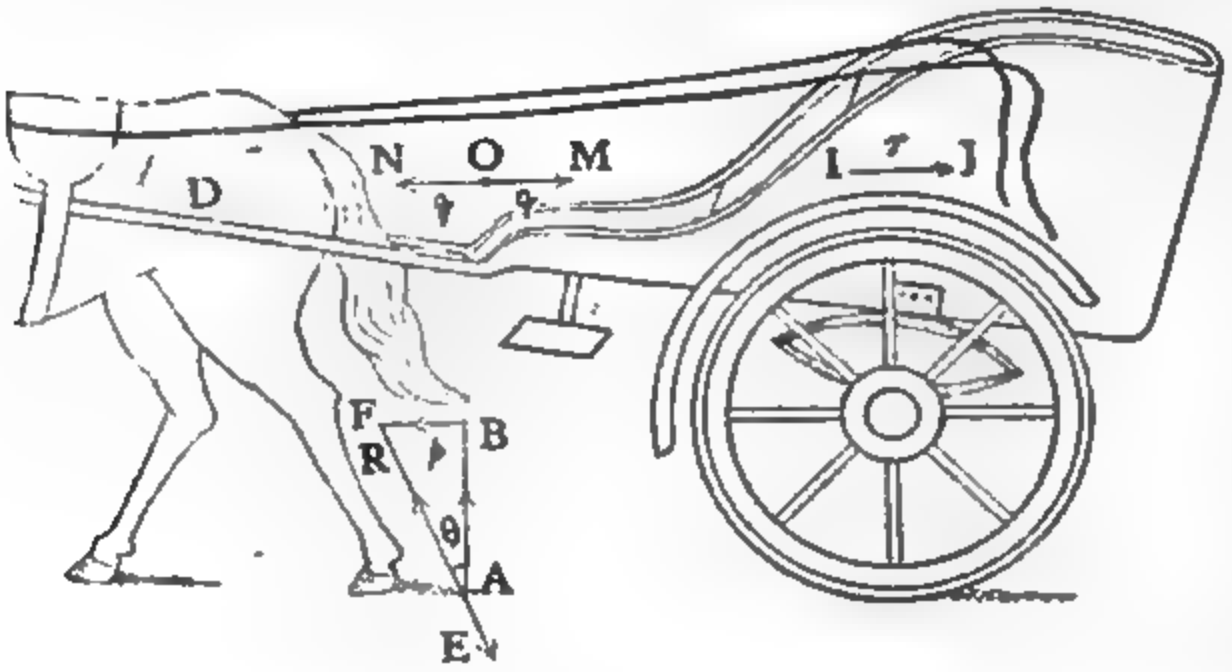

Fig. 61

To find the tension *q*, consider the carriage and the horse as two separate systems having a common acceleration. Then the resultant force acting on the carriage is $q - r$, producing the acceleration and the force $p - q$ producing the acceleration of the horse. On knowing their masses and acceleration, tension can be found.

(*d*) *Attraction and Repulsion*—These occur between two bodies acting at a distance. Thus, two opposite magnetic poles attract each other and two similar poles repel each other with equal and opposite forces.

(*e*) *Friction.*—This occurs when a body tends to slide over another body and always acts opposite to the direction of motion. Thus, when a body remains *at rest* on a *rough* inclined surface, the rough surface prevents the body from sliding down and exerts a force, called the *frictional force*, which is equal and opposite to the force tending the body to slide down. Again in walking, one presses against the ground in a *backward* direction with his feet and the frictional reaction of the ground gives him an equal *forward* force which thus sets him in motion. Hence it is difficult to walk on a very smooth surface.

The principle applies equally in case of *bodies in motion*. When we jump on the shore from a boat, the boat is found to be pushed backward and the monentum of the boat is equal and opposite to that of the man. Again when a bullet is fired from a rifle, the bullet is hurled forward violently by the force of the expanded gas. The bullet also exerts an equal and opposite force on the expanded gas,

which pushes the rifle backward. The momentum of the rifle is equal and opposite to the momentum acquired by the bullet, but the rifle, being much heavier than the bullet, its motion is much less. The backpush of a rifle, when fired, is a well-known fact.

The discharge of a rocket is due to the burning of the gun-powder contained in it. During combustion a stream of gas is given off downwards with a great force and due to the reaction the rocket shoots upwards. When streets are watered, water rushes out of the hose-pipe with a force, the backward reaction of which is felt by the water-man.

Examples :—

1. A force of 120 poundals acts for 2 seconds on a mass of 24 lb. initially at rest, and then ceases to act. Find the velocity acquired and the space moved over by the body during the time. How far will it move during the next half a minute?

A. According to equ. $P = mf$, $120 = 24f$ whence $f = 5$ ft. per sec.^{2} This shows that so long as the force acts upon the body, it moves with an acceleration of 5ft/sec.^{2} But the force acts for 2 secs, the initial velocity of the body being zero.

$\therefore\ v = u + ft. = 0 + 5 \times 2 = 10$ ft./sec. = final velocity after 2 seconds.

Again $s = ut + \frac{1}{2} ft.^2 = 0 + \frac{1}{2} \times 5 \times 4 = 10$ ft. = space moved over by the body in 2 seconds.

As soon as the force ceases to act on the body, there is no acceleration and the body moves with a constant velocity of 10 ft. per sec. Hence during next half a minute, it moves through $10 \times 30 = 300$ ft.

2. A shot of mass 40 lb. leaves a gun with a speed of 1000 ft./sec. If the velocity of recoil of the gun is $4\frac{13}{28}$ ft./sec. find the mass of the gun in tons.

(Utkal. U.—1951)

A. From the law of reaction, we know, that the momentum of the shot is equal and opposite to the recoil momentum of the gun.

Hence $40 \times 1000 = M \times 4\frac{13}{28}$ whence $M = \frac{40{,}000 \times 28}{125}$ lb.

$\therefore\ M = \frac{40{,}000 \times 28}{125 \times 28 \times 4 \times 20} = 4$ tons.

3. A shell of mass 40 lb. moving with a velocity of 12 ft./sec. explodes and bursts into two portions of mass 28 lb. and 12 lb, respectively. If the former be brought to rest owing to explosion, find the velocity of the latter.

A. Let the velocity of 12 lb. part be v ft./sec. after explosion.

The initial momentum of the entire shell $= 40 \times 12 = 480$ poundems.

The final momentum of 28 lb. part $= 28 \times 0 = 0$

The final momentum of 12 lb. part $= 12 \times v = 12v$ poundems.

From the principle of conservation of momentum, $480 = 12v + 0$

whence $v = 40$ ft./sec.

***33. Elastic Impact.**—If a marble ball is allowed to fall upon a cemented floor, it rebounds to a certain height, but if the same ball is allowed to fall upon an wooden floor, it rebounds to a much smaller height. If again, instead of a marble ball, an ivory ball be dropped

on the cemented floor from the same height, it rebounds to a larger height. The velocities before and after impact may be calculated from the distance fallen through and the height of rebound. As a result of experiments, Newton derived the following conclusion.

For the same pair of substances undergoing elastic impact, the velocity after impact bears a constant ratio to the velocity before impact and is independent of the mass of the ball.

This ratio which depends on the material of the ball and the floor, is known as the **coefficient of Elastic Restitution.** Suppose that a ball is colliding in a direction RO against a fixed plane PQ with a velocity u making an angle ϕ with the normal ON to the surface at the point of incidence and that it has no motion of rotation. After impact let the velocity be v in a direction making an angle θ with the normal. Now resolve the incident velocity into two components, $u \sin \phi$ parallel to and $u \cos \phi$ perpendicular to the colliding plane. In a similar manner resolve the velocity v into parallel and perpendicular components $v \sin \theta$ and $v \cos \theta$ respectively.

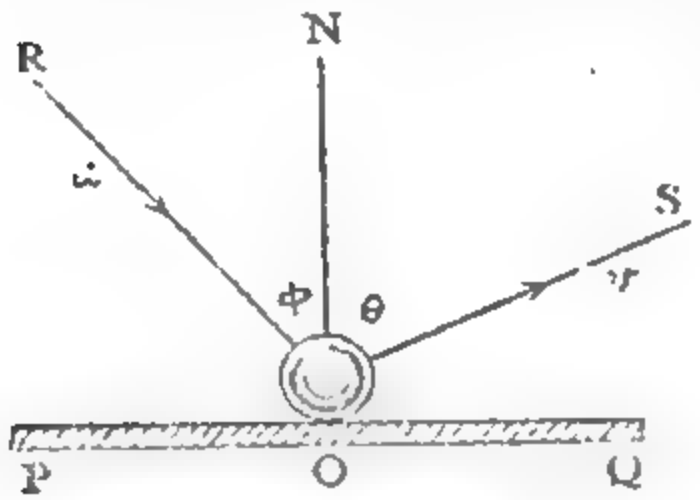

Fig. 52

Now as the plane is supposed to be perfectly smooth, there cannot be any force acting on the ball parallel to the plane during impact and consequently the parallel component of the velocity remains unchanged. Thus,

$$v \sin \theta = u \sin \phi \qquad \ldots (33,1)$$

The perpendicular component is altered by a certain ratio by Newton's law. Let the ratio be e; then,

$$v \cos \theta = e\, u \cos \phi \qquad \ldots (33,2)$$

$$\therefore \quad \frac{v \cos \theta}{v \sin \theta} = \frac{e\, u \cos \phi}{u \sin \phi} \qquad \text{whence } \cot \theta = e \cos \phi \qquad \ldots (33,3)$$

An ideal case is one in which the collision is perfectly elastic. In that case e becomes equal to unity and $\theta = \phi$. The ball makes equal angle with the normal before and after impact.

If m be the mass of the ball, then due to a single collision the change of velocity is $u \cos \phi + ev \cos \theta$ and the change of momentum is $m(u \cos \phi + ev \cos \theta)$. This change of momentum taking place within a very short time acts as an impulse, handed on to the plane surface during the act of collision. If the ball collides n times per sec. under similar conditions, then the total change of momentum per sec. is $nm(u \cos \phi + ev \cos \theta)$ and this quantity by the second law acts as the force exerted by the collisions on the plane surface.

Examples :—

1. A glass marble, whose mass is one ounce, falls from a height of 25 ft. and rebounds to a height of 16 ft., and the impulse and the average force between the

marble and the floor if the time during which they are in contact is 0·1 second; the acceleration of fall being 32 ft./sec.² [Nag. U.—1951]

A. Let v be the velocity of the marble at the instant of colliding against the floor. When falling from rest at a height of 25 ft.

$$v^2 = 0^2 + 2 \times 32 \times 25 \qquad \text{whence } v = 40 \text{ ft. per sec.}$$

When rebounding to a height of 16 ft, let v_1 be the velocity of start just after collision. Then,

$$0^2 = v_1^2 - 2 \times 32 \times 16 \qquad \text{whence} \qquad v_1 = 32 \text{ ft. per sec.}$$

The direction of velocity being reversed after collision, the change of velocity is evidently $40 + 32 = 72$ ft. per sec. and consequently the change of momentum of the ball is,

$\frac{1}{16} \times 72$ pound. ft. per sec. $= 4{\cdot}5$ poundems = impulse handed on by the marble to the floor during a period of 0·1 second.

Again since, force × time = impulse.

Force = impulse ÷ time = $4{\cdot}5 \div 0{\cdot}1 = 45$ poundals.

34. Composition of Forces.—If a particle is simultaneously acted on by two forces in the same straight line and in the same direction, their *resultant* is equal to the sum of the two forces and acts in the same direction. If the two forces act in opposite directions, the *resultant* is equal to the difference of the two and acts in the direction of the greater of the two forces. If two forces act in directions inclined to each other, their resultant is found by the Parallelogram Law as given below.—

Parallelogram of Forces.—*If two forces, acting simultaneously at a point, be represented in magnitudes and directions by the adjacent sides of a parallelogram drawn from the point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

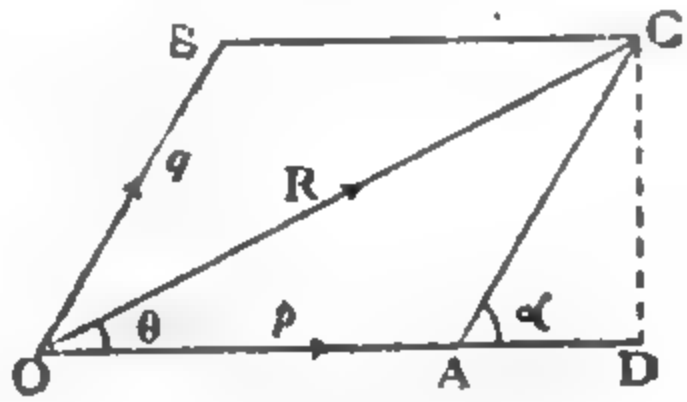

Fig. 53

Thus, if a particle at O be acted upon simultaneously by two forces p and q, represented in magnitudes and directions by OB and OB respectively (Fig 53), then the diagonal OC represents the resultant in magnitude and direction. Proceeding as in Art. 21, it can be shown that the resultant R is given by the equation—

$$R^2 = p^2 + q^2 + 2pq \cos \alpha \qquad \ldots (34,1)$$

where α is the angle between the directions of p and q.

If the direction of R is inclined at an angle θ to that of p,

$$\text{then,} \qquad \tan \theta = \frac{q \sin \alpha}{p + q \cos \alpha} \qquad \ldots (34,2)$$

When the two forces act at right angles to each other,

$$R^2 = p^2 + q^2, \quad \text{or} \quad R = \sqrt{p^2 + q^2} \text{ and } \tan \theta = q/p. \qquad \ldots (34,3)$$

Experimental Verification.—Take three strings knotted at a point O ; attach the other ends of the strings to three spring balances L, M and N (Fig. 54). Attach the spring balances to hooks fixed on a drawing board in such a way that the strings are kept well stretched. From the readings of the balances, find the forces acting along the strings and let them be represented by p, q and R respectively. Now mark on the paper fixed on the board the directions of p, q and R. Measure lengths OA and OB to represent p and q respectively. Complete the parallelogram OBCA and join OC. It is found that the length of the diagonal OC represents R in magnitude and direction in the same proportion. (Vide J. Chatterjee's Intermediate Practical Physics).

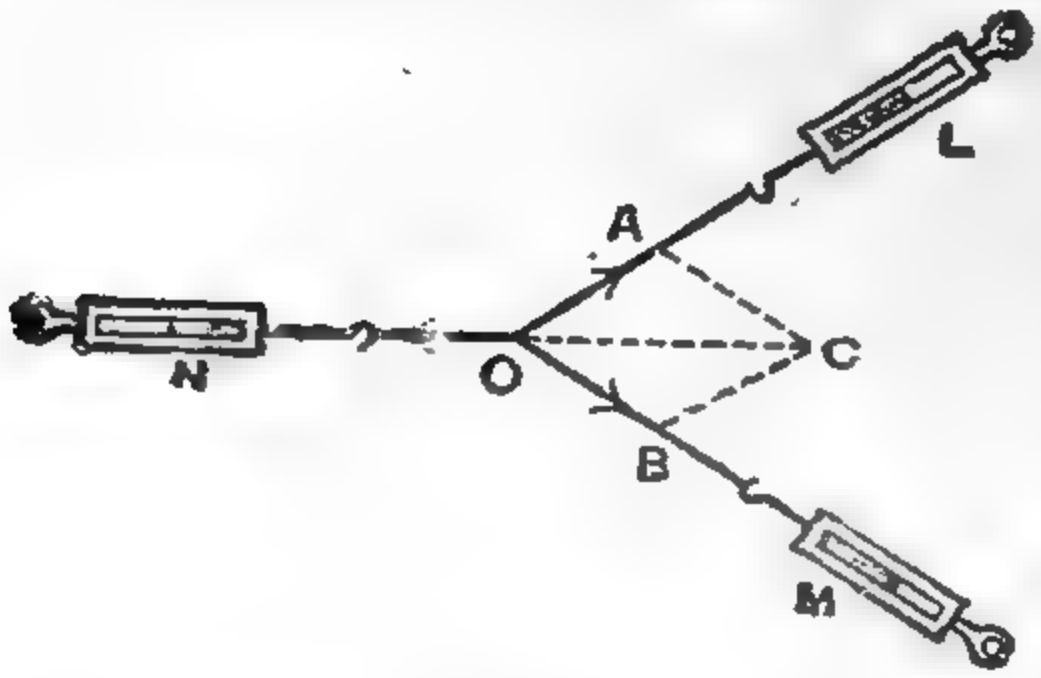

Fig. 54

The force R is called the **equilibrant** of P and Q, since the three forces together keep the point O in equilibrium.

Illustration of the Principle.—A ship being towed by two tugs in two different directions would not move in the direction of either but would take an intermediate direction, which can be obtained by the Parallelogram Law.

A bird while flying in the air strikes the air with wings and so air reactions act in the opposite direction (Fig. 55). If these reactions are represented in magnitude and direction by OA and OB respectively, the diagonal OC of the parallelogram OACB represents in magnitude and direction the force which make the bird advance. By varying the muscular effort of either wing, the corresponding reaction may be altered and hence the direction of the resultant *i.e.*, the direction which the bird flies may be changed.

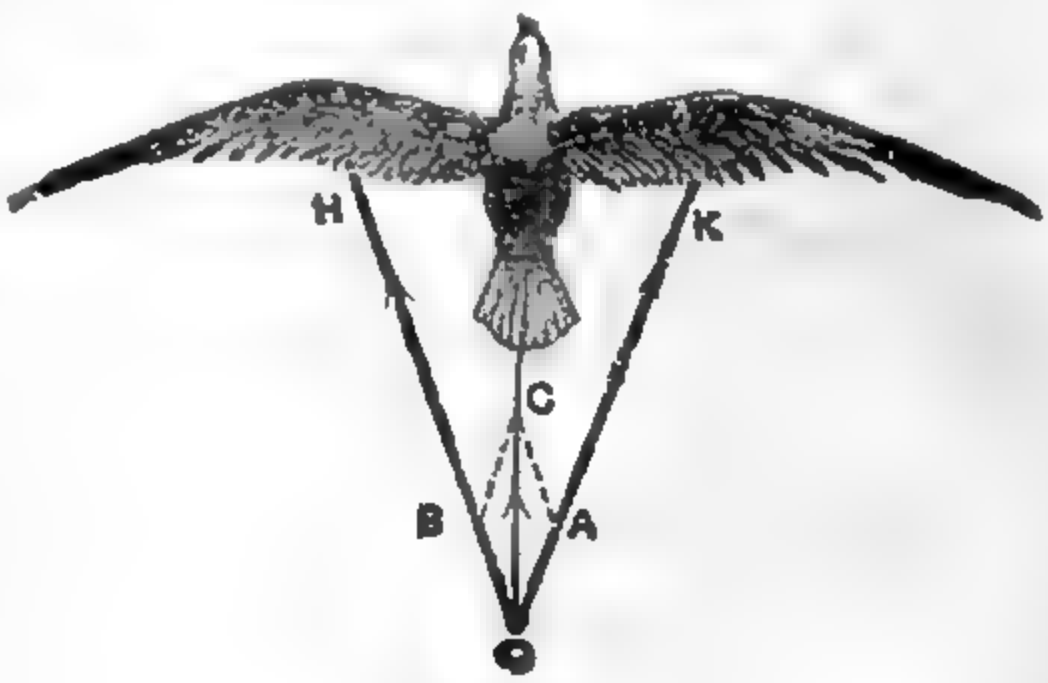

Fig. 55

35. **Resolution of Forces.**—A single force can be resolved into any two component forces. When two component forces act at right angles to each other, each component is termed a **resolute** or the *resolved part* of the given force in that particular direction. In fig. 44, a force R is represented by OS in magnitude and direction. If P and Q be the resolved parts represented by OP and OQ and $\angle SOP = \alpha$, it can be shown that $P = OS \cos \alpha$ and $Q = OS \sin \alpha$.

Some Practical Illustrations.—The following cases illustrate the resolution of forces.

Pulling of a Boat by a single Tug.—Suppose a boat is pulled by a single tug and let the pulling force be represented by OR (Fig. 56). This force can be resolved into two components—OA parallel to the length of the boat and OB, normal to it. The former moves the boat ahead, while the latter tends to move it sideways. The sideway motion is less effective since water exerts a large reaction to such a motion. Further, such a motion is partly counteracted by adjusting the position of the rudder D.

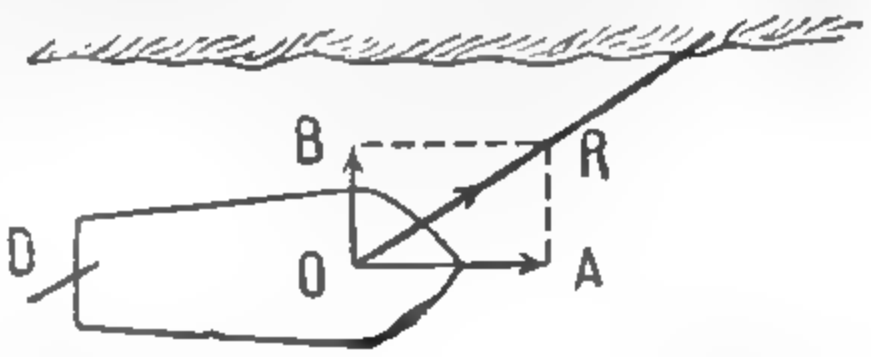

Fig. 56

Sailing of a Boat.—Let AB represent the length of the boat, CD that of the sail and WC that of the wind (Fig. 57). Let CE represent the force exerted by the wind on the sail in magnitude and direction. If this force is resolved into two components—CF, perpendicular to the surface of the sail and CD, along the surface, only the former component would be effective in exerting a pressure on the sail, while the surface component would glide past the sail. Let CF be now resolved into two components—CK, along the length of the boat and CH, at right angles to it. The component CK causes the boat to move forward, while CH tends to moves it sideways. But owing to great resistance offered by water to motion in this direction and also to that caused by proper inclination of the rudder, the component due to CH is ineffective and the boat moves forward. This shows how with the proper adjustment of the sail, a boat can advance even partly against the wind. In past days before the invention of steam power, ship had to be steered on high seas almost in any direction irrespective of the motion of wind by only adjusting a number of sails.

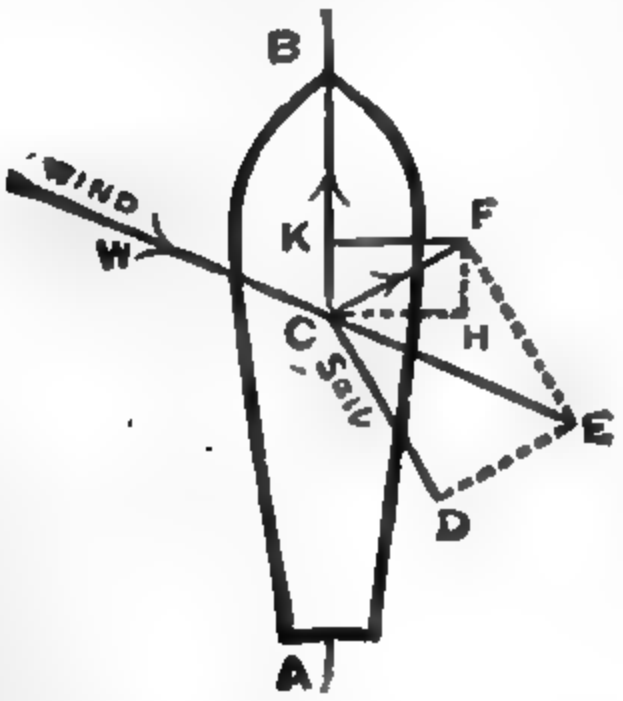

Fig. 57

Action of Bicycle Crank.—The effect of pressure applied by the foot on the pedal of a bicycle changes with the relative position of the crank. Let the foot press the pedal vertically downwards with a force represented by OR (Fig. 58). This can be resolved into two components, OA and OC, along and perpendicular to the crank respectively. Only the component OC is effective in driving the cycle, while the component OA is entirely passive. Evidently then, the more nearly does the pressure of the foot act normally to the crank, the more effective is the pedalling.

Moving a Lawn Roller.—When *pulling* the roller, the force exerted by the hand may be represented by OC (Fig. 59). This can be resolved into two components, OA and OB, parallel and

perpendicular respectively to the surface of the ground. The former is effective in dragging the roller while the latter acting vertically *upwards decreases* the pressure of the turf, caused by the weight of the roller. When *pushing* the roller, the force applied is given by

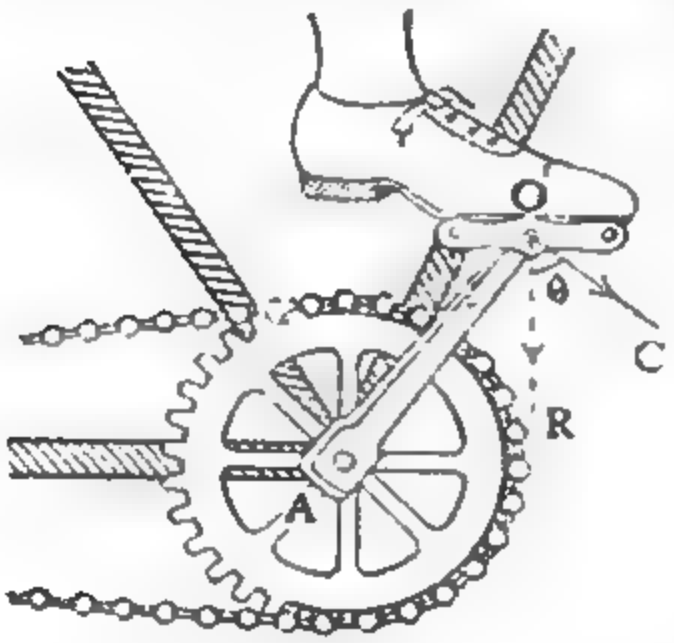

Fig. 58

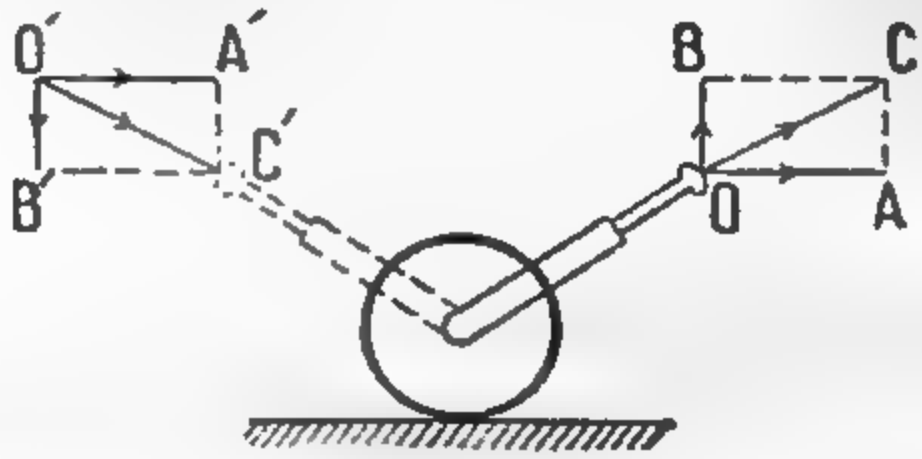

Fig. 59

O′C′ and the component O′A′ of the applied force is effective in moving the roller while the component O′B′ acting *downwards* adds to its weight. It thus *increases* the pressure exerted by the roller on the ground and makes it difficult to push the roller. Similarly, it is easier to pull a parambulator than to push it.

Flying of a Kite.—Suppose a kite K is flying in air (Fig. 60). The kite is acted on simultaneously by (*i*) its own weight W acting vertically downwards ; (*ii*) the tension T of the string and (*iii*) the pressure on its surface due to the wind. Let the effect due to the pressure of the wind on the surface of the kite be taken to be a single force acting at K and represented by DK. Let the weight W be represented by KA and the tension of the string by KB. The resultant of the two latter forces is represented by KC. The wind pressure, represented by DK, can be resolved into components—EK, along the face of the kite and the other perpendicular to it. If this latter component be exactly equal and opposite to the resultant KC, the kite remains in equilibrium in air.

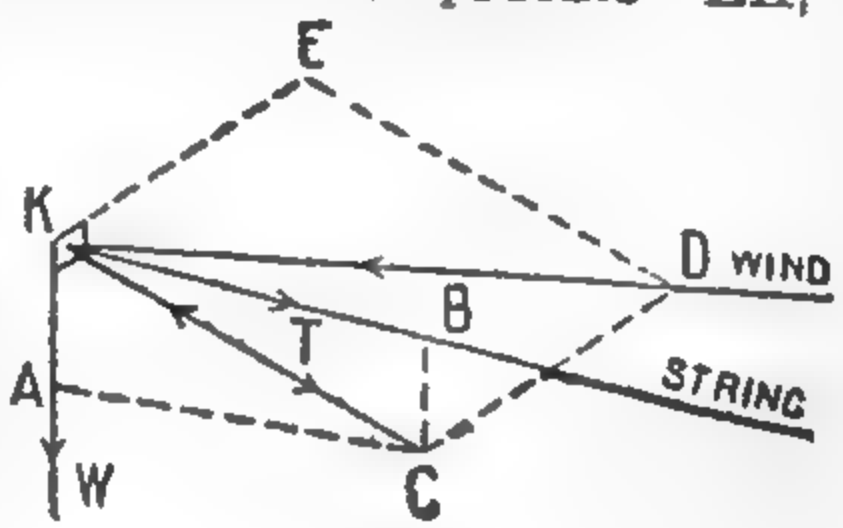

Fig. 60

If the wind pressure increases the normal component also increases and the kite rises up, until the resultant of T and W again becomes just equal and opposide to the normal compoment to produce equilibrium. The increase in the resultant takes place due to increase in the value of T and decreases of the angle AKB, as the kite rises. Again when the wind pressure decreases, the kite comes down. If the wind pressure remains the same and the string is let loose, the tension T decreases ; hence the resultant becomes *less* than the normal component of the wind pressure and so the kite will rise up. When the wind is gusty,

the magnitude and direction of its pressure change suddenly and kite moves irregularly. This is diminished by attaching a tail to the kite, which increases the inertia and opposes any sudden movement.

Near the ground, where the wind pressure is less, the man flying the kite is to run very fast to produce a sufficient wind pressure upon the kite, but when the kite rises to a height, the wind pressure is considerable and running may not be necessary.

36. Triangle of Forces.—If three forces, acting on a particle, be represented in magnitudes and directions by the sides of a triangle, taken in order, the forces are in equilibrium.

In fig. 39, the forces AB and BC acting at A can be represented by the sides AB and BC of the triangle ABC. By the parallelogram of forces, the resultant R of these two forces is represented by AC. Hence if a third force equal and opposite to R and represented in magnitude and direction by CA be impressed on the particle, it is bound to be at rest that is, the particle under the action of the three forces represented by the sides AB, BC and CA of the triangle ABC, taken in order, will be in equilibrium. The statement '*taken in order*' means that the direction of the three forces must go round the triangle, all in the same direction, clockwise or counter-clockwise. The triangle of forces can be verified by the same apparatus as is used to verify parallelogram of forces.

The *converse of Triangle of Forces* is also true and may be stated thus,—*If three forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle taken in order.*

Some Practical Illustrations.—The following cases illustrate the principle of triangle of forces.

A hanging Picture.—Fig 61 represents a picture suspended vertically by a cord passing over a single nail O and attached at the points A and B of the picture. The picture is in equlilibrium under the action of *three* forces, *viz.*, (*i*) the weight W of the picture (*ii*) the tension T_1 of the cord represented by AO and (*iii*) the tension T_2 of the cord represented by BO.

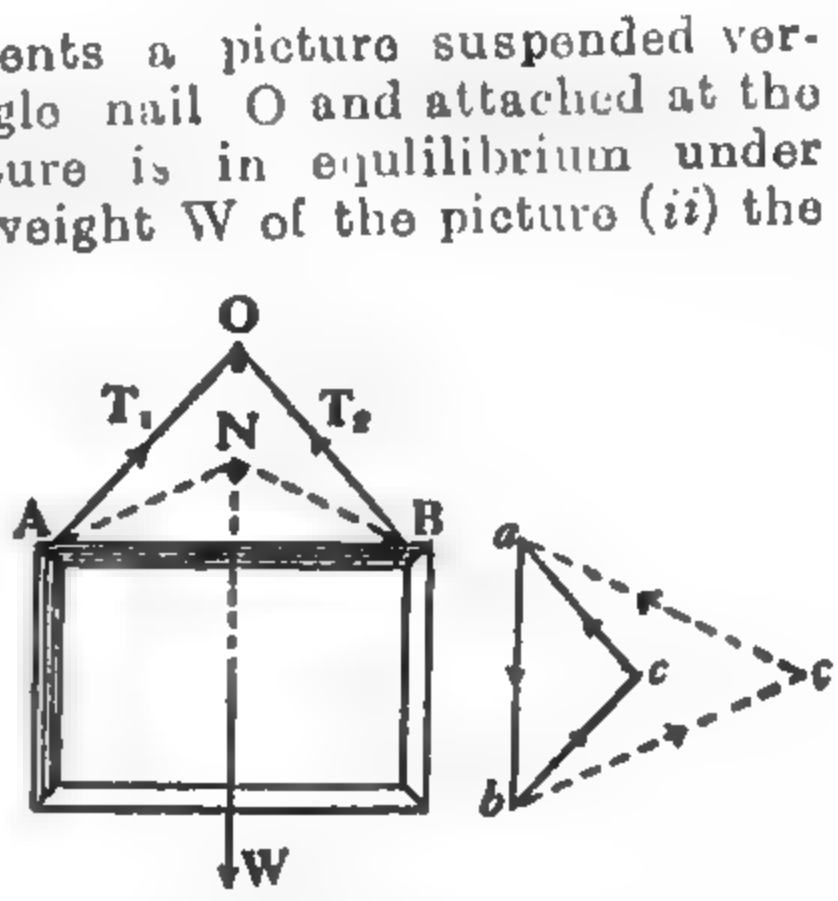

Fig. 61—Hanging Picture

Let *ab* represent the weight W of the picture in magnitude and direction. Draw *bc* parallel to AO and *ac* parallel to OB meeting each other at *c*. Then *bc* and *ac* represent tensions T_1 and T_2 respectively, as the same cord passes round O, tensions T_1 and T_2 are equal. If the cord is shortened, as indicated by ANB, the tension increases, for the triangle *abc* now takes the form of dotted triangle *abc*. Evidently if the cord is shortened too much, *it is more likely to break.*

A Simple Crane.—The top of the crane (Fig. 62) is in equilibrium under the action of *three* forces, *viz.*, (1) the load W hanging vertically downwards, (2) the thrust R of the jib and (3) the pull P of the tie-rope. Let *ab* represent W in magnitude and direction ; draw *bc* parallel to the tie-rope and *ac* parallel to the jib, meeting each other at *c*, then *bc* and *ac* represent P and R respectively. Thus the three forces being represented in magnitudes and directions by the sides of the triangle *abc*, taken in order, are in equilibrium. It is evident that heavier is the load pulled by the tie-rope, the greater is the thrust on the jib. So the jib rod must be very rigid and base of the crane must be very stout.

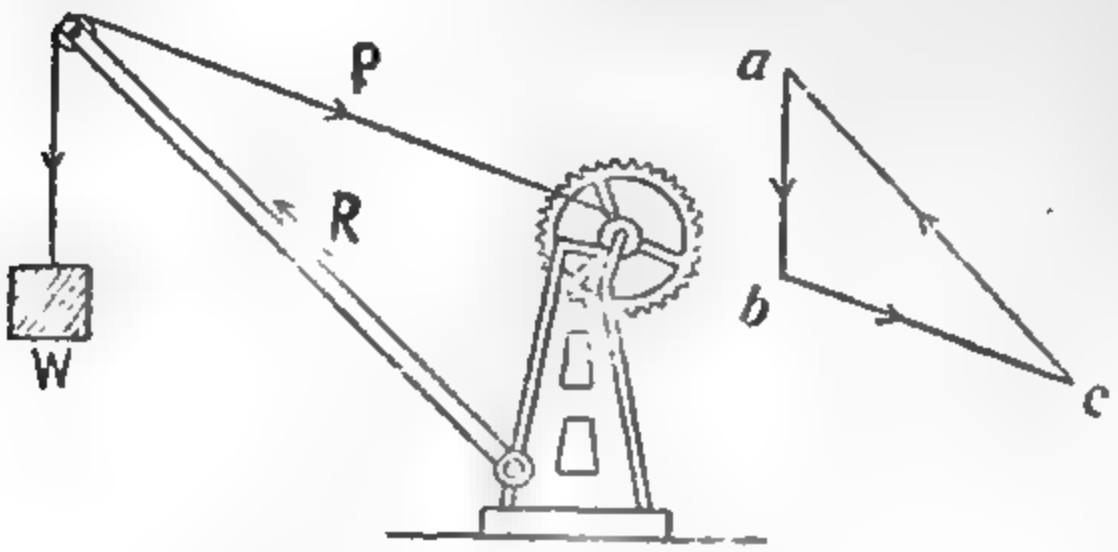

Fig. 62—A Simple Crane

37. Moment of a Force.—Let a body be fixed about a line or a point, about which it can rotate. A force properly applied to it may be able to rotate it about the fixed point or the line. The tendency of a force to produce rotation of a body is measured by the **moment** of the force.

In fig. 63 a rigid body is capable of rotating about an axis passing through the point O. The **rotatory effect** of a force in such a case depends on (*i*) the magnitude of the force and (*ii*) line of application of the force with respect to the axis or more properly the perpendicular distance of the line of action of the force from the *axis of rotation*.

Hence the *moment* of a force about an axis is measured by the product of the force and the perpendicular distance of its line of action from the axis. Thus if a body, whose axis of rotation is at O, be acted upon by a force F_1 and if OA be the perpendicular distance of its line of action from O, then the moment of the force is $F_1 \times OA$. The perpendicular distance is called the arm of the force. It is evident then the rotation is increased, if the force applied is increased or if the arm of the force is increased. Thus, it becomes easier to open or to close a door turning about its hinges, if the force is applied perpendicular to the surface of the door and as far away from the hinges as possible, Again, the moment of a force about a point vanishes, if (*i*) the force itself is zero or (*ii*) the line of its action passes through the point.

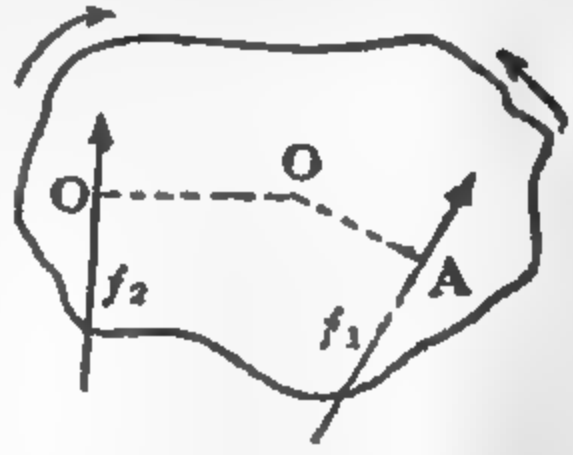

Fig. 63

According to the direction of rotation, the sense of the movement is described as *clockwise* or *anti-clockwise* and for mathematical expressions the former is usually taken as *negative* and the latter as *positive*. In the above figure the moment of the force F_1 is positive

and that of F_2 is negative. If the force and the distance from the axis are expressed in C. G.S. units, then the moment is given in a unit of *dyne cm*. In the F. P. S. system it is expressed as *poundal-foot*. The dimension of moment is the product of a force and distance or $MLT^{-2} \times L = ML^2T^{-2}$.

So if a rigid body movable about a fixed point, is acted on by two forces, such that their moments about the point are equal in magnitude but opposite in sense, the body remains at rest. If a body, under the action of any number of coplanar forces, be at rest, the *algebraic* sum of moments of all the forces about *any* point in the plane is zero.

*38. **Moment of Inertia.**—When a rigid body revolves about a certain axis, its momentum and kinetic energy may be expressed in terms of its angular velocity, if its mass be replaced by a factor known as the *Moment of Inertia*. If a body be imagined to be composed of n number of small fragments of masses $m_1, m_2, m_3 \ldots\ldots m_n$, at distances of $r_1, r_2, r_3 \ldots\ldots r_n$ from its axis of rotation, then,

$$m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots\ldots + m_nr_n^2 = \Sigma mr^2 \quad \ldots \quad (38,1)$$

where Σm = total mass of the body.

Here $\Sigma mr^2 = I$ = Moment of Inertia of the body about its axis of rotation. The unit of moment of inertia in C. G. S. is *gram. cm.*^{2} and its dimension is ML^2. If M be the mass of the body, then we can write $\Sigma mr^2 = Mk^2$ where the value of k depends upon the shape of the body and the position of its axis of rotation. The value of k is called the *radius of gyration* of the body about the given axis of rotation.

39. Parallel Forces.—Forces, whose lines of action are parallel, are termed *parallel forces*. Two parallel forces acting in the same direction are said to be **like** and those acting in opposite directions are termed **unlike** parallel forces.

(*i*) *Two like parallel forces.*—Let P and Q be two like parallel forces (Fig. 64) acting at A and B respectively. Join AB. The *magnitude* of the resultant R is evidently equal to the sum of the two forces, *i.e.* $R = P + Q$.

The *direction* of the resultant is parallel to that of either of the forces and its line of action lies between P and Q. It can be shown that the point of application of the resultant is a point G in AB, such that

$$P \times AG = Q \times BG \text{ or } \frac{P}{Q} = \frac{BG}{AG} \quad \ldots \quad (39,1)$$

i.e., the point G divides the line AB internally in the inverse ratio of the forces. By drawing aGb perpendicular to the lines the action of the forces, it can be shown that *the moments of P and Q about G are equal and opposite*.

(*ii*) *Two unlike parallel forces.*—Evidently the resultant is equal to the difference of the two forces and acts parallel to the direction of the greater force. It can be shown that the point of application of the

resultant passes through a point which divides AB externally in the inverse ratio of the forces.

Couple.—Two unlike parallel forces of equal magnitude constitute what is called a couple. The effect of a couple upon a body is to rotate it and the rotational tendency depends on (*i*) the magnitute of either of the forces constituting the couple, (*ii*) the perpendicular distance between the two forces. The perpendicular distance is called the **arm** of a couple.

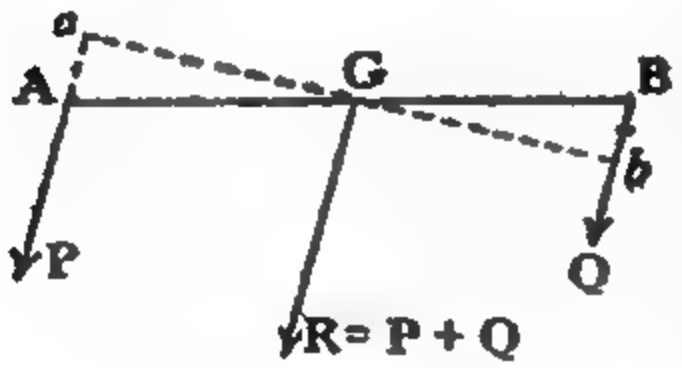

Fig. 64

Hence, the **moment of a couple** is measured by the product of one of the forces forming the couple and the perpendicular distance between their lines of action. This is also called the *Torque*. The moment of a couple in the anticlockwise direction is taken as *positive* and that in the clockwise direction as *negative*. A couple can be balanced by another couple when they act in the same plane or in parallel planes and the moment of each is equal and opposite to that of the other.

40. Application of Newton's Laws in Circular Motion.—Newton's laws of motion can be applied to bodies in circular motion. The laws may be stated thus.—

I. Every body continues in its state of rest or of uniform circular motion about an axis through its centre of mass, except in so far as it is compelled by some external impressed torque to change that state.

II. The rate of change of angular momentum is proportional to the applied torque and takes place in the same direction of rotation in which the torque acts.

III. To every rotational action there is an equal and opposite reaction.

Explanation.—The first law states that a material body cannot of itself change its state of rest or of *uniform circular motion*. If it is found to revolve, its speed of revolution persists. But if the speed of revolution is found to change, it must have been due to a torque acting upon it. Illustration of this law is obtained in continuous and uniform rotation of heavenly bodies. The speed of diurnal revolution of the earth remains the same always.

The second law gives a quantitative definition of a torque or the turning effect of a couple. If I be the moment of inertia of a body with reference to its axis of rotation and ω be its angular velocity, then the angular momentum of the body is measured by the product of the moment of inertia and angular velocity. Thus the angular momentum is $I\omega$.

Let a body having an initial angular velocity ω_1 be acted upon by a torque T for t seconds and let its final angular velocity be ω_2. Then change of angular momentum in t secs = $I\omega_2 - I\omega_1$.

∴ Rate of change of angular momentum.

$$= \frac{I\omega_2 - I\omega_1}{t} = I\frac{\omega_2 - \omega_1}{t} = I\alpha$$

where, α = angular acceleration of the body,

Thus according to the second law, $I\alpha \propto T$, or, $T = kI\alpha$. ... (40,1)

A unit torque is now defined to be that which produces a unit angular acceleration upon a body possessing unit moment of inertia. According to this definition, the constant k becomes unity. Hence $T = I\alpha$.

The third law illustrates the principle of conservation of angular momentum. Thus, if a revolving body of angular momentum $I_1\omega_1$ be coupled with another revolving body of angular momentum $I_2\omega_2$ then the *total angular momentum of the system* which is the sum of the two angular momenta *i e*, $I_1\omega_1 + I_2\omega_2$ *remains the same in any particular direction of rotation.*

Example :—

1. A flywheel, having a moment of inertia of 100 lb. ft.^{2} revolving at the rate of 400 r.p.m. is stopped by a brake in course of 5 seconds. Find the controlling torque. Calculate the force of friction on the brake when it is placed on the rim of the flywheel having a radius of 6 inches.

A. Initial speed = 400 revolutions per minute = $\frac{20}{3}$ revolutions per sec. = n

∴ Initial angular velocity = $2\pi n = \frac{2 \times 3{\cdot}14 \times 20}{3} = 41{\cdot}86$ radian/sec.

Final angular velocity = 0 and duration of interval $t = 5$ sec.

Angular acceleration $\alpha = \frac{41{\cdot}86}{5} = 8{\cdot}37$ radians per sec. per sec.

Thus, Torque $T = I\alpha = 100 \times 8{\cdot}37 = 837$ foot poundals.

Again, Force = Torque ÷ perpendicular distance = $\frac{837}{{\cdot}5} = 1674$ poundals.

41. Uniform Motion in a Circle.—Imagine a material particle to be moving uniformly in a circular path. At any intant at any point in its path, owing to its inertia, it has a tendency to move along the direction of the tangent at that point. For example, at the points A, B, C and D in its path of motion, it possesses tangential velocities represented by AM, BN, CP and DR respectively (Fig. 65). Illustrations of tangential motion are found in the following cases. Pieces of mud sticking to the rims of the wheels of a carriage are thrown off tangentially, when the wheels rotate rapidly. A piece of stone is tied at the end of a string and is whirled round over the head in a circle by holding the other end in the hand. If the string is suddenly released, it is found that the stone flies off in a straight line tangential to the circle. All these cases illustrate the fact that although speed over the circular path remains constant, the direction of motion is changing from point to point. Hence

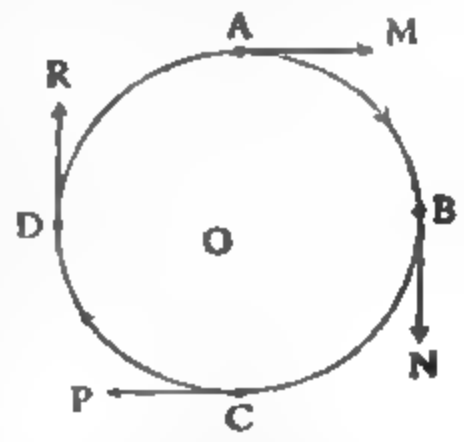

Fig. 65

there is a continuous change of velocity, as the particle moves over the circle. This change of velocity produces an acceleration towards the centre of the circular path.

Normal Acceleration in Uniform Circular Motion.—Let a particle move with uniform speed v along the circumference of a circle of radius r and having its centre at O. Let the particle describe a *small* arc AB (Fig. 66) in a *short* interval of time t.

At the point A, draw a tangent AD representing the direction of motion of the particle. Similarly draw another tangent BE at B. Let the two tangents meet at C. Cut off two equal lengths CD and CE from two tangents and let these represent velocities at A and B. Then the change in velocities during the period t in which the particle moves from A to B, according to triangle of velocities is represented in magnitude and direction by the third side DE.

Let $\angle AOB$ be θ ; further $\angle AOB = \angle DCE$

From C draw a perpendicular CF on DE. Since $\triangle DCE$ is isosceles and a perpendicular is dropped from the vertex C,—

$$\angle DCF = \angle ECF = \frac{\theta}{2}.$$

Now $DE = 2DF = 2CD \sin DCF = 2v \sin \frac{\theta}{2}$.

This value of DE represents the total change of velocity as it moves from A to B. If now we assume that the arc AB becomes smaller and smaller until it is a *very small arc* at A, the value of DE would then represent the change in velocity at A. Now as the arc becomes smaller, θ also assumes a smaller value and for an infinitely small arc $\sin \theta = \theta$ in circular measure. Thus the change in velocity

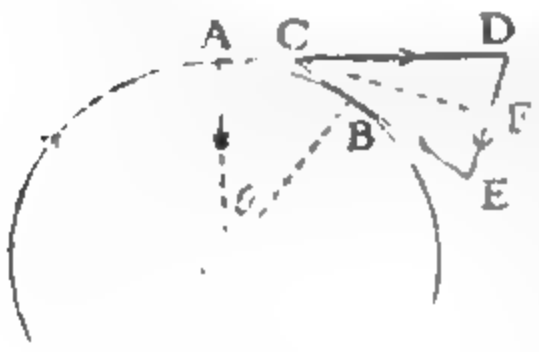

Fig. 66

$$\text{at A} = 2v \sin \frac{\theta}{2} = 2v \frac{\theta}{2} = v\theta,$$

$$\text{Thus acceleration at A} = \frac{\text{change in velocity}}{\text{time}} = \frac{v\theta}{t} = v\omega \qquad \ldots (41,1)$$

where ω is the angular velocity of the particle.

$$\text{Again } \angle ADF = 180° - \angle DFC - \angle DCF = 90° - \frac{\theta}{2}$$

When θ becomes infinitely small $\angle ADF$ is very approximately 90°, which means that the direction of acceleration sensibly coincides with AO. Hence the acceleration of the particle at any point in its path is towards the centre of rotation. This is called *normal* acceleration. Since $v = r\omega$,

$$\text{Normal acceleration} = v\omega = \frac{v^2}{r} = r\omega^2 \qquad \ldots (41,2)$$

42. Centripetal Force.—Whenever an acceleration of any mass in a certain direction is found, it must be assumed from Newton's

Second Law, that a force acts *in that direction*. Hence a force must be acting on the rotating particle in a direction towards the centre to generate its normal acceleration. The force, which acting on a body in a direction towards the centre of the circle makes it move in the circular path, is known as **the centripetal force** (Lat. *Peto*, to seek), because it always seeks or acts towards the centre. If the particle be of mass m, then the magnitude of the centripetal force is its mass × acceleration, which is mv^2/r. In the experiment with the stone and the string this force is exerted by the hand, placed at the centre of rotation on the revolving body through string in form of a tension towards the centre (Fig. 67).

It is to be carefully borne in mind that in all cases of a material body revolving in a circular path, the centripetal force is always to be supplied to the *revolving body* in a direction towards the centre. The nature of this *active* force may be any one of the following forms *e.g.* mechanical tension or thrust, gravitational attraction, electric or magnetic forces.

Since every action has an equal and opposite reaction, the particle moving uniformly in a circle exerts a force, directed away *from the centre* which is called the **centrifugal force** (Lat., *fugio*, to fly). The force is exerted by the rotating body at the centre of the circular path. In the experiment of the stone and the string, this force is exerted by the stone on the finger at O through the string, whereby the string is put under tension. It is to be noted that the centrifugal force is a *reaction* due to the centripetal force. With the stoppage of rotation, the centripetal force vanishes and so the centrifugal force ceases to act at once. The unit and dimension of a centrifugal force are similar to those of an ordinary force.

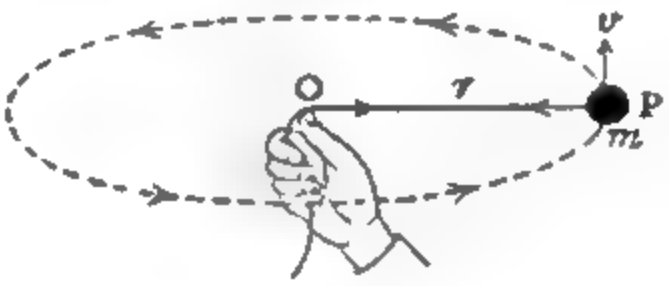

Fig. 67

Example :—

1. A stone of mass 200 gm. is tied with a thin string of length 50 cm. and is whirled round in a horizontal circle with centre fixed. It is found to execute 5 rounds in every two seconds. Find the normal acceleration and the tension of the string.

A. 5 rounds $= 5 \times 2\pi$ radians. $\therefore \quad \omega = \dfrac{10\pi}{2} = 5\pi$ radians/sec.

Linear speed v of the stone $= r\omega = 50 \times 5\pi = 250\pi$ cm./sec.

$\therefore$ Normal acceleration $= \omega^2 r = 25\pi^2 \times 50 = 1250 \times 9{\cdot}87 = 12337{\cdot}5$ cm/sec^2

Tension of the string = normal force $= 200 \times 12337{\cdot}5 = 2467{\cdot}5 \times 10^3$ dynes.

43. Some Practical Illustration of Centripetal Forces.—The following are a few cases of equilibrium in circular motion.

Motion of Planets.—It is well known that the planets revolve round the sun in definite orbits. For simplicity, let us assume that the

orbit is circular and that the sun forming the central mass is fixed in space. Let the mass of the sun be M and that of a planet, m revolving with a constant speed v in a circular orbit of radius r.

In order that the planet may execute uniform motion in a circle, a centripetal force mv^2/r must have to act on the planet in a direction towards the sun which means that *the sun has got to attract the planet with such an amount of force.* An investigation regarding the origin of this force led Newton to establish the universal law of gravitation, according to which there is mutual force of attraction between the sun and the planet of magnitude $G \times Mm/r^2$ (Chap. IV) where G is a constant. This is equal to the centripetal force mv^2/r acting on the planet.

$$\therefore \quad \frac{GMm}{r^2} = \frac{mv^2}{r} \text{ whence } M = \frac{v^2 r}{G} = \frac{v^2}{r^2} \times \frac{r^3}{G} = \frac{\omega^2 r^3}{G}$$

where ω is the angular velocity of the planet, which can be measured by an astronomical observation. Thus knowing r and G, the mass of the sun can be calculated. The example given above is one in which the centripetal force is of gravitational origin.

The picture of an atom is almost a miniature solar system. Within an atom minute electrically charged bodies, known as electrons, keep continually revolving round a central positively charged body, known as nucleus. In such cases the centripetal force supplied to the revolving electrons is of electrical origin.

All the above cases are illustrations of *dynamic* equilibrium ; the system as a whole does not change its configuration due to motion within it. If the motion stops, there is no centripetal force acting on the revolving body and the dynamic equilibrium is lost. The revolving body therefore comes directly to the central mass due to the mutual attractive forces.

Some very instructive conclusions may be derived out of the kind of equilibrium stated above. If, the body while rotating in a circular orbit, experiences an attracting force towards the centre greater than that required to keep it revolving in the particular orbit, the equilibrium would be lost and the body would be urged towards the centre with a force which is equal to the *difference of the attracting force and the centripetal force in that orbit.* The revolving body, therefore, having a constant tangential velocity throughout would come nearer the central mass in a path of gradually decreasing radius somewhat like a spiral. But during this process of advance as the distance r, diminishes, the centripetal force mv^2/r gradually increases ultimately when this force is again equal to the attracting force a fresh equilibrium sets and the body continues to revolve in a circular orbit. Therefore an increase in the force of attraction would make the body revolve in a circular orbit of a smaller radius, while a decrease of force would make it move over an orbit of a larger radius.

Bodies on the Surface of the Earth.—Let m be the mass of a body at any point on the equatorial region P on the surface of the earth

of mass M and let r be the radius of the earth in this plane (Fig. 68). Let the attractive force on the body exerted by the earth be w, when both are supposed to be at rest. This is the true weight of the body at that locality. When the earth revolves on its axis, the body shares an equal motion with it in a circular orbit of radius r with a speed v, say.

In order that m may revolve in a circular orbit, a centripetal force of magnitude mv^2/r would have to act upon m towards the centre. There being no other force present, this force *is derived out of the attractive force* w. Thus the apparent weight w of the body on the equatorial portion of the revolving earth is $w - mv^2/r$. The apparent weight of the body, is obtained with a spring balance. Therefore the loss in true weight of m depends upon the speed of rotation of m. Some suppose *from mathematical standpoint*, that this reduction is due to a force mv^2/r, always acting on m opposite to the direction of w, due to its circular motion. But this is not actually true from the physical point of view. Suppose that the speed of m is increased to such a value that $w = mv^2/r$, M will just cease to attract m and an infinitesimal increase in the circular motion of m would throw it off tangentially from any point on the circular path. This limiting condition is the dynamic equilibrium between two bodies, when one is revolving round the other.

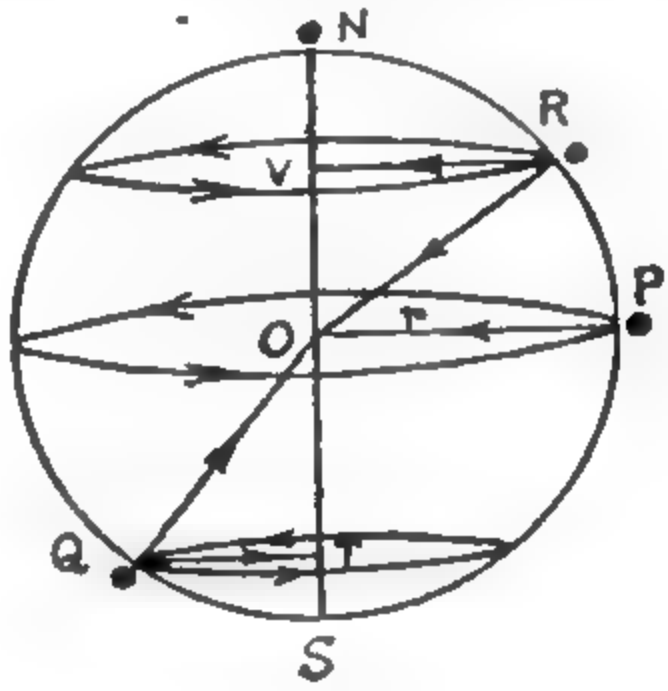

Fig. 68

For a discussion of apparent weight of the body placed at a region other than equatorial surface, the reader is referred to Art. 49, Chap. IV. If the body is placed at any higher latitude such as R, the body revolves round a smaller circle ; and consequently the tangential velocity to which it is subjected is smaller. Hence the part of the true gravitational force contributing to the centripetal force becomes less. At the poles this contributive force is nothing and hence there is no reduction in the weight of the body.

Motion of a Cyclist on a Circular path.—A cyclist while taking a bend along a circular path, automatically inclines his body out of the vertical towards the centre of the circular path. If the cycle and the rider be taken as one body of mass m, their weight mg acts vertically downwards (Fig. 69) through the centre of gravity (described later) of the system. The only other force is the *reaction* of the ground whose line of action passes through the track of the wheels and the centre of gravity of the system. The resultant of these two forces, for an equilibrium, is the necessary centripetal force generated. For simplicity, the ground reaction can be resolved into two components ; one, acting vertically upwards and the other P, acting horizontally towards the centre of curvature of the circular track. The vertical

component balances the weight of the system and the component P. gives the necessary centripetal force to the system to revolve in the circular path.

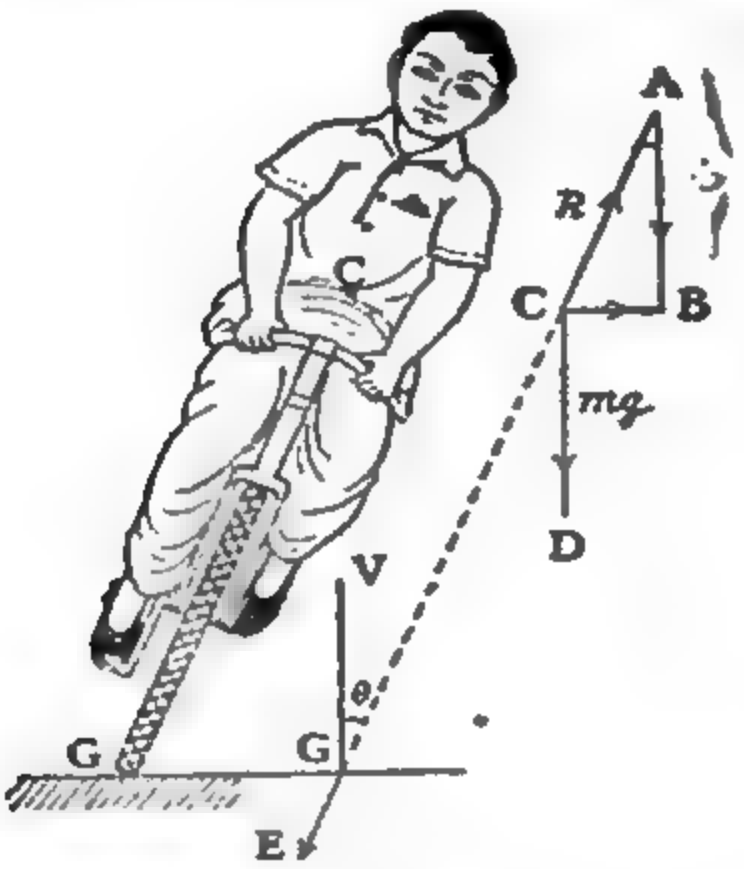

Fig. 69

Thus, the higher the speed or curvature, the greater is the centripetal force and the further has the cyclist to incline himself. If he leans too much or too little there will be a side slip. Similarly in a circus show when a horse is running along a circular path, both the horse and the rider are to incline their bodies inwards. If R be the ground reaction and θ the inclination of the cyclist to the vertical, then,

$$R \sin \theta = \frac{mv^2}{r} \text{ and } R \cos \theta = mg$$

$$\text{whence} \quad \tan \theta = \frac{v^2}{rg} \qquad \ldots (43,1)$$

Banking of Tracks.—The level at a bend in a railroad or in a racing track for motor cars is constructed a bit inclined so that the outer rail is a little higher than the inner one. The line HL is the level of a railway track at an inclination θ with the horizontal plane HH' (Fig 70). The reaction R of the rail on the wheel may be resolved into a vertical component DB and a horizontal component AD. The vertical component supports the weight of the carriage while the horizontal component supplies the necessary centripetal force mv^2/r.

If a small can filled with water is rapidly swung in a vertical circle the water will not fall down even when the bottom is at the uppermost position. In the laboratory, the crystals in a liquid are collected by putting the liquid into a pair of test tubes held in a suitable frame. As the frame is rotated rapidly, the test tubes revolve quickly and the solid particles, tending to fly away collect at the bottom of the test tubes. It is the principle of working of an apparatus known as a *centrifuge*.

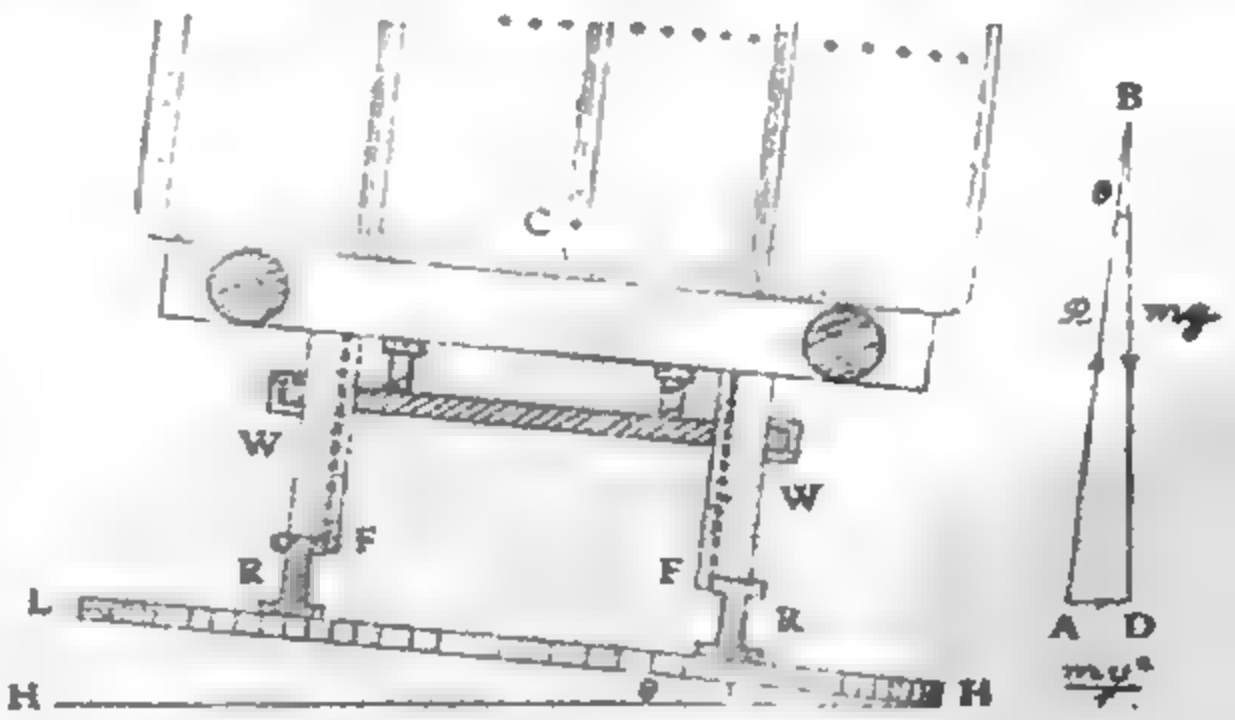

Fig. 70

A frame carrying a metal rod, along which two brass balls can slide, is taken. If the frame is rapidly rotated by means of a turning table, then any of the balls, if not exactly at the centre of rotation would move away towards the end.

Flattening of the Earth at the Poles.—The earth in its infancy consisted of a mass of molten matter. As the earth continued to revolve about its axis, every portion of matter forming the earth revolved,—the tangential velocity gradually increasing from the pole to the equator. To keep any portion revolving in a circular orbit, a centripetal force is necessary which is greatest at equator and is gradually decreasing towards the poles. The necessary centripetal force could not be supplied by gravitational and other forces of attraction at the equatorial region at the semi-solid stage, and as a result that portion of matter in tending to move away tangentially selected a greater distance from the polar axis, such that the reduced centripetal force was equal to the resultant attractive forces. Here a fresh equilibrium was set up. Consequently the earth became bulged at the equator and flattened at the poles. The outer layers of the earth, due to gradual cooling, have becomes solidified in course of time and have taken permanent flattened shape.

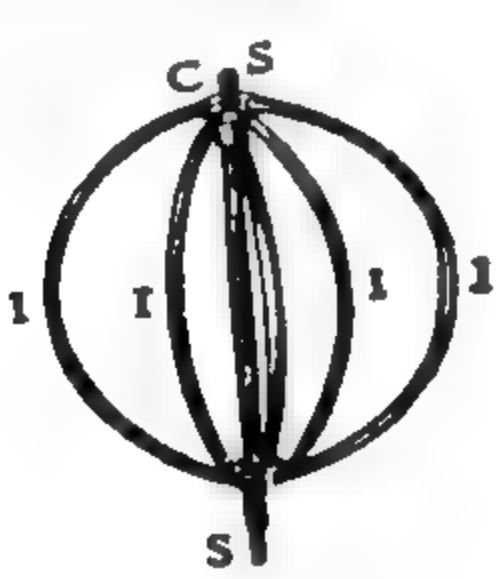

Fig. 71

Fig. 72

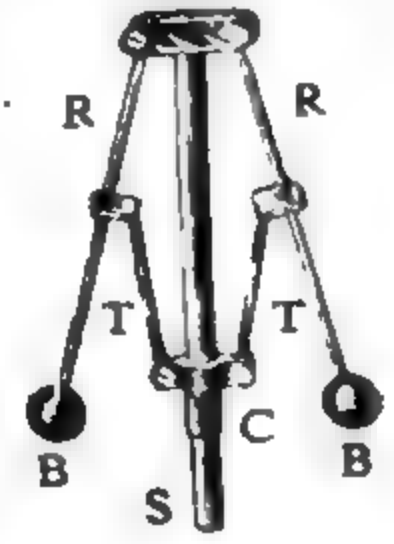

Fig. 73

The *model*, as shown in fig. 71, shows the nature of flattening of the earth due to rotation. It consists of a central rod SS and a circular hoop with some pieces of thin strips I attached to it. The hoop is fixed at the bottom and the top is attached to a collar C, which can slide up and down the rod. When put into rapid rotation, each particle of the strip tries to move outwards, as a result of which the collar slides down the rod; the extent of the sliding depends on the elasticity of the spring and the speed of rotation. Thus the hoop takes a form flattened at the ends and bulged in the middle (Fig. 72).

Speed Governor.—This is a device based on the principle of equilibrium in circular motion to regulate *automatically* the maximum speed of an engine. It consists of a vertical central spindle S at the top of which two other rods RR carrying balls BB are hinged. Each ball is partially supported by a rod T resting upon a collar C regulating a safety valve of the engine (not shown in the diagram). The weight of the balls supplies necessary thrust on the rods to close the valve (Fig. 73). As the speed of the engine increases, the vertical rod being geared with it, revolves more and more speedily. The balls also revolve about the rod as axis. The horizontal component of the thrust on the oblique rods resting upon the valve supplies the necessary centripetal

force for the circular motion of the balls. When the speed increases to such a limit that this component of the thrust is insufficient to keep the balls revolving in the particular path, the balls go farther apart raising the collar and thereby opening the valve partially. The gas pressure is thus partly released and the speed of the engine diminishes. Such automatic governors are always fitted with the revolving system of gramophone turn tables.

Examples :—

1. Supposing the sun to be a fixed star in space, calculate its mass in tons weight if the earth revolves round it on a circular orbit of radius 93×10^6 miles in a period of 365 days. [Given $G = 1{\cdot}066 \times 10^{-8}$ F. P. S.]

A. Angular velocity of earth $\omega = \dfrac{2\pi}{365 \times 24 \times 60 \times 60}$ radian/sec. $= {\cdot}2 \times 10^{-6}$

$\therefore$ Mass of the Sun $= \dfrac{\omega^2 r^3}{G} = \dfrac{{\cdot}04 \times 10^{-12} \times 93^3 \times 10^{18} \times 5280^3}{1{\cdot}066 \times 10^{-8}}$ lb.

$= 4263 \times 10^{27}$ lb. $= 1{\cdot}9 \times 10^{27}$ tons.

2. Calculate the inclination of a railway track at a bend of radius of curvature of 1000 ft. when the speed limit at the bend is assigned to be 45 miles per hour. [Given $g = 32$ F. P. S.]

A. $\tan\theta = \dfrac{v^2}{rg} = \dfrac{(45 \text{ m.p.h.})^2}{1000 \times 32} = \dfrac{66^2}{32000} = {\cdot}14$

Hence $\theta = \tan^{-1} {\cdot}14 = 8°$ approximately (from tan table).

ISAAC NEWTON (1642—1727)

Newton was born as a posthumous child at Woolsthorpe, Lincolnshire, England on the Christmas of 1642. At two years of age, he was left at the manor house under the guardianship of his grand mother since his mother married a second time. At the age of fifteen he was taken from Grantham grammar school and employed as a farmer, but for his inborn philosophic temperament he was readmitted to the school and in 1661, he entered the Trinity College, Cambridge.

The next six years of his life saw all the great discoveries. Kepler's Laws of planetary motion gave him the idea that some sort of attractive force must exist between any planet and the sun or a planet and its moon. The simple incident of the falling apple at the manor garden convinced him of the existence of some attractive force between earth and any material body on or near its surface. Thus he enunciated his famous Law of Gravitation.

Meanwhile in 1656, the great plague swept over England and he was compelled to retire to Woolsthorpe from Cambridge. In working out problems relating to gravitation, Newton found that mathematical equipments of those days were insufficient to tackle the relation between continuously varying quantities. He then set to develop mathematics and worked out the principles of Differential Calculus. He embodied all his findings in his immortal work "Principia" during the compilation of which, he got assistance from one of his great admirers, Edmund

Halley, a Professor of Mathematics at Oxford. The treatise appeared in July, 1687.

Newton afterwards devoted to the study of Optics and his discovery regarding the composite nature of light is of an outstanding character. He also developed a theory of propagation of light. His reflecting type of telescope came out in 1668. In 1669, he was employed as a Professor of Natural Philosophy at the Cambridge University. In 1672 he was elected a Fellow of the Royal Society. He was knighted in 1705. He died on March 20, 1727, at the age of eighty-five and was buried at Westminster Abbey.

Fig. 74—Isaac Newton

[Students are required to remember the following equivalence carefully]. Relation between a dyne and a poundal—**1 poundal = 13825 dynes.**

Summary

A force is that which, acting on a body, changes or tends to change the state of rest or of uniform motion of the body.

First Law.—*Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by external impressed force to change that state.*

Second Law.—*The rate of change of momentum of a body is proportional to the impressed force and takes place in the direction in which the force acts.*

Third Law.—*To every action there is always an equal and opposite reaction.*

Parallelogram of Forces.—*If two forces, acting simultaneously at a point, be represented in magnitude and direction by the adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

Triangle of Forces.—*If three forces acting at a point, be represented in directions and magnitudes by the sides of a triangle taken in order, their resultant vanishes.*

Uniform motion in a Circle.—A body of mass m moving with a uniform speed v in a circular path of radius r is subjected to a force mv^2/r always acting towards the centre.

EXERCISES ON CHAPTER III

Reference

Art. 29 1. State and explain the laws of motion.
(C. U.—1930; Dac. U.—1939, '40; U. P. B.—1942; Del H. S.—1952; *Cf.* Anna. U.—1950; P. U.—1945)

Art. 29 2. A 10 gm. bullet is shot from a 5 kgm. gun with a speed of 400 metres per second. What is the backward speed of the gun?
(Dac. U.—1935)

A. 80 cm. per sec.

Art. 29 3. A force of 100 dynes acts on a mass of 10 gm. for 5 secs. Calculate the change of momentum of the body.

A. 500 C.G.S. (Del. U.—1937, P. U.—1941)

Art. 29 4. What is meant by the term force? Define the units in which the force in measured.

Art. 29 5. A motor car of mass 400 lbs. is moving with a velocity of 30 miles per hour. By the application of brakes it is brought to rest in a distance of 40 ft. Find the average force resisting the motion.
(Del. U.—1943)

A. 9680 poundals.

Arts. 29 & 30 6. State Newton's second law of motion and show $P = mf$. Define a unit of force.
(Mad. U.—1951; Nag. U.—1950; Pat. U.—1925, '26, '30; P. U.—1932)

Arts. 29 & 31 7. Explain how Newton's second law of motion may be verified experimentally.
(Mad. U.—1950, '51; *Cf.* And. U.—1952)

Arts. 29 & 32 8. State Newton's laws of motion and explain how a carriage drawn by a horse moves on a level road.
(Pat. U.—1952)

Arts. 29 & 32 9. State Newton's third law of motion and explain it carefully. Show how this leads to the principle of conservation of momentum.
(Pat. U.—1953)

Art. 32 10. An one ounce rifle bullet leaves the rifle with a velocity of 100 ft. per second, the rifle tending to recoil with a velocity of 2 ft. per second. Find the mass of the rifle. Explain the principle employed.
(P. U.—1930)

A. 3·125 lb.

Arts. 30 & 33 11. Define impulsive force and explain how the impulse of an impulsive force is measured.
(Nag. U.—1951; *Cf.* Utkal.—U.—1951)

Arts. 33 12. A smooth sphere of mass m impinges obliquely on a fixed horizontal plane. If the co-efficient of restitution be e, find the velocity

Reference

and direction of the sphere after impact. Find also the impulse of the blow.

If the masses of two balls be as 2 : 1 and their respective velocities in the same line before impact be as 1 : 2 and in opposite directions and e be 5/6, show that each ball will after impact move back with 5/6 of its original velocity. (Nag. U.—1952)

13. Show that two spheres will interchange their velocities, if they are of equal mass and are perfectly elastic. (Utkal. U.—1951 ; Pat. U.—1953) Art. 83

14. State, explain and prove the principle of the parallelogram of forces. Art. 34

The wind blows from a point intermediate between north and east. The northerly component of its velocity is 5 miles per hour and the easterly component is 12 miles per hour. Find the total velocity. (C. U.—1934)

A. 13 miles per hour.

15. Four forces act at a point. The first is of 500 dynes acting due south the second of 50 dynes acting due west, the third of 400 dynes due north and the fourth of 100 dynes acting due east. What is the magnitude and direction of the resultant force ? Art. 84

A. 112 dynes in a direction 63°20′ S. of E.

16. Enunciate the law of parallelogram of forces and describe an experiment to verify it. (Anna. U.—1950 ; Mad. U.—1951 ; And. U.—1950) Art. 84

17. Explain why it is easier to pull a lawn-roller than to push it. (U. P. B.—1941) Art. 85

18. Explain with the aid of a diagram the flight of a kite. (P. U.—1927, '31) Art. 85

19. Enunciate and prove the proposition known as the triangle of forces. (Nag. U.—1952) Art. 86

20. Show that the resultant of two forces which act at a point diminishes as the angle between their lines of action increases. Arts. 84 & 86

Three forces in equilibrium act perpendicularly to the sides of a triangle ABC. Show that the forces are proportional to Sin A, Sin B and Sin C. [And. U.—1952]

21. A uniform bar AB is 4 ft. long and has weights of 10 lb. and 20 lb. hanging at 1 ft. and 3 ft. from the end A. If it balances at a point 1 ft. 9 in. from the end B, what is the weight of the iron bar ? (P. U.—1950) Art. 89

A. 10 lb.

22. Define moment of a force and that of a couple. (Nag. U.—1952 ; *Cf.* P. U.—1950) Art. 87

23. If a stone held by a string is whirled around uniformly in a horizontal circular path, find an expression for the acceleration which is produced. (Del. U.—1950 ; *Cf.* C. U.—1942 ; *Cf.* P. U.—1951 ; Del. H. S.—1954) Art. 41

Reference

Calculate the angular velocity in radians per second of a particle which makes 900 revolutions per minute. What is the linear velocity if the radius is 4 ft.

A. 10π radians/sec. 40π ft./sec. (C. U.—1942)

Art. 41 24. A body of mass 77 gm. travel in a circular path of radius 70 cm. with a uniform speed of 23 cm. per sec. What is the force that must act on the body to maintain the motion?

A. 581·9 dynes.

Art. 42 25. What are the centripetal and centrifugal forces and what are their relations with a body moving on a circular orbit?
(Del. H. S.—1951; Del. U.—1950; P. U.—1932; C. U.—1939; *Cf.* And. U.—1951; *Cf.* Pat. U.— 1952)

Art. 42 26. A stone of mass m is moving round a horizontal circle of radius r with a uniform speed v. Show that the force acting on the stone is mv^2/r. What is the direction of the force? (Del. U.—1942; Cf. Pat. U.—1952)

Art. 41 27. A stone weighing 6 lb. is rotated in a circle of radius 1 yd. with a speed of 10 ft./sec. Calculate the centripetal force in pounds weight.

A. 0·25 lb. weight. (P. U.—1951)

Art. 48 28. Explain the following statement bringing out the scientific principles involved. "If a small can filled with water is rapidly swung in a vertical circle, the water does not fall down." (U. P. B.—1941)

Art. 43 29. A motor racing track is in the form of a circle of diameter 400 yards and is banked so that the reaction between the wheels and the ground is normal to the track when the cars are travelling at 60 miles an hour. What is the slope of the track. If a car has a mass of 1 ton, what is the centrifugal force on it when going round this track at 60 miles per hour?

A. $\text{Tan}^{-1}\cdot 403$; $2\cdot 89 \times 10^4$ poundals.

Art. 43 30. A cyclist riding at a speed of 15 miles an hour takes a turn round a circular bend of radius 44 ft. What is his inclinnation to the vertical? ($g = 32$ ft./sec^2). (Pat. U.—1952)

A $\tan^{-1}\frac{11}{32}$

Art. 48 31. Enumerate the external forces which are acting on a bicycle and its rider when going round a curve, on a level road, with uniform speed and show how the acceleration is produced. Illustrate your answer with a diagram.

Calculate the angle which the bicycle and its rider must make with the vertical when going round a curve of 22 ft. radius at 10 miles an hour. What is the least value of the coefficient of friction between the tyres and the road which will prevent slipping? If the bicycle and the rider have a mass of 200 lbs. what frictional force must the ground exert on the wheels? (Gau. U.—1953)

A. $\tan^{-1}\frac{11}{98}$; $\frac{11}{80}$; $61\frac{1}{9}$ lb.

CHAPTER IV

GRAVITATION AND GRAVITY

44. Sciences in Ancient India.—During the period from the second century to the twelfth century A.D. there was a marked progress in India of the observational astronomy and its allied calculations Although there is no systematic record of the observational data, but from the results of the calculations, it reveals that Hindus of ancient India were in a very advanced stage in Astronomy, Medicine and Mathematics.

Of those who came to the forefront in Astronomy, the names of Arya Bhatta, Brahma Gupta and Bhaskar Acharya are most prominent. Arya Bhatta, a Hindu astronomer, observed in his Tantra sometime in fifth century A.D. that the planets revolve round the sun in elliptic orbits. He calculated the periods of revolutions of the planets such as Mercury, Venus, Earth, Mars, Jupiter and Saturn. His results have been found very nearly correct by modern astronomers. He also discovered that the earth is round and that the day and night effects on the earth are due to its diurnal motion. His findings were later on verified on telescopic observations by Kepler early in 17th century.

The greatest exponent of Hindu astronomy, Bhaskar Acharya flourished during 12th century. He compiled one treaties "Siddhanta Shiromani", which deals with various aspects of Astronomy and Mathematics. Here he calculates the possible height of the atmosphere which came out to be about 12 jojans (about 55 miles). The most astounding of all is his discovery of gravity and gravitation, which he explains in the following lines*,—

"Whenever any object appears to fall to the earth, it is due to the attraction of the earth on it. It is this mutual attraction between heavenly bodies which does not allow any one to leave its respective positions or orbits." This law was independently discovered by Newton in the later part of 17th century.

Such was the ancient glory of India, which faded gradually from the 13th century and became nearly extinct within the next four hundred years. This dwindling of Scientific culture was due to lack of the support of the State and co-ordinated enterprises to form schools of thought.

45. Law of Gravitation.—Johaan Kepler, after a continuous study of a motion of planets round the sun, came to the conclusion, that the orbit of a planet is not circular but is slightly elliptical with the sun at one focus. He formulated the laws of planetary motion. Newton in an attempt to explain the laws discovered the *Universal Law of Gravitation*, as stated below.

---

* आकृष्टि शक्तिश्च मही तया यत् । तत् स्वस्थं गुरु स्वाभिमुखं स्वशक्त्या ।
आकृष्यते तत् पततीव भाति । समे समन्तात् क्व पतत्वियं खे ।।

Every particle of matter in the universe attracts every other particle at all distances and in the line joining them, with a force which varies directly as the product of there masses and inversely as the square of the distance between them.

Fig. 75

If F is the force of attraction between two bodies of masses m and m' placed at a distance r from each other (Fig. 75),—

then $F \propto mm'$ also $F \propto \frac{1}{r^2}$

$$F \propto \frac{mm'}{r^2} \quad \text{or} \quad F = G\frac{mm'}{r^2} \qquad \ldots \quad (45,1)$$

where G is called the **gravitational constant,** which is independent of the nature of bodies. If we consider two bodies, each of unit mass and placed at a unit distance apart, the force of attraction between these bodies would obey the inverse square law. Then according to eqn. (45,1).

If $m = m' = 1$ and also $r = 1$, then $G = F$.

In other words, the gravitational constant G is numerically equal to the force of attraction between two bodies, each of unit mass and placed at a unit distance apart. The value of G, as accepted at present, is $6{\cdot}6576 \times 10^{-8}$ C. G S. unit, which was first accurately obtained by C. V. Boys in 1895. The determination has been carried to a still higher degree of precision by Heyl.

46. Determination of Gravitational Constant.—The first direct determination of G was made by Henry Cavendish in 1798. The apparatus consisted of a thin and light rod R of length of about 8 ft. from the ends of which two small identical lead balls *bb* of 2 inches in diameter, were suspended (Fig. 76). The rod was also suspended by a torsion wire FC from a support. Two large lead balls BB, each of 12 inches in diameter, could be placed on opposite sides of the smaller ones at equal distances, so that the centres of all the balls were on a horizontal plane. When the bigger balls were so placed as shown in the figure, the smaller ball was attracted by the nearer bigger ball due to gravitational pull in a horizontal direction joining the centres of the respective balls.

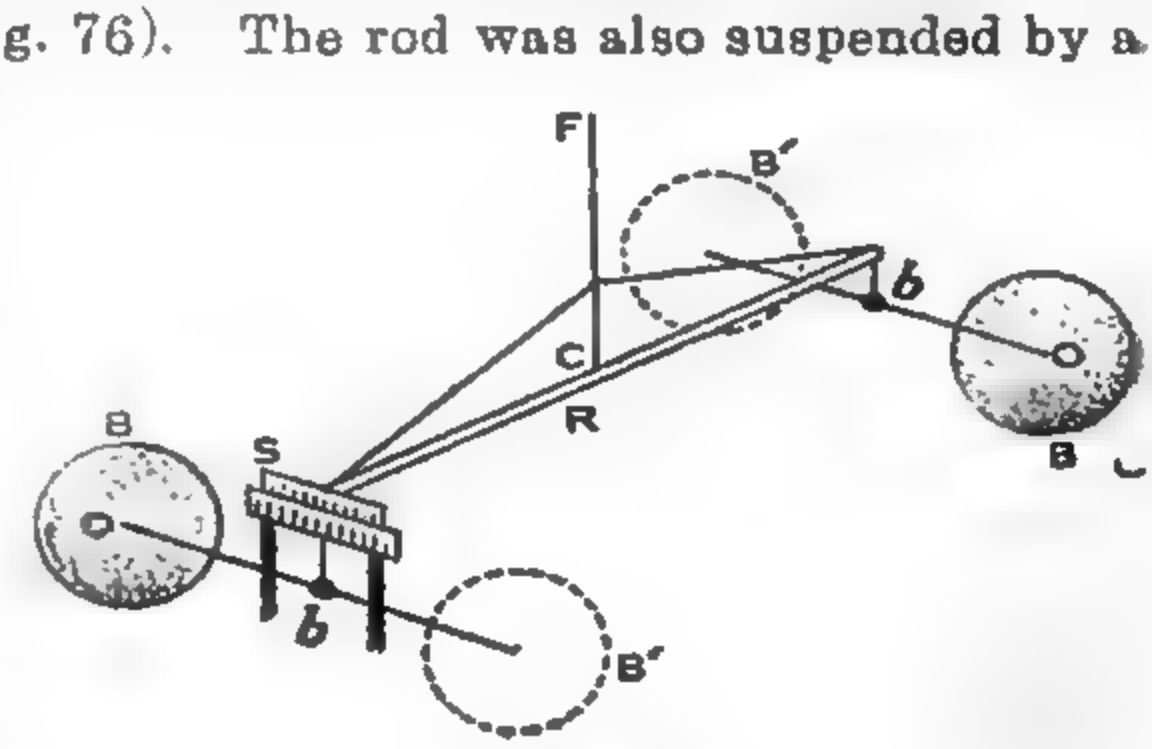

Fig. 76

The principle of measurement is as follows. The bigger balls are at first placed far away so as not to affect the smaller ones and the

position of the suspended rod is read with reference to the scale S. Then the larger balls are placed in position as shown, at equal distances from the corresponding small balls. Due to the attraction on the two small balls, a couple is exerted on the rod and this couple moves the rod through a small angle twisting the suspension wire. The angle of rotation can be measured by knowing the length of the rod and the amount of its movement on the scale S.

If d be the distance between the centres of a large ball and the corresponding smaller one, M and m the masses of the balls, then the attracting force at each end of the rod is $\frac{GmM}{d^2}$. The two equal, parallel and oppositely directed forces constitute the deflecting couple on the rod whose moment is $\frac{GmM}{d^2} \times l$ where l represents the length of the rod. If the rod is deflected through an angle θ, then the restoring torque due to twisting of the wire is $T\theta$, where T is the torque per unit twist. Thus for equilibriuin,

$$G\frac{Mm}{d^2}l = T\theta \text{ whence } G = \frac{T\theta d^2}{Mml} \quad \ldots (46,1)$$

Thus knowing all the terms on the right hand side of the equation, the value of G may be found.

Examples :—

1. In one of Cavendish experiments each of the larger lead balls had a mass of 30 Kgm. and was placed 40 cm. from the corresponding small balls each of mass. 200 gm. The length of the suspension rod was 3 metres. The deflection on the scale was found to be ·15 mm. If the restoring torque of the suspension fibre be 750 dynes per radian, calculate the gravitational constant.

If θ be the angular deflection of the suspension rod, then according to the condition of the experiment $\tan\theta = \frac{\text{linear deflection of one end}}{\text{half the length of the rod}} = \frac{\cdot 015}{150} = 10^{-4}$

The value being so small, $\tan\theta = \theta = 10^{-4}$ radian

According to the eqn (46,1)

$$G = \frac{T\theta d^2}{Mml} = \frac{750 \times 10^{-4} \times 40^3}{30 \times 10^3 \times 200 \times 300} = \frac{75 \times 16 \times 10^{-1}}{180 \times 10^7} = 6 \cdot 66 \times 10^{-8} \text{ C.G.S. unit.}$$

2. Calculate the gravitational force in poundals between two masses each of 1 lb. placed 1 foot apart, given that $G = 6 \cdot 6576 \times 10^{-8}$ C.G.S. Two persons each of mass 140 lb. are sitting side by side with their centres of mass 2 ft. apart. Calculate the gravitational pull between them.

A. 1 lb. $= 453 \cdot 59$ gm. and 1 foot $= 30 \cdot 48$ cm.

$$\therefore F = 6 \cdot 6576 \times 10^{-8} \frac{453 \cdot 59 \times 453 \cdot 59}{30 \cdot 48 \times 30 \cdot 48} \text{ dynes} = \frac{6 \cdot 6576 \times 10^{-8}}{13825} \times \frac{453 \cdot 59^2}{30 \cdot 48^2} \text{ poundal} = 1 \cdot 0672 \times 10^{-8} \text{ poundal}$$

Thus when masses are expressed in lbs. and distance in feet, the value of $G = 1 \cdot 0672 \times 10^{-8}$ F.P.S. unit.

According to the condition of problem, $m_1 = m_2 = 140$ and $r = 2$ ft.

$\therefore$ $F = 1{\cdot}0672 \times 10^{-9} \times \frac{140^2}{2^2} = 5{\cdot}3 \times 10^{-9}$ poundal.

47. Gravity.—Gravitation is a general term, indicating the force of attraction between any two bodies. In the particular case, between the earth and some object lying on or near the surface of the earth, the force of attraction is termed *gravity*. As the mass of the earth is very very large in comparison to the mass of any terrestrial object, earth's attraction predominates and due to the mutual attraction, the object is pulled in a direction towards the centre of the earth.* If the mass of the earth be M, the mass of an object on or near the surface of the earth be m and if the distance between the centres of earth and the body be d,—

Then the *force of gravity* $F = G\frac{Mm}{d^2}$ ... (47,1)

It is due to this force of attraction that all bodies show their weights and tend to fall downwards. For a similar reason a plumb line, which consists of a long piece of thread with a bob at one end, when suspended, is attracted *towards* the centre of the earth and it hangs in a perfectly vertical line.

Acceleration due to Gravity.—We know that a force acting on a body produces acceleration on it. So the force of *gravity* produces acceleration on all bodies falling towards the surface of the earth.

From eqn. (47,1) force of gravity F on a mass $= \frac{GMm}{d^2}$

$\therefore$ Acceleration produced on the mass $= g = \frac{F}{m} = \frac{GM}{d^2}$ ... (47,2)

The *acceleration due to gravity* is usually represented by the letter 'g'. As G and M are constants, the value of g remains constant, so long as d remains the same. Remembering that d is the distance between the body and the centre of the earth, *the value of 'g' is constant at all equidistant places from the centre of the earth.* The value of g at any place can be determined by Atwood's machine or more accurately by a pendulum.

Variation of g.—It is evident from eqn. (47,2), that g at any place varies inversely as the square of its distance from the centre of the earth. Hence the value of g, depends upon the height above the sea level. But the *variation* in g for ordinary differences of height is inappreciable. Again owing to the fact that the earth is not perfectly round, all points on its surface are not at equal distances from its centre,—the *equatorial radius* is the longest while the *polar radius* is shortest. Hence the value of 'g' varies *slightly* with the *latitude* of

* In considering the gravitational effect of the earth upon bodies lying on or outside its surface, the whole mass of the earth may be supposed to be concentrated at its centre.

the place on the surface of the earth. It is least at the equator and gradually increasing becomes maximum at either of the poles. This fact was first verified by Edward Halley, a British astronomer, by a pendulum in 1677, during his voyage from London to St. Helena. The variation over the earth's surface, however, is very small ; the value of g at the equator is about 978 cm. per sec ^{2} and that at the poles is about 983 cm. per sec ^{2}. The *mean value of g* is taken to be 981 cm. per sec.^{2} or 32·19 feet per sec.^{2} at latitude 45° and at sea level, which is accepted as the standard of reference.

48. Weight of a Body.—The weight of a body is due to the attractive force on the body by the earth. If the mass of the body is increased, it is evident from eqn. (47,1) that the force of attraction would increase in the same proportion. Hence *the weight of a body at a place is proportional to its mass.* Again force = mass × acceleration ; so the weight W of a body of mass m is given by,—

$$W = mg \quad \ldots (48,1)$$

Mass and Weight.—The word 'weight' is sometimes loosely used to denote mass of a body but *mass* of a body is the quantity of matter contained in it while *weight* is the force with which it is attracted towards the centre of the earth. The *mass* of a body remains constant at all places, but its *weight* varies *very slightly* from place to place on the earth's surface with the variation of g. The variation in the weight of a body is dealt with elaborately in next Art.

Gravitational Unit of Force.—This is the force with which a unit mass is attracted by the earth towards its centre and is equal to the weight of a unit mass.

Hence in the F. P. S. system, the gravitational unit of force
= the weight of 1 lb = $1 \times g = 1 \times 32{\cdot}19$ or $32{\cdot}2$ poundals.

In the C.G.S. system, the gravitational unit of force
= weight of 1 gm. = $1 \times g = 1 \times 981 = 981$ dynes.

1 dyne = weight of 1 milligram nearly.

∴ 1 lb wt. = g poundals. Also 1 gm. wt. = g dynes, or we can say that a gravitational unit of force = g × absolute unit of force.

So m lb wt. = mg poundals and m gm. wt. = mg dynes.

Again 1 *poundal* = $\frac{1}{32{\cdot}2}$ × wt. of 1 lb = wt. of $\frac{16}{32{\cdot}2}$ oz.

= *wt. of half an ounce nearly.*

As the value of g varies on the surface of the earth, the gravitational unit of force is not constant at all places.

Example :—

1. A car of mass 1 ton is moving with a speed of 30 miles per hour. A constant force of 28 lb. opposes its motion. Calculate (*a*) the time required to bring the car to rest and (*b*) the distance it will travel in stopping. (Mys. U.—1951)

A. Mass of 1 ton $= 28 \times 4 \times 20$ lb. and a force of 28 lb. $= 28 \times 32{\cdot}2$ poundals.

$\therefore$ Retardation f of the car is obtained from the relation, $P = mf$

or, $28 \times 32{\cdot}2 = 28 \times 4 \times 20 \times f$ whence $f = \dfrac{32{\cdot}2}{4 \times 20}$ ft. per sec.^{2}.

(*a*) Initial velocity of the car $= 30$ *m.p.h.* $= 44$ ft. per sec. When at rest, final velocity is taken to be zero. Hence,

$$0 = 44 - \frac{32{\cdot}2}{80} t \quad \text{whence} \quad t = 109{\cdot}3 \text{ seconds.}$$

(*b*) $0^2 = 44^2 - 2 \times \dfrac{32{\cdot}2}{80} \times s$ whence $s = 2405$ ft.

49. Variation in the Weight of a Body.—As the weight of a body is proportional to g, it varies from place to place on the surface of the earth and at different altitudes.

(1) *Weight of a body at a Height above the surface of the Earth.*— If the mean radius of the earth be r and its average density is ρ, then the mass M of the earth is equal to the product of its volume and density. Thus, $M = \frac{4}{3}\pi r^3 \rho$.

Let a mass m be placed at a height h above the surface S of the earth (Fig. 77). The force of attraction due to gravity of the mass or the weight of the body is given by

$$F = G\frac{mM}{(r+h)^2} = G \times \tfrac{4}{3}\pi r^3 \rho \times \frac{m}{(r+h)^2} \quad \ldots \quad (49{,}1)$$

Let the acceleration due to gravity at a height h be g_h,

$$\text{Then} \quad g_h = \frac{F}{m} = G \times \frac{\frac{4}{3}\pi r^3 \rho}{(r+h)^2} \quad \ldots \quad (49{,}2)$$

Hence the acceleration due to gravity and consequently the weight of the body decreases with height from the surface of the earth, being inversely proportional to the square of the distance of the body from the *centre* of the earth. Thus at a height of 4000 miles above the surface of the earth a body would weigh $\frac{1}{4}$ of its normal weight. At the top of a mountain 4 miles high a mass of 1000 gm. would weigh about 998 gm.

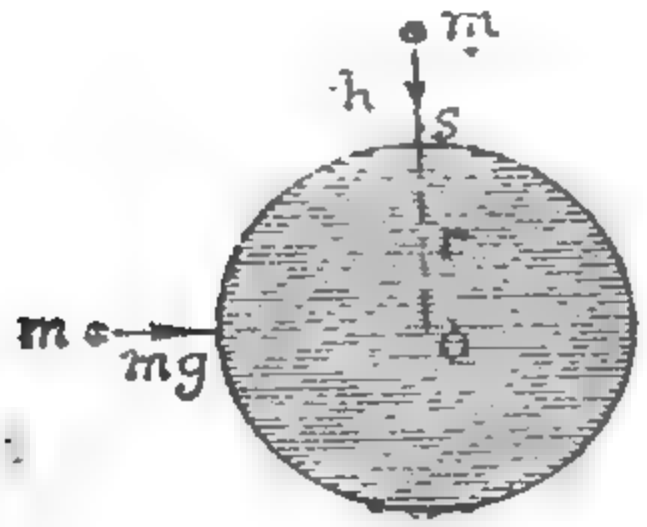

Fig. 77

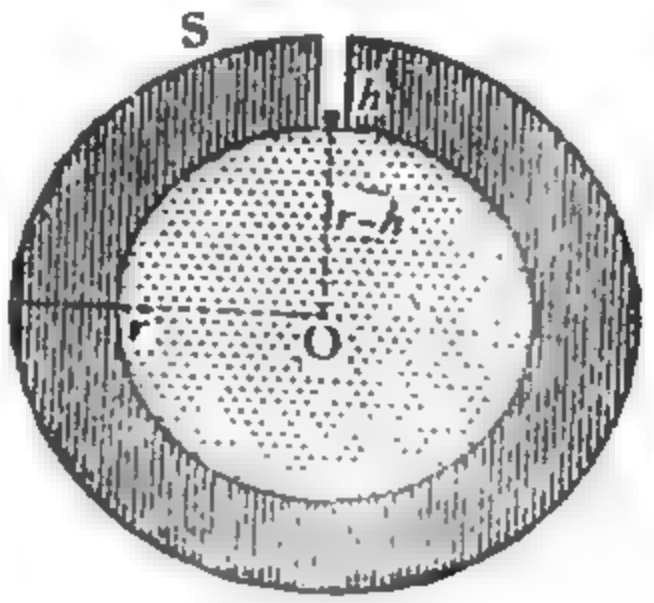

Fig. 78

(2) *Weight of a body at a Depth below the surface of the Earth*—

Let us take a body of mass m, say at the bottom of a mine at a depth h below the surface of the earth (Fig. 78). Let r be the mean radius of the earth. Imagine a sphere, concentric with the earth, to be drawn with a radius equal to $(r - h)$. The earth may now be considered to be divided into two portions—one, an inner sphere of radius $(r - h)$ and the other, a hollow outer shell of thickness h. Then the given body lies on the surface of this inner sphere but just inside the hollow outer shell.

As the gravitational force inside a hollow shell is zero, the force of attraction due to this portion of the earth is nothing. Hence the force of attraction on the given body would be only due to the inner solid sphere of radius $(r - h)$, being directed towards the centre of the earth.

Now mass of the inner sphere $= \frac{4}{3}\pi(r - h)^3\rho$

$\therefore$ the force of attraction on m due to this sphere which is also the weight of the body at this region is given by,—

$$F = G\tfrac{4}{3}\pi(r - h)^3\rho \times \frac{m}{(r - h)^2} = \tfrac{4}{3}\pi G(r - h)\rho m \quad \ldots \quad (49,3)$$

So the acceleration due to gravity g_d at a depth h is given by

$$g_d = \frac{F}{m} = \tfrac{4}{3}\pi G(r - h)\rho. \quad \ldots \quad (49,4)$$

Hence the *acceleration due to gravity* inside the earth varies *directly* as its distance from the centre of the earth. Thus, if we go farther and farther inside the earth, the value of g gradually decreases. Hence, at the *centre of the earth*, the acceleration due to gravity is zero and there is no weight of the body.

*(3) *Variation in the weight of a Body with different Latitudes on the surface of the Earth.*—The weight of a body varies with latitude on the surface of the earth for two reasons :—

(*i*) The peculiarity in the shape of the earth.

(*ii*) The rotation of the earth about its axis.

The radius of the earth is longest at the equator and it decreases continuously with latitude becoming least at the poles. The polar radius is less than the equatorial radius by about $13\frac{1}{2}$ miles. Consequently, the force of gravitational attraction is greatest at the poles and least at the equator, and so the weight of a mass increases from the equator to the poles in the same proportion.

Again, owing to the diurnal rotation of the earth about its axis, every body on its surface revolves in a circular orbit. We also know that for a circular motion to take place a centripetal force must have to act on the body. A *small* part of the gravitational force of the earth is utilised in supplying the necessary centripetal force. The former thereby falls short of the required amount. Therefore the observed weight of the body is less than its true weight.

Due to the diurnal rotation of the earth about its axis, a point near the pole describes a circle of smaller radius than one near the equator. Suppose that NS represents the polar axis of the earth (Fig. 79). Since rotation of all parts is completed in one day, a body at a point P near the equator would move on larger circle than a body at a point Q nearer a pole. So the former would move with a greater linear speed. Hence the centripetal force is greatest at the equator and zero at the poles. Therefore the loss of weight due to the diurnal rotation is least at the

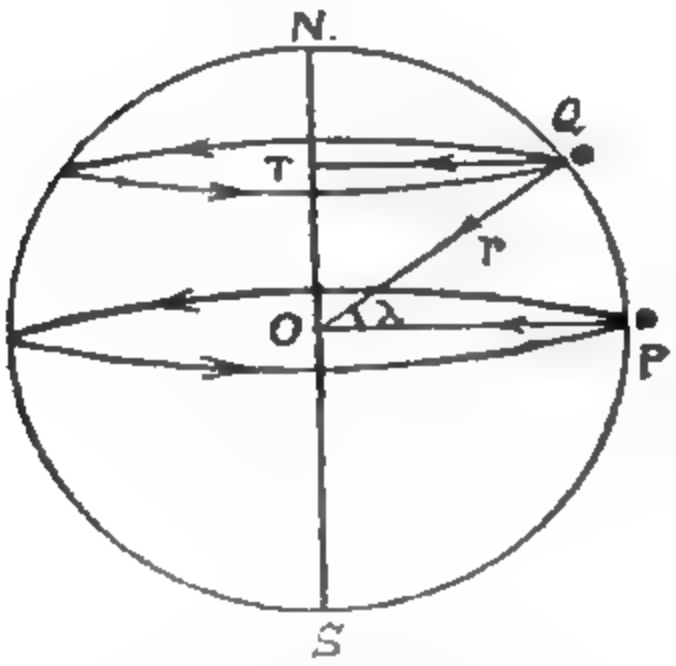

Fig. 79

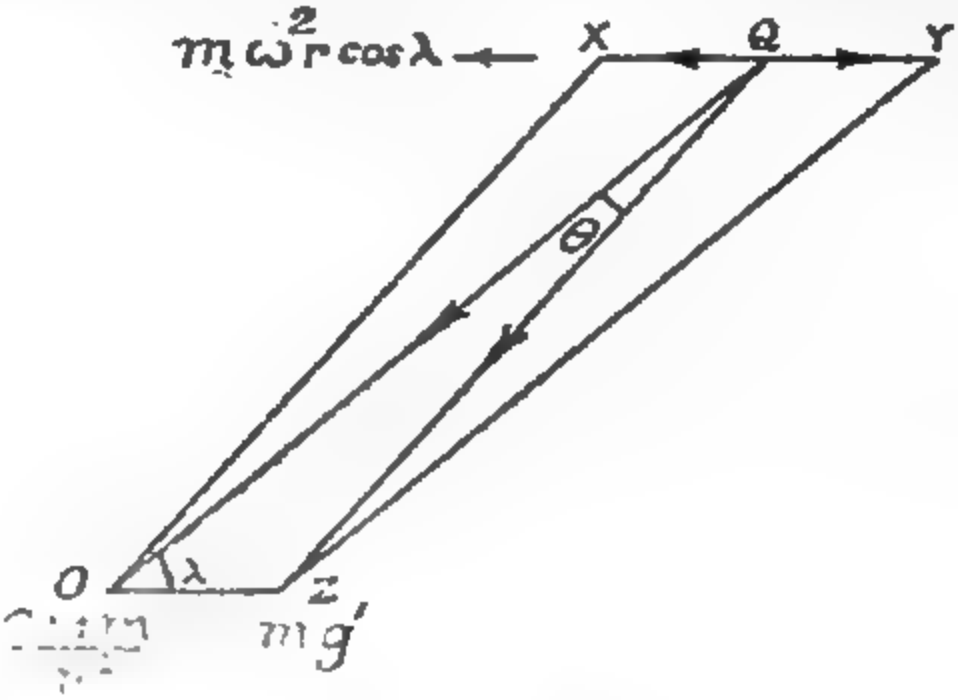

Fig. 80

poles and greatest at the equator and so the observed weight of a body is greatest at the poles and least at the equator. Thus for both the causes, *the weight of a body increases when taken from the equator to the poles.*

The actual variation in the weight of a mass with latitude due to diurnal rotation may be found from the following considerations. Let a mass m be placed on the surface of the earth at a region Q at latitude λ (Fig. 79). The mass m is undergoing a uniform circular motion with centre T, NS being the axis of the earth with centre O. If r is the radius of the earth, supposed to be constant for the present purpose, and ω the angular velocity of the particle at Q in latitude λ, then the centripetal force necessary to revolve it is given by,

$$m\omega^2 QT = m\omega^2 r\cos\lambda$$

Fig. 80 is an enlarged diagram of the component forces : QX represents the centripetal force and QO, the gravitational force $G\frac{Mm}{r^2}$ making an angle λ with QX or OZ. On completing the parallelogram QZOX, we get, $\overrightarrow{QZ} + \overrightarrow{QX} = \overrightarrow{QO}$ or, $\overrightarrow{QO} - \overrightarrow{QX} = \overrightarrow{QZ}$.

The component QZ which represents in magnitude and direction the *apparent weight* of the mass m at Q, may be found in a convenient manner by reversing the component QX along QY. Then QZ forms the resultant force due QO and QY which are inclined to each other at an angle $180° - \lambda$.

$$\text{Thus } QZ = \sqrt{QO^2 + QY^2 - 2QO.QY\cos\lambda}$$

$$= m\sqrt{\left(\frac{GM}{r^2}\right)^2 + \omega^4 r^2\cos^2\lambda - 2\frac{GM}{r}\omega^2\cos^2\lambda}$$

$$= mg', \text{ say and writing } g = \frac{GM}{r^2},$$

$$g' = \sqrt{g^2 - 2g\omega^2 r\cos^2\lambda + \omega^4 r^2\cos^2\lambda}$$

Since the angular velocity ω of the earth's rotation is of the order of 10^{-4} radian the term containing ω^4 may be neglected.

$$\text{Hence } g' = g\left[1 - \frac{2\omega^2 r\cos^2\lambda}{2}\right]^{\frac{1}{2}}$$

$$= g - \omega^2 r\cos^2\lambda \text{ approximately} \quad \ldots \quad (49,5)$$

If θ = inclination between the real and apparent gravitational forces, then from the $\triangle QOZ$,

$$\tan\theta = \frac{m\omega^2 r\cos\lambda\sin\lambda}{mg - m\omega^2 r\cos^2\lambda} = \frac{\omega^2 r\cos\lambda\sin\lambda}{g - \omega^2 r\cos^2\lambda} \quad \ldots \quad (49,6)$$

The variation in the weight of a body at different places cannot, however, be detected by a common balance for the body as well as the 'weight' used are equally affected due to the variation of g. It can, however, be detected by means of a very delicate *spring balance*, for the body being attracted by the earth with different forces at different places would cause the pointer to come to different positions on the dial.

Examples :—

Taking the earth to be a uniform sphere of radius 4000 miles and the true value of acceleration due to gravity to be 982·2 C.G.S., find the apparent values of the same at the equator and at latitude 45° due to effect of earth's rotation. Find also the inclination between the real and apparent gravitational forces at latitude 45°.

A. Since $\omega = \frac{2\pi}{T}$ and T for earth is 24 hours,

$$\omega = \frac{2\times 3\cdot 14}{24\times 60\times 60} = \cdot 0000726 \text{ radian per sec.}$$

$r = 4000\times 1760\times 3\times 12\times 2\cdot 54$ cm.

$g = 982\cdot 2$ cm. per sec^2 and $\lambda = 45°$.

Thus from eqn. (49,5)

$$g' = 982\cdot 2 - \cdot 0000726^2\times 4000\times 1760\times 3\times 12\times 2\cdot 54\times\cos^2 45°$$

$$= 982\cdot 2 - 1\cdot 7 = 980\cdot 5 \text{ cm/sec}^2 \text{ at latitude } 45°$$

Again, at the equator $\lambda = 0$

$$g' = 982\cdot 2 - \cdot 0000726^2\times 4000\times 1760\times 3\times 12\times 2\cdot 54$$

$$= 982\cdot 2 - 3\cdot 4 = 978\cdot 8 \text{ cm/sec}^2$$

At latitude 45°, let θ = inclination, then $\tan\theta = \frac{1\cdot 7}{980\cdot 5} = \cdot 001734$

whence $\theta = 0'$ approximately.

50. Law of Falling Bodies.—When a body falls freely from rest, subject only to the action of gravity, then for all practical purposes, (the height not being too large) the acceleration due to gravity may be assumed to remain constant throughout the fall. Then the motion is governed by the following simple laws :—

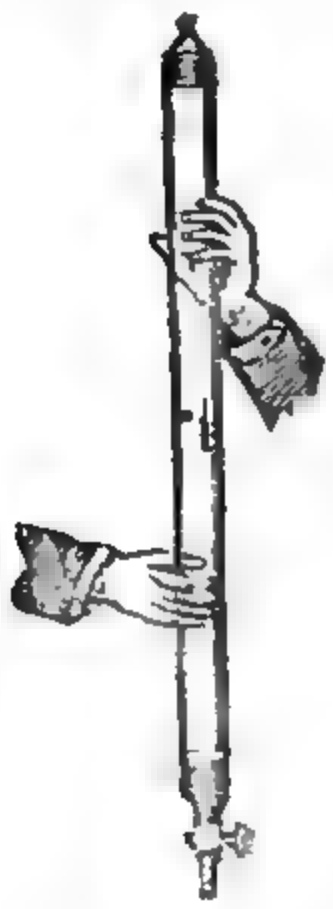

Fig. 81

(*1*) *In vacuum, all bodies starting from rest, fall with equal rapidity.*

(*2*) *The velocity, acquired by a falling body in a given time, is directly proportional to the time.*

(*3*) *The space traversed by a falling body in a given time is directly proportional to the square of time.*

Analysis of the Laws.—The *first law* seems contrary to our common observation, as a heavy body appears to fall to the ground more quickly than a lighter body, when dropped from the same height. Galileo first showed in 1589 by dropping balls of different sizes and materials from the top of the leaning tower of Pisa, that they reached the ground in practically the same time. He pointed out that the slight observed differences in the rate of fall between a light and a heavy body were due to the resistance offered by the air. He then argued that all bodies would fall at the same rate, if the resistance due to air could be avoided, and stated the laws as above. After the invention of the air pump in 1650, the law was conclusively verified by Newton in the well-known *guinea and feather experiment*.

Guinea and Feather Experiment.—A glass tube, about a metre long with a cap screwed at one end and a stop cock at the other, is taken (Fig. 81). A small coin and a piece of paper are introduced into the tube. The air is then exhausted from the tube by an air-pump and the stop cock is closed. On suddenly inverting the tube, the coin and paper are found to be falling together and reach the other end simultaneously. If air is now reintroduced by opening the stop-cock and the tube is inverted, the two are found to fall separately, the coin reaching earlier. The experiment was originally done with a guinea and a piece of feather within the tube and hence the name was derived.

When a body falls freely through a height h in the time t, then $h \propto t^2$. Thus, if the body falls through a height x ft. in the 1st second, the height fallen through in the 2nd., 3rd., 4th., etc., seconds are $4x$, $9x$, $16x$ etc., respectively. According to the second law if v be the velocity acquired in falling for a time t, then $x \propto t$. Thus, if the velocity obtained in the 1st second be v, that in the 2nd, 3rd, 4th seconds, are $2v$, $3v$, $4v$, etc. respectively. The laws may be verified by experimenting with *Atwood's machine* or Inclined Plane. These laws also follow from the Equations of Motion given in the Chapter II.

51. Vertical Motion of Bodies due to Gravity.—When a body falls vertically downwards, then provided the distance fallen through is

not very large, the acceleration g due to gravity may be supposed uniform.

Bodies projected downwards.—If a body is projected *downwards* with a initial velocity u then,—

$$\left.\begin{aligned} &\text{Velocity } v \text{ after a time } t && = u + gt \\ &\text{Height } h \text{ fallen through} && = ut + \tfrac{1}{2}gt^2 \\ &\text{and hence } v^2 && = u^2 + 2gh \end{aligned}\right\} \quad \ldots (51,1)$$

If the body *starts from rest*, then $u = 0$, and

$$\left.\begin{aligned} &\text{So } v = gt \quad i.e., \quad v \propto t \ \ldots\text{2nd Law.} \\ &h = \tfrac{1}{2}gt^2 \ i.e., \quad h \propto t^2 \ \ldots\text{3rd Law.} \end{aligned}\right\} \quad \ldots (51,2)$$

Bodies projected upwards.—Here the acceleration due to gravity is opposite to the direction of motion and is so taken to be negative.

$$\left.\begin{aligned} &\text{Hence } v = u - gt \\ &h = ut - \tfrac{1}{2}gt^2 \\ &v^2 = u^2 - 2gh \end{aligned}\right\} \quad \ldots (51,3)$$

When the body reaches the highest point, its velocity becomes zero.

$\therefore$ From (51,3), time to reach the highest point, $t = \dfrac{u}{g}$ ... (51,4)

The time to reach the ground from the highest point is also u/g. Therefore the total time of flight is $2u/g$.

If h is the maximum height reached by the body.

$$0 = u^2 - 2gh \text{ or } h = \frac{u^2}{2g} \quad \ldots (51,5)$$

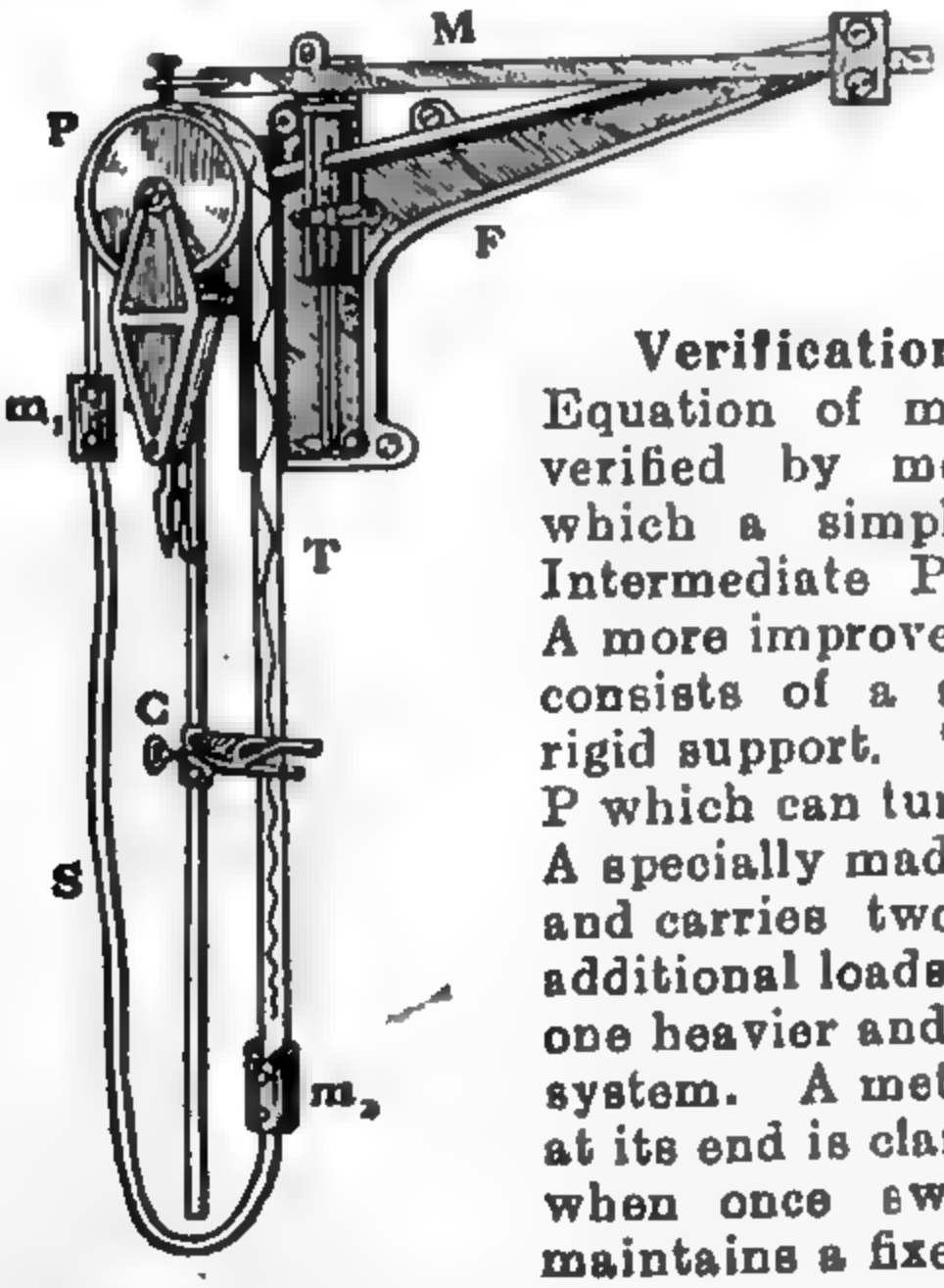

Fig. 82—Atwood's machine

Verification of Laws of Falling Bodies.—Equation of motion as given in Art. 51 can be verified by means of an Atwood's machine of which a simpler type has been described in the Intermediate Practical Physics by J. Chatterjee. A more improved form is shown in fig. 82, which consists of a stout metal rod fitted vertically to a rigid support. The top of the rod carries a pulley P which can turn very freely without any friction. A specially made paper tape T runs over the pulley and carries two equal loads at its ends. Small additional loads can be put on either loads to make one heavier and hence to start the motion of the system. A metal reed M carrying an inked brush at its end is clamped to a cross-piece F. This reed when once swung through a small range always maintains a fixed period of vibration.

To verify the law, that for accelerated motion, $v \propto t$, a small additional load is put on m_2 and

the combined load is raised and placed upon the clamp C at any suitable height. The period of oscillation T of the reed is known from its specification or may be found out by comparing its vibration with that of a standard chronometer.

The brush is inked and the reed is made to vibrate on the paper tape. The brush moves to-and-fro over a line at right angles to the length of the paper. The clamp C is released and m_2 is at once found to descend, while on the paper tape a wavy curve as shown in fig. 83 is traced by the inked brush. The curve becomes longer as the load m_2 descends more showing at a glance that the speed increases continuously. Each complete curve as that between a and b marks a unit of time *i.e.*, the time T of vibration of the reed. The tape is taken out and placed on the table. Count a number of such curves, say n, from the beginning. The time t from the start of the load to the termination of count is nT. Measure the straight length of a complete wave about this point, which is, say l. Then the average velocity v at this point is l/T. Take various intervals and the corresponding velocities. It would be found that in each case that,

$$\frac{l}{T} \propto nT \quad \text{or} \quad \frac{l}{nT^2} = \text{constant.}$$

Since T for a particular length of the reed is constant l/n is constant which verifies the law.

To verify the law that in an accelerated motion of a body, $s \propto t^2$, the experimental procedure is the same. Measure a length s covering a few wavy curves from the start. Count the number of complete waves in this length, let it be n. Then the time to cover this distance is nT. Measure a few lengths and count corresponding times. It would be found that in each case,

$$s \propto n^2T^2 \quad \text{or} \quad \frac{s}{n^2T^2} = \text{constant.}$$

To measure 'g'.—From the second part of the experiment, it is easy to measure the acceleration of the system. We know that for a system starting from rest and moving with uniform acceleration, $s = \frac{1}{2} ft^2$. Hence acceleration f is $2s\ t^2$ or $2s\ n^2T^2$. If the small additional load put upon m_2 be of mass m, then the downward force exerted by this load is mg. If the total mass including m_1, m_2 and m be M

$$\text{Then } Mf = mg \quad \text{or,} \quad g = \frac{Mf}{m}.$$

This apparatus may also be used to study the uniform motion or even to verify the conservation of momentum of two bodies.

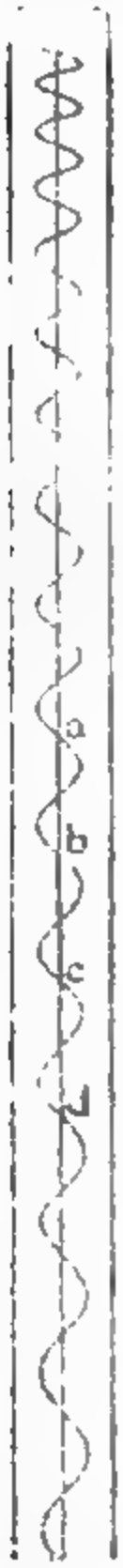

Fig. 83

52. A Body projected horizontally.—Let a body be projected from O with a uniform velocity u along OX (Fig. 84). As soon as the body leaves O, it is subjected to the uniform acceleration g due to gravity acting vertically downwards, along OY.

Let the positions of the body at the end of successive seconds be at P_1, P_2, P_3, etc. if it were moving along OX with a uniform velocity u alone ; then $OP_1 = P_1P_2 = P_2P_3 = u$. If the body starts from rest at O and moves with uniform acceleration g, its positions along OY at end of successive seconds would be respectively at Q_1, Q_2, Q_3, such that $OQ_1 = \frac{1}{2}g \times 1^2$, $OQ_2 = \frac{1}{2}g \times 2^2$, $OQ_3 = \frac{1}{2}g \times 3^2$, etc. Hence due to the two motions combined, the actual position of the body at the end of the first second will be at R_1 which is the end of the diagonal OR_1 of the parallelogram $O P_1 R_1 Q_1$; similarly, the actual positions of the body at the end of successive seconds will be at R_2, R_3, etc. Taking the intervals to be very small, the actual path of the body will be along the continuous curve $OR_1 R_2 R_3 \ldots$

It can be shown that *the path* described by the body is a *parabola*.

The horizontal distance travelled in time t is given by

$$x = ut \qquad \ldots (1)$$

The vertical distance fallen through in the same time is

$$y = \tfrac{1}{2}gt^2 \qquad \ldots (2)$$

Eliminating t from (1) and (2), $y = \frac{g}{2} \times \frac{x^2}{u^2}$ ... (52)

which represents a relation of a parabola in x and y.

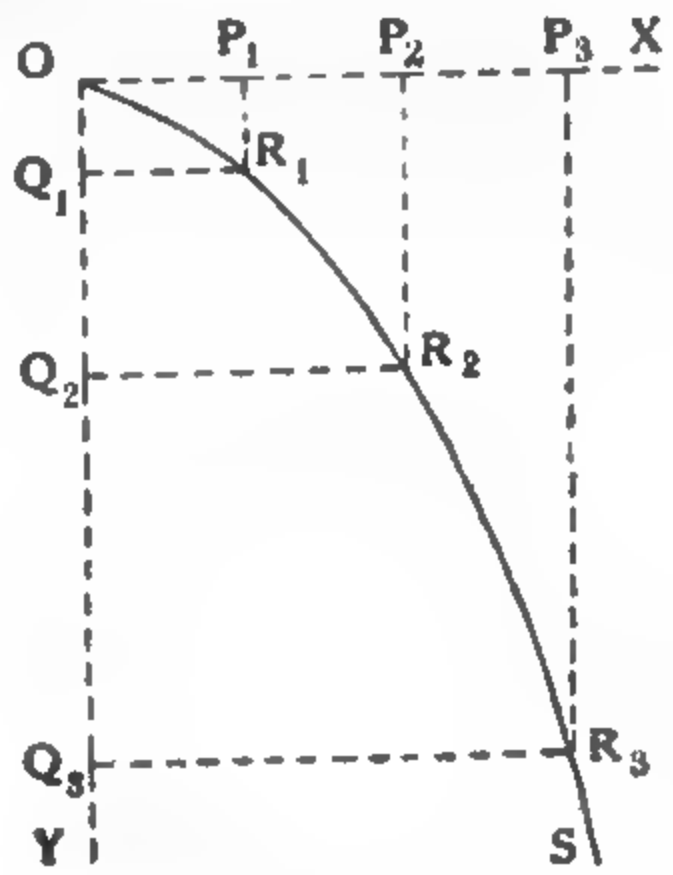

Fig. 84

The path of the body projected in direction making an angle with the horizontal plane is also a parabola with its vertex at the top-most position. Of course, we assume here that the resistance it encounters with air during its flight is negligibly small. The projected particle is called a *projectile* and its path is termed a *trajectory*. The path of a cannon ball after it has left the muzzle of the gun or of a bomb dropped from an aeroplane moving horizontally, furnishes instances of projectiles.

Examples :—

1. A stone is dropped from a tower with no initial velocity. It is observed to strike the ground after 3·5 seconds. Find the velocity of impact and the height of the tower. [given $g = 32$ F. P. S.]

A. Here $u = 0$, $g = 32$ and $t = 3{\cdot}5 = \frac{7}{2}$ sec.

$\therefore$ $V = u + gt = 32 \times 3{\cdot}5 = 112$ ft./sec.

Again $h = ut + \frac{1}{2}\ ft.^2 = \frac{1}{2} \times 32 \times \frac{7^2}{2} = 196$ ft.

2. A body dropped from a balloon at rest clears a tower 81 ft. high during the last quarter second of its motion. Find the height of the balloon and the velocity of the body when it reaches the ground. ($g = 32$ ft. per sec. per sec.)

[And. U.—1950]

A. Let t be the period of fall in seconds. Since the body falls from rest, the initial velocity $u = 0$. If s be the distance fallen through in t sec. Then,

$$s = \tfrac{1}{2}\ gt^2 = 16 \times t^2 \qquad (1)$$

Again if S_1 be the distance fallen through quarter of a second earlier,

$$\text{Then,}\quad S_1 = \tfrac{1}{2}g(t-\tfrac{1}{4})^2 = 16\left(\ t^2 - \frac{t}{2} + \frac{1}{16}\ \right) = 16t^2 - 8t + 1 \qquad \ldots \quad (2)$$

But $S - S_1 = 81$ ft. $= 8t - 1$ whence $t = 10\tfrac{1}{4}$ seconds.

$$\text{Thus}\quad S = \tfrac{1}{2}gt^2 = 16 \times \frac{41^2}{4^2} = 1681 \text{ ft.}$$

Again final velocity $v = gt = 32 \times 10\tfrac{1}{4} = 328$ ft./sec.

3. It is required to pierce a war balloon at an elevation of $\frac{1}{4}$ mile by means of a rifle bullet fixed immediately under it. If, to pierce the balloon, the bullet must have a velocity of 40 ft. per sec. on reaching the balloon, with what velocity must it leave the muzzle of the gun? Given $g = 32$ ft./sec.^{2} (Pat. U.—1932)

A. Let the initial velocity of the bullet $= u$ ft. per sec.

Height attained by the bullet $= \frac{1}{4}$ mile $= 1320$ ft.

From eqn. (57.3), $40^2 = u^2 - 2 \times 32 \times 1320$

Whence $u^2 = 86080$ or $u = 293{\cdot}4$ ft. per sec.

4. Two stones are projected vertically upwards at the same instant. One attains 32 ft. higher than the other, and returns to earth 2 seconds later. Find the velocities of projection of the stones, ($g = 32$ ft. per sec. per sec.)

(C. U.—1933)

A. Let the velocities of projection of the stones be u_1 and u_2 respectively. If h_1 and h_2 be the maximum heights attained by the stones, then from eqn. (57.1)

$$h_1 = \frac{u_1^2}{2g} \quad \text{and} \quad h_2 = \frac{u_2^2}{2g}$$

$$\therefore\ h_1 - h_2 = \frac{u_1^2}{2g} - \frac{u_2^2}{2g} = 32 \text{ or } u_1^2 - u_2^2 = 32 \times 64 \qquad \ldots \quad (1)$$

If t_1 and t_2 be the times of flight respectively, then

From eqn. (57.1) $t_1 = \dfrac{2u_1}{g}$ and $t_2 = \dfrac{2u_2}{g}$

$$\therefore\ t_1 - t_2 = \frac{2u_1}{g} - \frac{2u_2}{g} = 2 \text{ or } u_1 - u_2 = 32 \qquad \ldots (2)$$

Dividing (1) by (2) $u_1 + u_2 = 224$

And (2) and (3) $u_1 = 128$ ft./sec. and $u_2 = 96$ ft./sec.

5. A bomber aeroplane is travelling horizontally with a velocity of 60 miles per hour and when it is at a height of 1936 ft. a bomb is dropped from it. Find where the bomb strikes the ground and the time of its fall, ($g = 32$ ft. per sec.^{2})

A. Horizontal velocity of the plane $= 60$ miles per hour $= 88$ ft./sec.

∴ The bomb falls vertically with an acceleration of 32 ft per sec.^{2} and also travels horizontally with a uniform velocity of 88 ft. per sec.

Let the time of fall of the bomb $= t$ seconds.

Then from $h = \frac{1}{2}gt^2$, $1936 = \frac{1}{2} \times 32 \times t^2$, whence $t = 11$ seconds.

∴ The horizontal distance covered by the bomb $= 88 \times 11 = 968$ ft.

Hence the bomb strikes the ground at 968 ft. from a point on the ground vertically below the point from which it was dropped.

6. Calculate the mass of the Sun, given that the distance between the Sun and the Earth is $1{\cdot}49 \times 10^{13}$ cm. and $G = 6{\cdot}66 \times 10^{-8}$ C. G. S. units. Take the year to consist of 365 days. [P. U.—1942]

A. Let mass of Sun $= M$ and mass of Earth $= m$.

Distance between the two, which is the radius of the earth's orbit $= 1{\cdot}49 \times 10^{13}$ cm.

$\therefore$ Gravitational force of attraction $= G.\frac{Mm}{d^2} = 6{\cdot}66 \times 10^{-8} \times \frac{Mm}{(1{\cdot}49 \times 10^{13})^2}$ dynes.

Time of revolution of the Earth $= 365$ days $= 365 \times 24 \times 60 \times 60$ sec.

or, Angular velocity ω of the Earth $= \frac{2\pi}{T} = \frac{2 \times 3{\cdot}1415}{365 \times 24 \times 60 \times 60}$ radians/sec

$\therefore$ Centripetal force on the Earth $= \frac{mv^2}{r} = m\omega^2 r = m \times \left(\frac{2 \times 3{\cdot}1415}{365 \times 24 \times 60 \times 60}\right)^2 \times 1{\cdot}49 \times 10^{13}$ dynes.

The centripetal force must be equal to the gravitational force for equilibrium.

Hence, $6{\cdot}66 \times 10^{-8} \times \frac{Mm}{(1{\cdot}49 \times 10^{13})^2} = m \times \left(\frac{2 \times 3{\cdot}1415}{365 \times 24 \times 60 \times 60}\right)^2 \times 1{\cdot}49 \times 10^{13}$

whence $M = 19{\cdot}72 \times 10^{24}$ gm. $= 4{\cdot}347 \times 10^{24}$ lb.

53. Some Problems—

(1) Motion of two bodies connected by a String passing over a smooth Pulley.—Let two bodies of masses m_1 and m_2 be connected by a light inextensible string passing over a smooth pulley P and both of them hanging vertically (Fig. 85). If $m_2 > m_1$, we are to find the resulting motion of the system and the tension of the string. As the pulley is smooth, the tension of the string is the same throughout its length. Again as the string is inextensible, the downward motion of m_2 is equal to the upward motion of m_1; hence their acceleration is the same.

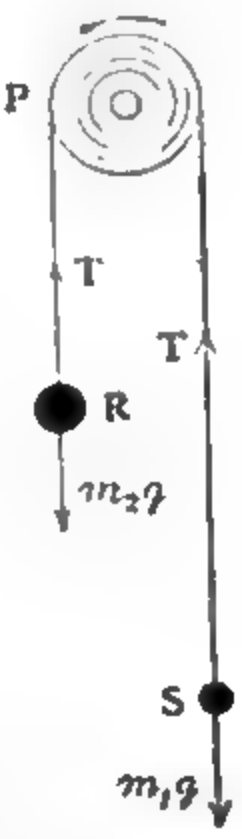

Fig. 85

Let common acceleration of the system be f and the tension of the string be T in absolute units.

Now the resultant *downward* force on $m_2 = (m_2g - T)$

And the resultant *upward* force on $m_1 = (T - m_1g)$

$\therefore$ $m_2g - T = m_2f$ and $T - m_1g = m_1f$

Adding, $(m_2 - m_1)g = (m_1 + m_2)f$

whence $$f = \frac{m_2 - m_1}{m_1 + m_2}g \quad \ldots (53,1)$$

Also $$T = m_1(f + g) = \frac{2m_1m_2}{m_1 + m_2}g \quad \ldots (53,2)$$

Observing the acceleration f of the system and using the eqn. (53,1) the value of g may be roughly determined.

(2) Reaction of a moving Lift.—When a man is standing on a

lift either at rest or moving up or down with a uniform velocity, the reaction at the floor of the lift is equal to the weight of man. But when the lift is moving with an acceleration, the thrust is not equal to the weight of the man. It is however equal to the thrust then exerted by the man on the lift, which may thus be called the *apparent weight* of the man on the accelerating lift.

Let m be the mass of the man, R the thrust between the man and the floor and f, the acceleration of the system. When the lift is *accelerating up*, R must be greater than the weight of the man.

$$\therefore \quad R - mg = mf \qquad \text{or} \quad R = mg + mf = mg\left(1 + \frac{f}{g}\right)$$

i.e., the man appears to be heavier by $\frac{f}{g}$ of his weight.

When the lift is *accelerating down*, the weight must be greater than the thrust R.'

$$\therefore \quad mg - R' = mf \qquad \text{or} \qquad R' = mg - mf = mg\left(1 - \frac{f}{g}\right)$$

i.e., the man appears to be lighter by $\frac{f}{g}$ of his weight.

(3) Mass of the Earth.—Let the mass of the earth be M gm. and its radius be R cm. Then by the law of gravitation.—

The force of attraction on a mass m on the earth $= \frac{GMm}{R^2}$ dynes.

This force being equal to the weight of the body $= mg$ dynes.

$$\therefore \quad \frac{GMm}{R^2} = mg \qquad \text{whence} \quad M = \frac{gR^2}{G} \qquad \ldots \quad (53,3)$$

Taking R = 4000 miles, $G = 6{\cdot}65 \times 10^{-8}$ and $g = 981$ cm. per sec.^{2}

$$M = \frac{981 \times (4000 \times 1760 \times 3 \times 30{\cdot}48)^2}{6{\cdot}65 \times 10^{-8}} = 6{\cdot}16 \times 10^{27} \text{ gm.}$$

$$= 1{\cdot}36 \times 10^{25} \text{ lb.} = 60{\cdot}7 \times 10^{20} \text{ tons.}$$

If ρ is the mean density of the earth, then $M = \frac{4}{3}\pi R^3 \rho \qquad \ldots \quad (53,4)$

$$\therefore \quad \text{From (i) and (ii)} \quad \rho = \frac{3g}{4\pi GR}$$

$$= \frac{3 \times 981 \times 7 \times 10^8}{4 \times 22 \times 6{\cdot}65 \times 4000 \times 5280 \times 30{\cdot}48} = 5{\cdot}46 \text{ gm. per c. c.}$$

The value of G was first determined by Cavendish whence the mass and density of the earth were calculated. It is thus said that 'Cavendish weighed the Earth' in his balance.

Examples :—

1. A light string passes over a smooth pulley and has masses of 240 gm. and 250 gm. attached to its ends. Calculate the value of g, if the system starting from rest moves a distance of 160 cm. in 4 seconds. (Anna. U.—1960)

A. Since the two loads are unequal, the combination would have an acceleration with the heavier load descending downwards. Let the common acceleration be f cm. per sec^2. Then,

$$160 = 0 + \tfrac{1}{2} f \times 4^2 \qquad \text{whence} \qquad f = 20 \text{ cm./sec}^2.$$

Let T be the tension of the string which is throughout equal.

Then $250\ g - T = 250 \times 20$ and $T - 240\ g = 240 \times 20$

Adding the two equations, we get,

$$10g = 490 \times 20 \qquad \text{whence} \qquad g = 980 \text{ cm./sec.}^2$$

2. A mass of 10 lb. is hung from a spring balance attached to a lift. The lift is (*a*) ascending with an acceleration of 4 ft. per sec per sec., (*b*) ascending with a uniform velocity of 4 ft. per sec. Calculate how the reading of the spring balance will be affected in each case. The value of $g = 32$ ft. per sec per sec.

[Pat. U.—1953]

A. When the lift is ascending with an acceleration of 4 ft. per sec.^{2} the reaction R of the mass of 10 lb. on the spring balance, is given by,

$$R - 10 \times 32 = 10 \times 4 \text{ poundals} \qquad \text{whence} \qquad R = 360 \text{ poundals}$$

Hence the spring balance will record $360 \div 32 = 11{\cdot}25$ lb., that is $1{\cdot}25$ lb. more than its original reading.

Again when the lift is moving with a uniform velocity, whatever may be its magnitude, there is no change in the reaction of moving parts and so the spring balance records the same reading of 10 lb.

54. Spring Balance.—A spring balance (Fig. 86) is an instrument with which weights of bodies can be determined and compared. It consists of a *spiral spring* of steel S, enclosed in a metallic case, with the upper end fixed to a ring at the top of the case. The lower end is attached to a straight rod R which has a hook H at the lower end from which the body to be weighed can be hung. The front part of the case is covered by a plate with a long narrow slit C, which is graduated either in grams or in pounds along the slit. A pointer attached to the rod passes out of the slit and rests against the scale. When a body is hung from the hook, the spring extends in proportion to the amount of pull exerted by the weight of the body. The pointer accordingly moves along the scale and indicates the weight of a body from the reading on the scale.

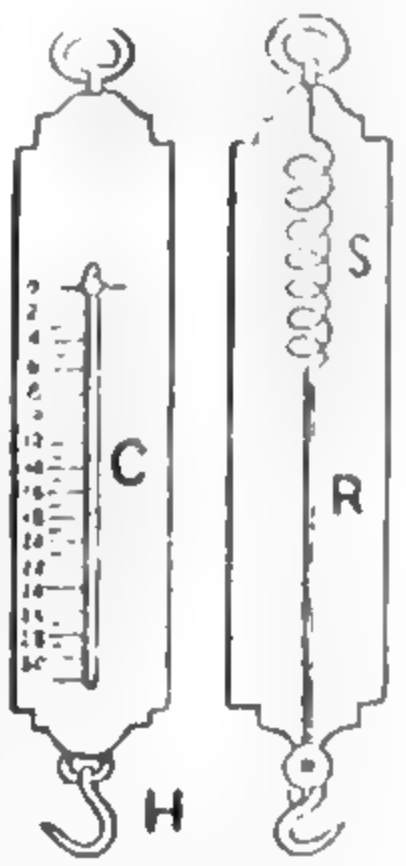

Fig. 86

As the spring extends by the pull exerted on the body by the earth, the spring balance directly gives the weight of a body at a place. If we move to a different place, the weight of the body changes and so the pull on the spring also changes. Hence from theoretical principles the instrument may be calibrated so as to record the weight of a body.

Thus, in a common balance we compare different masses, while in a spring balance we compare different weights. If taken to different places, the bodies and the 'weights' used are equally affected by the

variation of 'g' and so it will not be possible to detect any change in the weight of a body by a common balance. A spring balance on the other hand is differently affected due to the difference in the pulls and may indicate the variation in the weight. In practice however the variation is so small that it does not affect an ordinary spring balance.

*55. **Kepler's Laws of Planetary Motion.**—During the last decade of the 16th century, the Danish astronomer Tycho Brahe and Johaan Kepler made a systematic observation on the nature of the orbit of the planet Mars around the Sun. After the death of Tycho Brahe, Kepler enunciated the three following laws,—

1. *Each planet moves in an elliptic orbit with the sun at one of the foci.*

2. *The radius vector joining the planet to the sun describes equal areas in equal times.*

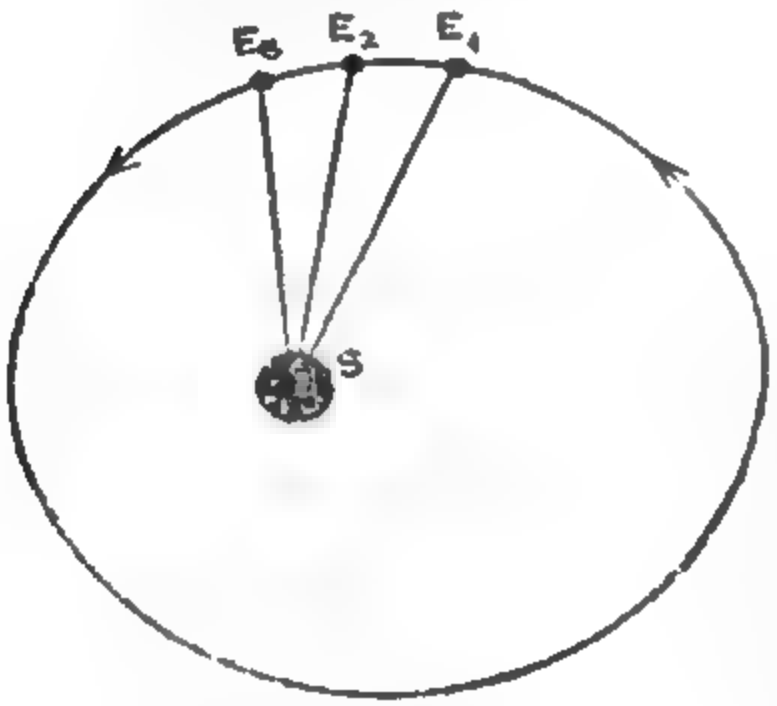

Fig. 87

3. *The square of the periodic time of a planet varies as the cube of its mean distance from the sun.*

Explanation of Laws.—Let fig. 87 represent the elliptic orbit of a planet. Then the sun must be at any one of the foci of the ellipse, say at S. The first law may be verified, in case of the earth's orbit by a systematic measurement of the variation of the distance of the sun from the earth throughout one year. On December 31, the distance becomes minimum and on July 1, the distance is maximum. The eccentricity of the ellipse is very small having a value '0168, which means that the orbit is nearly a circle.

The second law states that if E_1, E_2 and E_3 represent the positions of a planet at equal intervals of time, then the area of the sector E_1SE_2 is equal to the area E_2SE_3. If the interval of time be taken small, for example one day, then arc E_1SE_2 is very approximately $E_1S \cdot \theta$ where θ is the angle E_1SE_2 and the area E_1SE_2 becomes $\frac{1}{2} SE_1{}^2\theta$. Knowing SE_1 and measuring θ, the law can be verified for the two sectors E_1SE_2 and E_2SE_3.

Kepler's Third Law and the Law of Gravitation.—In order to explain the third law, Newton had to assume the existence of some sort of attracting force between the planet and the sun, which varies as the product of the two masses and inversely as the square of the distance. Let M be the mass of the sun and m that of the planet at a mean distance r from the sun. For simplicity assume that the orbit is circular.

If v be the tangential velocity of the planet at any point in its orbit, the centripetal force necessary for the rotation of the planet is mv^2/r.

The origin of this centripetal force is the gravitational force. If T be the periodic time of the planet,

$$\text{Then } v = \frac{2\pi r}{T} \text{ or, } \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2}$$

If the centripetal force is equal to the gravitational force.

$$\text{Then, } \frac{4\pi^2 mr}{T^2} = \frac{GMm}{r^2} \text{ or, } T^2 = \frac{4\pi^2}{GM} r^3 \quad \ldots (55,1)$$

$$\text{whence } T^2 \propto r^3$$

Thus the third law can be proved by assuming an inverse square law of gravitational force.

Comparison of Masses of Heavenly Bodies.—Kepler's third law gives us a clue to compare masses of heavenly bodies *which have statellites at known distances and of known periodic times.* Transforming eqn. (55,1), we can write,

$$\frac{T^2 M}{r^3} = \frac{4\pi^2}{G} = \text{a constant.}$$

If there are two heavenly bodies of masses M_1 and M_2, having two statellites of periodic times T_1 and T_2, placed at a distance of r_1 and r_2 from their respective parent bodies, then

$$\frac{T_1^2 M_1}{r_1^3} = \frac{T_2^2 M_2}{r_2^3} = \frac{4\pi^2}{G}$$

$$\text{whence, } \frac{M_1}{M_2} = \frac{T_2^2}{r_2^3} \times \frac{r_1^3}{T_1^2} \quad \ldots (55,2)$$

Example :—

1. A satellite of Jupiter moves round the planet in a circle of radius $1{\cdot}161 \times 10^6$ miles completing one revolution in 16·6 days, while Earth's moon on an orbit of radius $2{\cdot}24 \times 10^5$ miles completes one revolution in 27 days. Compare the masses of Jupiter and Earth.

A. Let mass of Jupiter $= M_1$, orbital radius of the satellite $= r_1 = 1{\cdot}161 \times 10^6$ miles and period of revolution of the satellite $= T_1 = 16{\cdot}6$ days. Those for the earth bear subscript 2.

$$\text{Then, } \frac{\text{mass of Jupiter}}{\text{mass of Earth}} = \frac{M_1}{M_2} = \frac{27^2}{(2{\cdot}4 \times 10^5)^3} \times \frac{(1{\cdot}161 \times 10^6)^3}{16{\cdot}6^2} = \frac{299}{1}$$

56. Requisites of a Good Balance.—The description of a physical balance has already been given in Chapter I. A good balance must be (1) *true* (2) *sensitive* (3) *stable.*

(1) A balance is said to be **true**, if the beam remains horizontal when both the pans are empty or are loaded with equal masses.

The *conditions* to be satisfied by a balance to be true are :—

(i) The two arms of the beam must be of equal length.

(ii) The scale pans must be of equal mass.

(iii) The centre of gravity of the beam must be vertically below the fulcrum when the beam is horizontal.

To test whether a balance is true, a body is placed on one of the pans and weights are placed on the other, till the beam is horizontal. The body and the weight are then interchanged ; if they still balance *i.e.*, if the beam horizontal, the balance can be taken to be true.

(2) A balance is said to be **sensitive,** when the beam is appreciably displaced from the horizontal position, even for a very small difference in the weights of the bodies placed on the two pans. A sensitive chemical balance indicates a difference of one-tenth of a milligram.

The *conditions* to be satisfied by a *sensitive* balance are :—

(i) The C. G. of the beam should be very near the fulcrum.

(ii) The beam should be light.

(iii) The arms of the beam must be long. If this condition is satisfied, even a small excess in the weight on one pan would produce sufficiently large moment to cause appreciable inclination of the beam.

(3) A balance is said to be **stable,** if the beam, after being disturbed, quickly comes back to its position of equilibrium. The *condition for stability* requires that the C. G. *of the beam must be* [illegible] *the fulcrum.* This condition is opposed to one of the conditions for sensitiveness. In other words, *stability is increased at the cost of sensitiveness.* A balance should be *rigid, i.e*, the beam should not bend when weights are placed on the pans.

57. **Methods of Weighing by a Balance.**—Different methods are adopted in these accurate weighing under different circumstances.

Method of Oscillations.—If the balance is *true*, the most common method of accurate weighing is the *method of oscillations.* With the pan empty the balance is made free and the oscillations of the pointer [illegible] should oscillate through equal number of divisions on either side of the middle division. The body to be weighed is placed on the left pan; weights are placed on the right pan, till on making the balance free, the pointer again oscillate through equal number of divisions on either side. The mass of the body is then equal to the mass of the total weights placed on the pan. (For details vide J. Chatterjee's Intermediate Practical Physics).

Method of Double Weighing.—Even when the balance is ***not true***, the mass of a body may be determined by the following methods of *double weighing.*

(1) **Borda's Method** (*method of substitution*).—The body to be weighed is placed on one of the pans, say the right-hand one and is counterpoised with small shots, sand, etc. The body is then removed and standard weights are placed on that pan, till equilibrium is restored. Evidently the sum of these standard weights is equal to the weight of the given body, for whether the balance be *true* or *false*, these weights and the body, separately placed on the same pan, balance the sand, etc. on the other pan.

(2) Gauss' method.—This method is applied, if only the arms are of unequal lengths but the balance is otherwise correct. The body is first placed on one of the pans, say the left-hand one and is exactly counterpoised in the usual way. The body is then placed on the right-hand pan and is counterpoised again.

Let the arms of the balance = a, b ; true weight of the body = W ; observed weight with body on one pan = W_1 ; observed weight with body on the other pan = W_2.

Then $Wa = W_1 b$ and $W_2 a = Wb$ ...(57,1)

$$W^2 ab = W_1 W_2 ab \text{ whence } W = \sqrt{W_1 W_2} \quad ...(57,2)$$

Thus *the true weight is the geometrical mean of the two observed weights.*

To find the ratio of the arms :—

From eqn. (57,1), $\frac{a}{b} = \frac{W_1}{W}$ also $\frac{a}{b} = \frac{W}{W_2}$

$$\text{Hence } \frac{a}{b} = \sqrt{\frac{W_1}{W} \times \frac{W}{W_2}} = \sqrt{\frac{W_1}{W_2}} \quad ...(57,3)$$

Suppose a tradesman uses a false balance with unequal arms a and b. A customer takes a quantity of some article, which appears to weigh W_1, being weighed in one pan. The balance is turned and another quantity also appearing to weigh W_2 is weighed in the other pan. Here the customer really gets a quantity $W_1 + W_2$ in place of 2W.

$$\text{Now } W_1 + W_2 - 2W = W\frac{a}{b} + W\frac{b}{a} - 2W$$

$$= W\frac{a^2 + b^2 - 2ab}{ab} = \frac{W}{ab}(a-b)^2$$

As a and b are unequal. $(W_1 + W_2) > 2W$

i.e., the tradesman cheats himself by the amount $\frac{W}{ab}(a-b)^2$

Hence it is profitable for a customer to purchase always a quantity of substance in two instalments, weighing half on one pan and the other half on the other in a false balance.

Examples :—

1. A tradesman sells his article weighing equal quantities alternately from the two pans of a balance having unequal arms. If the ratio of the lengths of the two arms be 1·025, what is his percentage loss or gain ? [Pat. U.—1952]

A. Let the true weight (mass) of the body W and its observed weights when placed on the two pans be W_1 and W_2. Then,

$$\frac{a}{b} = \sqrt{\frac{W_1}{W_2}} = 1{\cdot}025$$

When equal quantities are weighed alternately from the two pans of the balance, the customer gets a quantity in excess of W by,

$$W\frac{(a-b)^2}{ab} = W\frac{a^2+b^2-2ab}{ab} = W\left(\frac{a}{b}+\frac{b}{a}-2\right) = W\,(1{\cdot}025 + {\cdot}9756 - 2)$$

$= {\cdot}0006W$

Hence the tradesmnn cheats himself or loses by $\frac{{\cdot}0006W}{W} \times 100 = {\cdot}06\%$.

2. The turning points of a balance were observed to be successively 13, 8, 11. With the body on the left pan and 24·82 gm. on the right pan, the turning points were 14, 9, 12. On adding 10 more milligrams to the weights, the turning points became 10, 3, 8. Calculate the correct weight of the body. (Mys. U.—1952)

A. Supposing the scale of the pointer to begin from left, it is evident from the sets of readings of the turning points, that for any set two readings were taken to the right and one to the left. Hence resting points in each case is to be taken in the following way.—

(*i*) With empty pans, $\frac{13+11}{2} = 12$; $\frac{12+8}{2} = 10 =$ Resting point.

(*ii*) With body on the left and 24·82 gm. on the right pan,

$\frac{14+12}{2} = 13$; $\frac{13+9}{2} = 11 =$ Resting point.

This shows that the body, which is kept on the left pan, is heavier than the weights placed (= 24·82 gm.) on the right pan by a load which shifts the resting point by one division only.

(*iii*) With addition of 10 mgm. = ·01 gm.

$\frac{10+8}{2} = 9$; $\frac{9+3}{2} = 6 =$ Resting point.

Thus 5 divisions of resting points are shifted for ·01 gm.

∴ 1 division of resting point is shifted for ·002 gm.

Hence the true weight of the body = 24·82 + ·002 = 24·822 gm.

58. Centre of Gravity.—Any material body may be supposed to consist of a large number of particles bound together. Due to gravity every such particle is attracted towards the earth with a force proportional to the mass of the particle (Fig. 88). All the component forces act towards the centre of the earth ; but since the centre is at a large distance from the surface of the earth, all the forces may be considered to be acting vertically downwards. The resultant of these parallel forces is thus equal to the weight W of the body acting vertically downwards at a single point. This point is called the *centre of gravity* (written as C.G.). Thus in fig. 88 the point G is centre of gravity of the body.

When the body is rotated to a different position the attractions on the particles still remain the same in magnitude and direction, being parallel to one another and so the resultant passes through the *same point* as before.

Hence the ***centre of gravity*** *of a body is a point fixed relative to it, through which the resultant force due to earth's attraction on all the particles constituting the body, always passes in whatever way the relative position of the body changes, provided the size and shape of it remain unaltered.* In other words, the centre of gravity of a body is the point through which the line of action of the weight always passes.

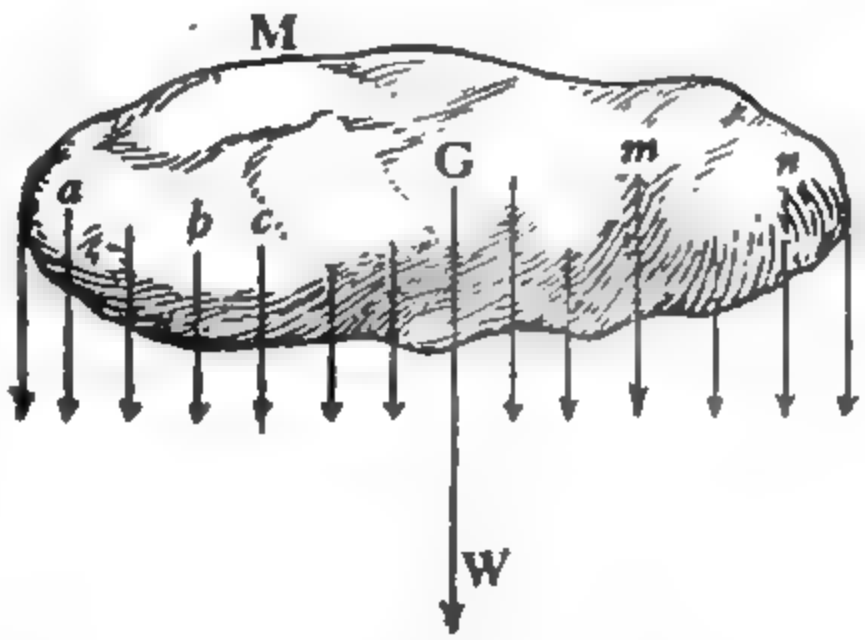

Fig. 88

As the line of action of the weight of the body passes through its C. G., a force equal to the weight, being applied through the C. G. in a vertically upward direction, would keep the body in equilibrium. Thus, if a body is suspended by a string, the C. G. lies vertically below the point of suspension.

Centre of Mass.—If we consider the masses of the particles constituting a body and if a system of forces, proportional in magnitude to the masses of the particles, be drawn. the point at which the resultant of all these parallel forces may be supposed to act, is called the **centre of mass** or *mass centre* of the body. Hence it is the point at which the whole mass of the body may be supposed to be concentrated. If the forces, proportional to the masses, be all drawn in the vertically downward direction, the centre of mass coincides with the C. G. of the body.

C. G. of Symmetrical Bodies.—The position of the C. G. of a body can be determined from the principle of Statics. But in some simple cases, it may be inferred from consideration of symmetry, the bodies in these cases being supposed to be of uniform density throughout.

Thus, the centre of gravity of,—

(1) a *uniform straight rod*, such as a piece of wire or a beam, etc., is at the middle point of its axis ;

(2) a *uniform circular lamina* or a *sphere* is at the *geometrical centre* ;

(3) a *uniform circular ring* is at the centre of the circle.

(4) a *uniform parallelogram* or a *regular parallelopiped*, is at the intersection of its diagonals ;

(5) a *uniform cylindrical body*, solid or hollow, is at the middle point of its axis ,

(6) a *uniform triangular plate*, is at its centroid.

59. Determination of the C. G. of a Lamina.—When a body is freely suspended from a point by a string, the C. G. of the body lies somewhere on the prolongation of the string by which the body is suspended.

Take a thin sheet of a metal piece and suspend it by a string attached at some point near its edge (Fig. 89). Hang a plumb line from the point of suspension and trace on the sheet the vertical line through the point of suspension. Similarly, suspend the sheet at two or three different points and find the vertical through the point of suspension in each case as before. It is found that the verticals cut one another at a single point. This point G is the required *centre of gravity* of the sheet.

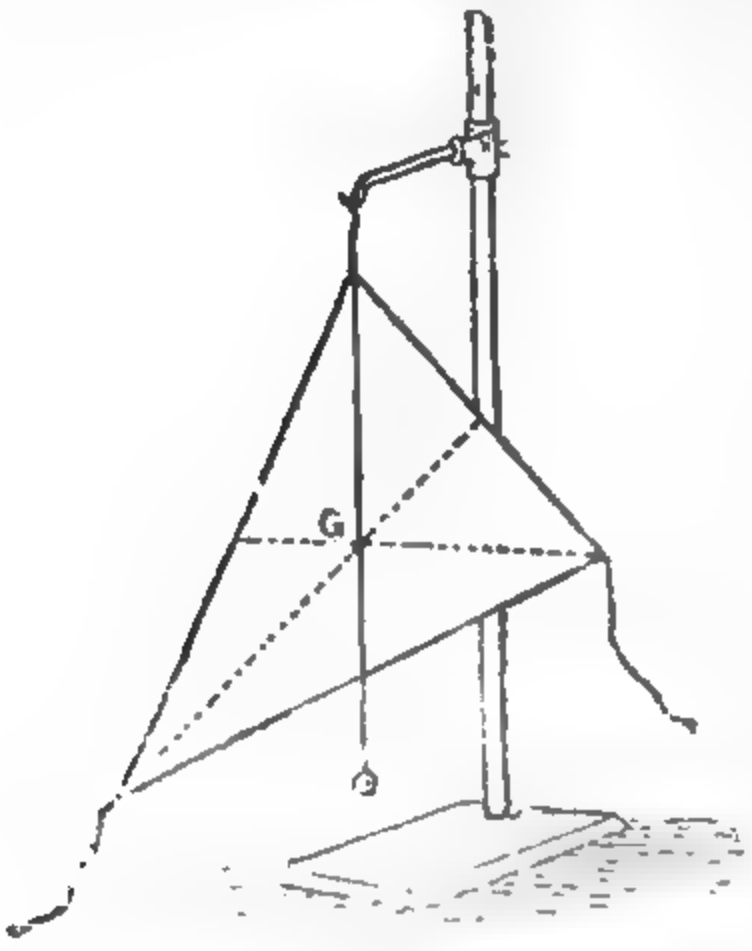

Fig. 89

The centre of gravity of a lamina may also be found by *balancing it* on a horizontal edge. The plate is placed on a table and moved towards the edge, until it is just on the point of toppling over. A line is then drawn on the under-surface of the lamina, along the edge of the table. The C. G. of the plate lies somewhere on this line. By turning the plate and proceeding as before, another line is obtained. The point of intersection of these two lines gives the position of the C. G. of the plate. If the plate is supported at the point, it remains balanced. The centre of gravity of an object of small thickness such as a bicycle may be determined by the method of suspension or balancing. But in any case the steering arm must be fixed rigidly, as any change of position of the front wheel will alter the position of the centre of gravity of the bicycle.

60. **Equilibrium of Bodies.**—If a body remains at rest under the simultaneous actions of two or more forces, it is said to be in equilibrium. As the weight of a body acts vertically downwards through its C. G., the resultant of the reactions at the points of support of the body in equilibrium must act vertically upwards through the C. G. of the body. So in the case of a body, *supported at one point*, for example, when suspended by a string attached at one point or when balanced on a point, equilibrium is possible, when the C. G. of the body coincides with the point or is above or below the point in the vertical line; further the force supporting the body or the *force of reaction* at the point of support must be equal and opposite to the weight of the body. If a body is *supported at two points*, the resultant reaction passes through some intermediate point and so for equilibrium, the vertical through the C. G. of the body must lie between the two points of support.

In the case of a body *supported at three or more points*, for example, tripod stand or a glass tumbler placed on a table, the area obtained by joining successively the outermost points of support is called the *base* of the body. Equilibrium is only possible, when the vertical through the C. G. falls within the area of base, for in that case resultant re-

action may possibly pass through the C. G. of the body. If it falls outside, the body overturns and it takes a new position of equilibrium, in which the vertical falls within the base. If a rectangular block of some heavy material is placed on an inclined plane and the inclination is gradually increased, it will be found that so long as the vertical through the C. G. of the block falls within the base area, the body will not topple over, whatever be its inclination.

Evidently then the wider is the base, the greater is the stability of the body ; for even if the body is sufficiently inclined, the vertical may still fall within the base. This is clearly shown by the *Leaning Tower* of Pisa and a certain building at the Benares Ghat. Again a quadruped stands or walks more stable than a man. A table with an extended base, may not turn over, even when a heavy weight is placed on one side of the top. To make it more stable, a mucilage bottle is given a conical shape with a wider base. In the ordinary upright position the C. G. of a man is near about the navel ; under different conditions, the man changes his form to come to a stable position. Thus a man carrying a heavy load at one hand, say the right, bends his body towards the left and often extends the arm on the side at full length. Here the C. G. of the man with the load is shifted towards the side of the load, but by the bending of the body or the extension of the hand towards the left, the C. G. is shifted. In fact it is easier to walk with two equal loads, one in each hand, than with the two in one hand. If the man carries the load on the back, he bends his body forward. When a man is seated on a chair with the legs hanging down, he is in an equilibrium position. If he inclines himself backwards, so that the chair rests on the two hind legs, the man has to extend his legs forward as far as possible. In the circus, while walking on a rope, the performer holds in his hand a long pole or an open umbrella, which helps him to keep the C. G. vertically above the rope.

61. Different States of Equilibrium.—It has been explained that a body remains in equilibrium, whenever the resultant upward reaction passes through the C. G. of the body. The *static equilibrium* *i.e.*, the equilibrium of a body at rest may be of three kinds, namely *stable*, *unstable* and *neutral*.

A body is said to be in a **stable equilibrium,** if on being slightly displaced, it tends to come back to its original position. Thus, a table or a cube resting on its side, a right circular cone (Fig. 90 P) or a funnel resting on its base, a pendulum at rest are examples of stable equilibrium.

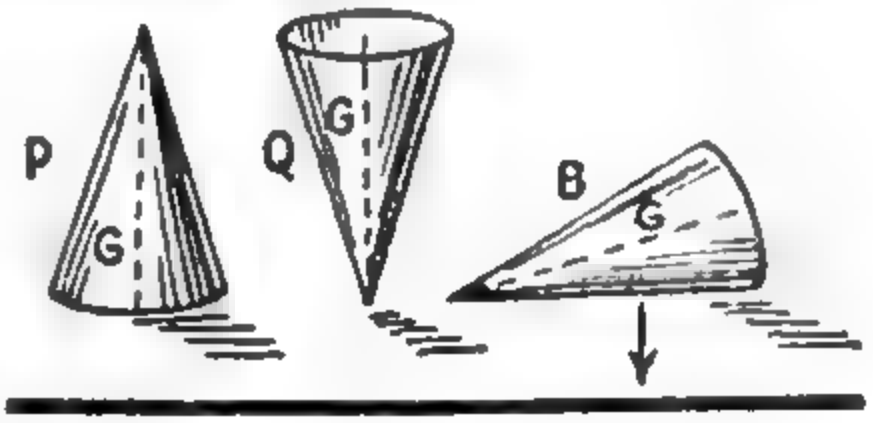

Fig. 90

A body is said to be in **unstable equilibrium** if, on being slightly displaced from that position, it does not come back to its original position but tends to move further away from it. Thus a cone resting

on the apex (Fig. 90Q), a stick balanced on the finger and an egg standing on its end are examples of unstable equilibrium.

A body is said to be in **neutral equilibrium,** if on being slightly displaced, it neither returns to its original position nor moves further away from it, but remains in its new position as before. A uniform sphere on a flat surface, a right cone (Fig. 90, B) and a cylinder resting on its side are examples of neutral equilibrium.

A body always tends to keep the position of the C. G. as low as possible and the three states of equilibrium depend on the relative position of the centre of gravity. In all cases of *stable* equilibrium, the C. G. occupies the *lowest* possible position, so that a slight displacement tends to *raise* the position of the C. G. Hence the body returns to its original position as soon as it is free to do so. In all cases of *unstable* equilibrium, the C. G. occupies the *highest* possible position, being always higher than the point of support ; so a slight displacement tends to *lower* the C. G. and thus to bring the body to a more stable position. In all cases of *neutral* equilibrium, to the C. G. is neither raised nor lowered by a slight displacement of the body and so the body remains in any position to which it may be displaced.

In case of bodies supported at one point, the point of support is higher than the C. G. in case of *stable* equilibrium, *lower* than in case of *unstable* equilibrium and *coincides* with the C. G. in case of *neutral* equilibrium. To keep the C G. at the lowest possible position a ship is loaded with ballast and a wagon loaded with heavy material at the bottom ; the lower deck of a double-decker omnibus should be loaded more. A cork cylinder weighed with a piece of lead hemisphere attached at the base or an ink-stand loaded at the base furnishes intersting examples of stable equilibrium position. Even when tilted to one side it would spring back to its equilibrium position when set free. The C. G. of the whole mass is raised by tilting and so the body comes back to its initial position when free.

Static Equilibrium and Potential Energy.—A body is in equilibrium position when its potential energy is either a *maximum* or *minimum*. If the potential energy is *maximum*, any disturbance gives rise to a couple tending to displace the body further. Thus, *when the potential energy of the body is maximum, its equilibrium is unstable.* Again when the potential energy is *minimum* any disturbance sets up a couple tending to bring back the body to its equilibrium position. Thus *when the potential energy of a body is minimum, its equilibrium is stable. In state of neutral equilibrium,* the potential energy of the body remains *constant* for any small displacement.

Dynamic Equilibrium.—A body *in motion* may be in stable equilibrium in a position, in which it is unstable when at rest. The equilibrium in this case depends on the motion of the body and the body is said to be in a state of *dynamic equilibrium.* A rotating top and a cyclist in motion furnish examples of this.

62. Simple Harmonic Motion.—A body is said to execute a *Simple Harmonic Motion*, when it shows the following characteristics,* —

(*i*) *The motion is periodic*, which means that the body in motion passes through the same succession of positions after each definite interval of time.

(*ii*) *The motion takes place in a straight line*, such that the acceleration of the body is always directed towards a fixed mean position called the position of equilibrium.

(*iii*) *The magnitude of the force acting upon the body at any point in its path is proportional to its displacement from the fixed mean position.*

The motion of a simple pendulum oscillating through a small arc, illustrates simple harmonic motion very approximately.

63. Pendulum.—A *simple pendulum* is defined to be a heavy point mass, suspended by a *weightless*, *inextensible* and *perfectly flexible* string from a fixed point about which the pendulum oscillates *without friction*. Evidently, it is not possible to have such a pendulum in practice, for we cannot isolate a single material particle, nor can we get a perfectly inelastic and weightless string which can oscillate without friction at any point. The nearest approach to such a pendulum consists of a small heavy metal ball, suspended by a very thin and inelastic thread. A *compound pendulum* consists of any heavy mass supported in such a way that it is capable of oscillating freely in a vertical plane about a fixed point or a fixed line as axis. A practical form of a compound pendulum is that of a clock, which consists of a rigid rod having at its lower end a heavy metallic mass and capable of turning about an axis at the upper end.

The heavy spherical mass A (Fig. 91) suspended at the end of the string is called the bob of the pendulum. The distance between the point of suspension O and the centre of gravity of the bob, is called the *effective length* of the pendulum. One complete to-and-fro movement of a pendulum is called an *oscillation*. A pendulum is said to execute **one complete oscillation,** when starting from any point in its path it comes back to the same point, moving in the same direction. Thus when the bob starts from A, moves to B, back to A, then to C and back to A, it makes one complete oscillation. The maximum displacement AB or AC of the bob on either side of its position of equilibrium is called the **amplitude** of vibration. The time to perform one complete oscillation is called the *time period* or simply the **period** of oscillation.

The number of complete oscillations performed in one second by a vibrating body is called the **frequency** of vibration. If the period be T, and the frequency of vibration be n, then

$$nT = 1 \quad \text{or} \quad n = \frac{1}{T}$$

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* For a detailed study of Simple Harmonic Motion, the student is referred to Chapter II of the part on **Sound** of this Book.

The **phase** of a body executing S. H. M. gives the state of motion at a particular instant and thus determines the *position* of the body in the path and *direction* of motion at the instant.

64. Motion of a Simple Pendulum.—Let a spherical bob A of mass m be suspended by a string of length l, from the point O (Fig. 91) When at rest at A, the weight of the bob is just balanced by the tension of the string

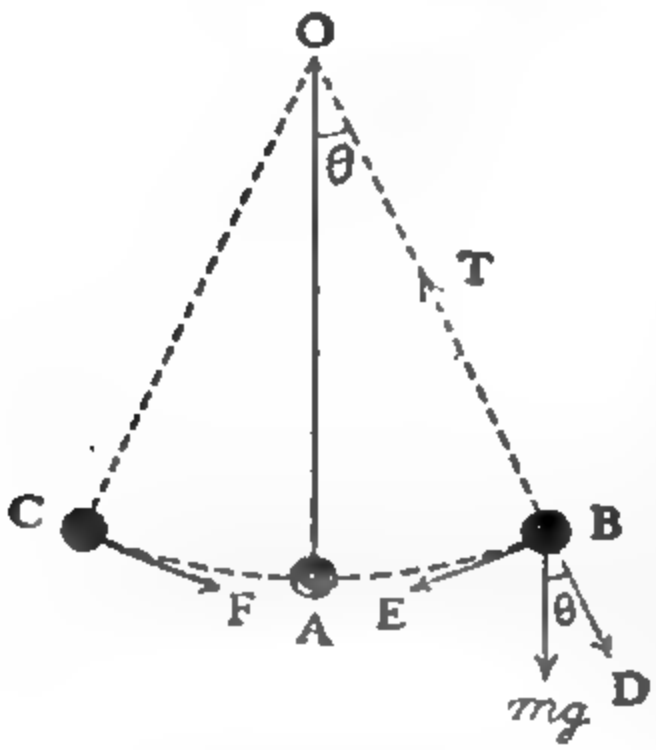


Fig. 91

Let the pendulum be now displaced through a small angle θ to the position OB. The weight mg of the bob acting vertically downwards can be resolved into $mg \cos \theta$ along BD, the direction of the string and $mg \times \sin \theta$ along BE tangential to the path AB. The component $mg \cos \theta$ is balanced by the tension T of the string. The tangential component $mg \sin \theta$ is the only effective force which tends to move the pendulum towards the position of equilibrium A.

If θ is very small, $\sin \theta = \frac{\text{arc}}{\text{radius}} = \frac{\text{displacement}}{\text{length of pendulum } (l)}$

$\therefore$ The effective force $= \frac{mg}{l} \times$ displacement.

Thus the acceleration of the bob at the instant under consideration

$$\frac{g}{l} \times \text{displacement} \qquad \ldots(64,1)$$

As g and l are constants, the acceleration along the tangent is proportional to the displacement from the mean position of rest; *hence the motion of the bob is simple harmonic in nature.*

It can be proved that $T = 2\pi\sqrt{\frac{l}{g}}$ ... (64,2)

65. Laws of Pendulum.—The oscillations of a simple pendulum are governed by the following laws :—

Law I. **Law of Isochronism.**—*The oscillations of a simple pendulum are isochronous, i.e.*, they are effected in equal intervals of time, provided the amplitudes are small. The first law was discovered in 1582 by Galileo by casually noticing the swings of a lamp hanging by a chain from the roof of the Leaning Tower at Pisa. He timed the oscillations by counting his pulse and found these to occur in equal intervals, whether the lamp oscillated through smaller or greater extent.

Law II. **Law of Length.** -*The period of oscillation of a simple pendulum varies directly as the square root of its effective length,*

i.e., $T \propto \sqrt{l}$. Thus if the length of the pendulum be increased to 4, 9 or 16 times, the period will be 2, 3 or 4 times respectively. As the length of a pendulum changes with temperature, the period will *slightly* be affected by changes of temperature.

The law of length was also discovered by Galileo from the same observation. The laws were afterwards verified by him by more accurate observation. In 1658, Huygens first applied a pendulum to regulate the motion of clocks.

Law III. **Law of Gravity.**—*The period of oscillation of a simple pendulum is inversely proportional to the square root of the acceleration due to gravity at the place of observation ; i.e.,* $T \propto 1/\sqrt{g}$.

If the pendulum be taken from one place to another, where the value of g is greater the period will be smaller *i.e.*, the pendulum will vibrate more rapidly.

Law IV. **Law of Mass.**—*The period of oscillation of a simple pendulum is independent of the mass or the material of the bob, provided the effective length remains the same.*

The period will remain the same, so long as the distance from the point of suspension to the centre of gravity of the bob remains the same even if the bob be of different sizes and of different materials.

If a simple pendulum be made of a thin string and a hollow spherical bob, then its effective length is the distance between the point of suspension and the centre of the bob. If now the bob is half-filled with mercury, the centre of gravity of the bob and hence that of the system lowers down resulting in a larger effective length. Consequently its time period becomes greater. But if it is completely filled, its C. G. returns to the centre of the bob and there is no change in the period.

It is found from eqn. (64,2) that

If l and g are constants,	T is constant	... Law I & IV
If g is constant,	$T \propto \sqrt{l}$	... Law II
If l is constant,	$T \propto 1/\sqrt{g}$	... Law III

66. Verification of the Laws.—To verify the laws, take a small metal ball suspended by a fine thread from a hook attached to a suitable stand (vide J. Chatterjee's Intermediate Practical Physics.)

First Law.—Set the pendulum into oscillation in a vertical plane, taking care that the amplitude is very small. By means of a stop watch, note the total time required to perform a number of complete oscillations, say 20. Then dividing the total time by the number of oscillations, calculate the time period. Repeat the experiment with different amplitudes upto the limit of 4° and get the period in each case. It will be found that within the limits of the experimental errors the time period in all cases is equal.

Second Law.—Take a pendulum, the length of which can be altered. Measure the diameter of the bob at several places by a slide

callipers and get the mean radius : add to this the length of the string ; the sum gives the effective length of the pendulum. Note the time required for a number of complete oscillations and hence find the time period. Change the length of the string to different values and find the time period in each case. Square the time period (T) and divide by the corresponding length (l).

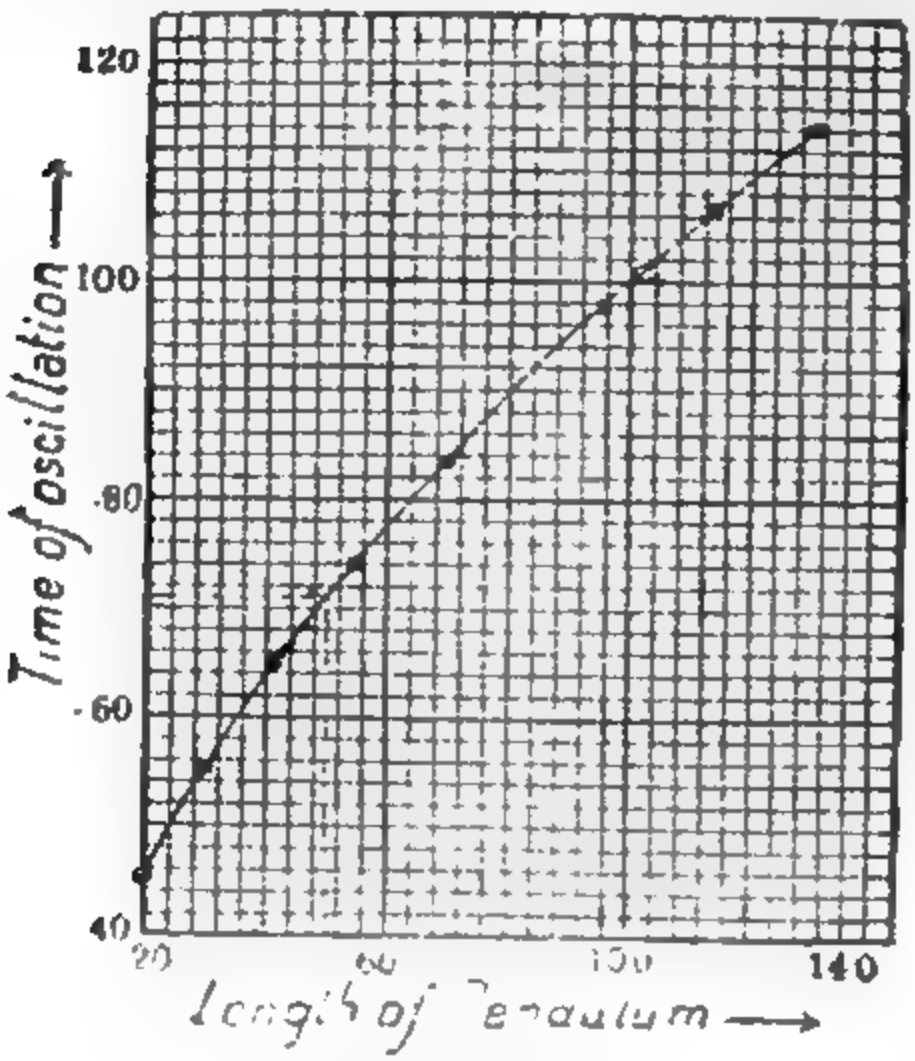


Fig. 92

It will be found that, $\frac{T^2}{l}$ is constant, *i.e.*, $T \propto \sqrt{l}$

The relation between the length of a pendulum and the time period can be represented by a graph (Fig. 92), in which the length (l) is measured along the abscissa and the corresponding time period (T) along the ordinate. The graduations along the abscissa represent length in cm. and those along ordinate one-hundredth of a second. The graph will be found to be parabolic in nature as is also evident from the mathematical relation between l and T. When a pendulum clock gains or loses, it is regulated by altering the effective length of the pendulum by lowering or raising the bob by a screw nut below it.

Third Law.—This law can be verified by taking a given pendulum to different places on the surface of the earth or to different altitude at the same place. It will be found that for each place the product of the period of oscillation and the square root of the acceleration due to gravity is constant,

i.e., $T \sqrt{g} = \text{constant}$ or $T \propto \frac{1}{\sqrt{g}}$

Take pendulums of different materials and of different lengths by altering the length of the string keep the effective length of the pendulum the same in each case. Determine the time period of each of these pendulums in the usual way, which will be found to be equal in each case.

Seconds Pendulum.—A simple pendulum which swings from one extremity to another in one second, or in other words, whose time period is two seconds is called a *seconds pendulum*. Hence the time period of a seconds pendulum is 2 seconds.

From eqn. (64,2), $2 = 2\pi \sqrt{\frac{l}{g}}$ whence $l = \frac{g}{\pi^2}$, ...(67,1)

As the value of g varies from place to place on the surface of the earth and at different altitudes, the length of the seconds pendulum is

different at different places. Taking the average value of g as 981 cm. per sec. per sec., the corresponding length of the seconds pendulum is given by

$$l = \frac{981}{\pi^2} = \frac{981}{9{\cdot}87} = 99{\cdot}39 \text{ cm.}$$

Taking $g = 32{\cdot}2$ ft. per sec.^{2}, $l = \frac{32{\cdot}2}{\pi^2} = \frac{32{\cdot}2}{9{\cdot}87}$

$= 3{\cdot}26$ ft. $= 39{\cdot}12$ inches.

68. Determination of the value of 'g' by a simple Pendulum.—If is evident from eqn. (64,2) that a simple pendulum furnishes a very convenient method of determining the value of the acceleration due to gravity at a place.

From eqn. (64,2), $T = 2\pi\sqrt{\frac{l}{g}}$ $\therefore$ $T^2 = 4\pi^2\frac{l}{g}$

whence $g = 4\pi^2\frac{l}{T^2}$ ...(68,1)

Taking a simple pendulum, its length (l) is carefully measured and the mean time period (T) is accurately determined from a number of observations. Substituting in eqn. (68,1), the value of 'g' is calculated.

Instead of taking the mean value of T, the value of l and T, may be plotted in a graph as in fig. 92, from which the value of T, corresponding to any suitable value of l is obtained. Thus on substituting in eqn. (68,1), the value of 'g' is calculated.

If the experiment is performed at the same place but with bobs of *different* substances, it will be found that as the length of the pendulum remains the same, the time period will be the same at the same place. This shows that acceleration due to gravity at the same place is the same for all bodies.

Variation in the value of T at different Places.—As the value of g varies from place to place, the value of the time period T of a simple pendulum of given length also changes; thus, if g decreases, T increases and *vice versa*. It has been explained in art. 47, that the value of g increases from the equator to the poles. Hence if a pendulum clock be taken from the equator to the poles, T decreases and so the clock *gains i.e.*, it goes faster. Again if at the same place, the clock is taken from the surface of the earth to the top of a mountain or to the bottom of a mine, the value of g decreases and so T increases; hence the clock *loses i.e.*, it goes slower.

Examples :—

1. Find the length of a simple pendulum beating $\frac{1}{2}$th of a second at a place where the value of g is 980 cm. per sec. per sec.

A. Let the length of the pendulum $= l$ cm.

The time period (t) of this pendulum $= 2 \times \frac{1}{2} = \frac{3}{2}$ seconds.

$$\therefore \quad \tfrac{3}{2} = 2\pi\sqrt{\frac{l}{980}} \text{ or } l = \tfrac{9}{4} \times \frac{980}{4 \times (3{\cdot}1416)^2} \text{ whence } l = 55{\cdot}84 \text{ cm.}$$

2. A faulty seconds pendulum loses 20 seconds per day. Find the required alteration in its length so that it may keep correct time; given $g = 32$ ft. per sec. per sec. [Pat. U.—1953]

A. Let the length of the faulty seconds pendulum be l and the alteration in its length to make it correct be x. Since it is losing time, its effective length must have increased and so its correct length should be $l - x$.

A correct seconds pendulum beats seconds and hence,

$$1 = \pi\sqrt{\frac{l-x}{g}} = \pi\sqrt{\frac{l-x}{32}} \qquad \ldots \quad (1)$$

The faulty pendulum loses 20 seconds a day of 86400 seconds and so its number of beats in one day $= 86400 - 20 = 86380$. Its time for single beat is therefore,

$$\frac{86400}{86380} = \pi\sqrt{\frac{l}{32}} \qquad \ldots \quad (2)$$

From (2), $\left(\frac{86400}{86380}\right)^2 = \pi^2 \frac{l}{32}$ or, $l = \left(\frac{86400}{86380}\right)^2 \times \frac{32}{\pi^2}$

From (1), $1 = \pi^2 \frac{l-x}{32}$ or, $l - x = \frac{32}{\pi^2}$

Substituting the value of l, we get

$$\left(\frac{86400}{86380}\right)^2 \times \frac{32}{\pi^2} - x = \frac{32}{\pi^2}$$

$$\text{or, } x = \frac{32}{\pi^2}\left[\left(\frac{86400}{86380}\right)^2 - 1\right] = \frac{32}{3{\cdot}1416^2}\left[\left(1 + \frac{20}{86380}\right)^2 - 1\right]$$

$$= \frac{32}{9{\cdot}8696}\left[1 + \frac{40}{86380} - 1\right] = \frac{32}{9{\cdot}8696} \times \frac{40}{86380} = 0{\cdot}0015 \text{ ft.}$$

3. A pendulum which keeps correct time at the foot of a mountain loses 10 [illegible] a day [illegible] taken to the top. Find the height of the mountain. (Neglect [illegible] the mountain and take the radius of the earth as 21×10^6 [illegible].

[illegible] the height of the mountain be h ft.; let the acceleration due to gravity [illegible] and the top of the mountain be g and g' respectively. Then

$$\frac{g}{g'} = \frac{1}{(21 \times 10^6 + h)^2} \text{ or } \frac{g}{g'} = \left(\frac{21 \times 10^6 + h}{21 \times 10^6}\right)^2 \qquad \ldots \quad (1)$$

[illegible] clock make 86400 and 86384 oscillations respectively at the bottom and [illegible] the mountain, is a day *i.e.*, in 86400 seconds.

$$\frac{g}{g'} = \left(\frac{86400}{86384}\right)^2 \text{ and } \frac{86400}{86384} = 2\pi\sqrt{\frac{l}{g'}} \text{ whence } \frac{g}{g'} = \left(\frac{86400}{86384}\right)^2 \qquad \ldots \quad (2)$$

From (1) & (2), $\left(\frac{21 \times 10^6 + l}{21 \times 10^6}\right)^2 = \left(\frac{86400}{86384}\right)^2$ or $1 + \frac{l}{21 \times 10^6} = 1 + \frac{16}{86384}$

whence $h = 3890$ ft. approximately.

GALILEO GALILEI (1564—1642)

Galileo was born at Pisa on February 18, 1564. He belonged to a noble family of Florence and many members of this family had filled many positions of honour in the State. But Galileo's father Vincenzo was a mathematician and a lover of music, although he was very poor. Galileo was sent to the University of Pisa to study medicine at the age of seventeen. From early boyhood he was interested in mathematics and toy making.

In 1583, Galileo, attending a lecture on Euclid, became facinated with the subject and began to devote all his spare time to it. Afterwards in 1585, he gave up medicine and returned to his father's house.

Fig. 93—Galileo Galilei

Here he seriously studied mathematical problems and in 1589 became a lecturer in mathematics at the University of Pisa.

While at Pisa he discovered the famous laws of falling bodies in contradiction to the theory of Aristotle, who taught that bodies fell in a time proportional to their weight. Galileo dropped balls of different sizes from the leaning tower of Pisa and showed that they all came down in equal time. But people of those days, unwilling to accept any experimental demonstration against popular belief, so much disfavoured him that he had to leave Pisa. In 1592, Galileo was appointed Professor of Mathematics at Padua University.

In 1604, the appearance of a bright new star in the constellation of Serpentarius, gave him an inspiration of Astronomy. The difficulty of observing stars and planets with naked eyes was very embarrassing to him. In 1609, hearing of the invention of a Dutch optician for bringing distant objects near, Galileo in one night succeeded in working out the principle and at once set to work to construct a telescope and having produced a satisfactory one turned it towards the heavens with results, which in his days, were truly astonishing. He observed the motion of Sun spots, phases of Venus and Mercury and discovered the rings of Saturn and satellites of Jupiter. In this way he established the truth of Copernican system which holds that the Earth and Planets revolve round the Sun. Before his time and even during his time there was universal belief that all heavenly bodies revolved round the Earth. Galileo's assertion of Copernican system could not be tolerated by the Church, and as a result he was tried by the Inquisition and [illegible] before which he made a formal abjuration of his theory. He returned to his village home at Arcetri, near Florence as a broken and disappointed old man. He died in January 8 1642.

Summary

Every particle of matter in the universe attracts every other particle with a force acting along the line joining them, with a force which varies directly as the product of their masses and inversely as the square of the distance between them.

Gravity is the force of attraction exerted by the earth on bodies lying on or near its surface.

Value of acceleration due to gravity g is constant at the same place but varies from place to place. The value of g increases from the equator to the poles but decreases with height.

The weight of a body is the attractive force exerted on the body by the earth. It is proportional to the mass of the body. Hence it varies in the same way as the value of g, although the mass of a body remains constant.

The centre of gravity of a body is a point fixed relative to it through which the line of action of the weight of the body passes.

A common balance is used to measure the mass by the method of counterpoise. A Spring balance measures the weights directly.

States of Equilibrium (i) *stable*, (ii) *unstable*, and (iii) *neutral*.

Laws of Falling Bodies.—(1) In vacuum all bodies starting from rest fall with equal rapidity. (2) The velocity acquired by a falling body starting from rest in a given time is directly proportional to the time. (3) The space traversed by a falling body in a given time starting from rest is directly proportional to the square of the time.

A Simple pendulum in practice consists of a small heavy metal sphere suspended by a very light and inelastic thread.

The *time period* of a simple pendulum is given by $T = 2\pi\sqrt{l/g}$.

From the above relation, the value of g at a place can be determined by a simple pendulum.

A Seconds pendulum is a simple pendulum whose time period is two seconds.

Variation of T.—At a place above or below the surface of the earth the value of g decreases and so T increases. The value of g increases from the equator to the poles and so T *decreases*.

EXERCISES ON CHAPTER IV

Reference.

1. What is meant by acceleration due to gravity? How does it vary from place to place? How does the rotation of the earth affect this quantity? (And. U.—1951) — Art. 47

2. Distinguish between mass and weight of a body and state the units in which they are expressed in C. G. S. system (C. U.—1941; *Cf.* Nag. U.—1950) — Art. 49

3. Explain the distinction between the absolute measure and the gravitational measure of force and show how one may be expressed in terms of the other. — Art. 48

Express wt. of 10 kgm. in dynes and the value of a dyne in gramme weight. (Dac. U.—1942)

A. $10{,}000g$ dynes; $1/g$ gm. wt.

4. Distinguish between mass and weight. How are the mass and weight of a body affected by variations of latitude? Is weight an essential property of matter? (Pat. U.—1930, '32) — Art. 48 & 49

5. State the laws of falling bodies and illustrate them by suitable examples. (C. U.—1941) — Art. 50

6. A stone is dropped from a rising balloon at a height of 200 ft. above the ground and it reaches the ground in 6 secs. What was the velocity of the balloon just at the moment when the stone was dropped? (C. U.—1942) — Art. 51

A. 62·6 ft. per sec. upwards.

7. A body of mass 50 gm. is allowed to fall freely under the action of gravity. What is the force acting upon it? Calculate the momentum and the kinetic energy it possesses after five seconds. ($g = 980$ cm. per sec.^{2}) (Del. H. S.—1951; C. U.—1937) — Art. 51

A. 49×10^3 dynes; 245×10^3 gm. cm/sec.; $60{\cdot}025 \times 10^7$ ergs.

Reference

Art. 51 S. A stone is let fall from the top of a tower 180 ft. in height and at the same time another is projected vertically upwards with a velocity of 60 ft. per sec. Find where and when the two will meet.

A. 36. ft. above the ground, 3 seconds.

Art. 52 9. A ball is thrown horizontally from the top of a tower with a velocity of 80 ft. per sec. Show that the curve traced by the ball is a parabola. (Del. U.—1942)

Art. 53 10. Weights of 40 and 50 gm. are connected by a string passing over a smooth fixed pulley. If the system travels 2·18 metres in the first two seconds from start, what is the value of 'g' in cm. per sec. per sec.? (P. U.—1932)

A. 981 cm. per sec.²

Art. 53 11. A man weighing 12 stones on earth is standing on a lift. Find his weight as recorded by a weighing machine (or spring balance) when (*a*) the lift moves upwards with a uniform velocity of 10 ft./sec. (*b*) the lift moves downwards with a uniform acceleration of 4 ft./sec.² and (*c*) lift moves upwards with a uniform acceleration of 4 ft./sec.²

A. 12 st. 10·5 st. 13·5 st. (U. P. B.—1942)

Art. 53 12. Two masses of 80 and 100 gm. are connected by a string passing over a smooth pulley. Find the tension of the string when they are in motion. Find also the space described in 4 seconds g being 981 C. G. S. (Mad. U.—1951)

A. 87200 dynes; 872 cms.

Art. 53 13. A man weighing 10 stones is sitting on a lift which is moving vertically with an acceleration of 8 ft. per sec. per. sec. Prove that the pressure on the base of the lift is greater when it is ascending than when it is descending, and compare the pressures. (Pat. U.—1931)

A. $R_1 : R = 5 : 3$.

Art. 54 14. Describe and explain the action of a spring balance.

Compare the action of a spring balance with that of a common balance. (C. U.—1950; *Cf.* Nag. U.—1952; Dac. U.—1929)

Art. 56 15. Explain why a balance which is sensitive cannot be stable. (P. U.—1952)

Art. 56 16. What are the requisites of a good balance? Explain them. (Raj. U.—1952; C. U.—1950; Dac. U.—1934; Del. H. S.—1950; Pat. U.—1952; U. P. B.—1952; P. U.—1952)

Art. 56 & 57 17. Explain the term sensitiveness of a balance. Describe a common balance and indicate how the correct weight of a body can be found by the method of vibrations. (Mys. U.—1952; Cf. And. U.—'51)

Art. 56 & 57 18. What are the requisites of a good balance? A balance with unequal arms is used for weighing. The apparent weights of the same body when placed in the two pans are 158·00 and 158·25 grammes respectively. Find the ratio of balance arms. (Dac. U.—1934; C. U.—1950)

A. $\sqrt{633} : \sqrt{632}$.

Art. 57 19. A body is placed on one pan of a balance, whose arms are unequal and is found to weigh W_1 gm. It is then removed to the

Referenc

other pan and weights W_2 gm. Show that the actual weight is $\sqrt{W_1 W_2}$ gm. (U. P. B.—1926)

20. A dealer has correct weights, but one arm of his balance is $\frac{1}{20}$ th part shorter than the other. If he sells two quantities, each apparently weighing 9·5 lb. at Rs. 40 per pound, weighing one in one scale and other in the other, what will he gain or lose. (Nag. U.——1952) Art. 57

A. Re. 1. (loss).

21. The only fault in a balance being the inequality in the weights of the scale pans, what is the real weight of a body which balances 10 lbs. when placed in one scale-pan and 12 lbs. when placed in the other? (Dac. U.—1933) Art. 57

A. 11 lbs.

22. Define centre of gravity of a body. How is the C. G. of a plane lamina determined? (Mad. U.—1951) Art. 58 & 59

23. Distinguish clearly between centre of gravity and centre of mass of a body. (Utkal. U.—1951) Art. 58

24. Define simple harmonic motion. Prove that a simple pendulum when disturbed executes simple harmonic motion. How can you use it to determine the force of Earth on a unit mass? (Dac. U.—1943) Art. 63 & 64

25. Explain S. H. M. and the terms amplitude, frequency and phase associated with it. Give an example of S. H. M. and illustrate your answer with reference to it. (C. U.—1949) Art. 61 & 63

26. A pendulum 96 cm. long loses 20 seconds per day. Find how much must it be shortened to keep correct time. Art. 69

A. $\frac{4}{9}$ mm.

27. A seconds pendulum which keeps correct time on the surface of the earth is taken to the surface of the moon. Find the time period there, taking the mass of the earth to be 81 times that of the moon and the radius of earth to be 4 times that of the moon. Art. 65

A. 4·5 seconds.

28. A pendulum that beats seconds at a place where $g = 980$ cm./sec.^{2} is taken to a place where $g = 975$ cm./sec.^{2} How many seconds per day does it lose or gain? (Mad. U.—1950) Art. 65

A. It loses 220·4 sec.

29. A pendulum which beats seconds at a place where $g = 32{\cdot}02$, is taken to a place where $g = 32$. How many seconds does it gain or loss per day? Art. 65

A. It loses 27 seconds each day.

30. What is a simple pendulum? State laws of oscillation of a simple pendulum and state how would you verify them experimentally. What is meant by effective length? (Pat. U.—1953; C. U.—1943, '47, '49; Mad. U.—1950) Art. 65

31. A simple pendulum has a hollow spherical bob attached to its thread. Will the period alter if the hollow bob is half-filled with mercury? (C. U.—1950) Art. 65

Reference

Arts. 66 & 67

32. State the laws of vibration of a simple pendulum and describe an experiment to verify the relation between the period and the length of the pendulum. Calculate the length of a seconds pendulum, assuming 'g' to be 980. **(Dac. U.—1931, '49)**

A. 99·29 cm.

Art. 68

33. Give an accurate method of measuring the acceleration due to gravity and explain the method of calculating the result. Why does the weight of a body change when taken from Delhi to Madras? **(Del U.—1943)**

Art. 68

34. Describe an accurate method of measuring the acceleration due to gravity and explain the principle. **(Del. U.—1952, '53; U. P. B.—1951; *cf.* Del. H. S.—1953)**

Art. 68

35. Will a pendulum clock gain or lose when taken to the top of a mountain? **(C. U.—1949)**

Art. 68

36. Will the periodic time of a simple pendulum vary, if it is taken from the equator to the north pole? Explain your answer. **(C. U.—1948)**

Art. 68

37. Explain why a pendulum clock loses time when taken to the top of a hill. **(U. P. B.—1941)**

Art. 68

38. A pendulum clock was regulated to give correct time at Dehradun. When taken to Mussoorie it was found to lose time. Why? **(U. P. B.—1944)**

CHAPTER V

WORK AND ENERGY

69. Work.—The term *work* gives us an idea of *something being done.* Work is said to be done by a force when the point of application of the force is displaced along some direction. A man in raising a load does work, a horse in drawing a carriage along a road or an engine in pulling a wagon does work. In such cases, the body moves in the direction in which the force acts and also against an equal and opposite force. So work may also be said to be done *against the opposing force.* Thus, the man in raising the load does work *against* the force of gravity : and the horse, in drawing the carriage, does work *against* the force of friction.

Measurement of Work.—*The magnitude of the work done, is given by the product of the force and the distance through which the point of application of the force is moved,* when the direction of the force and that of displacement coincide. Thus, if a force P acting on a body displaces it through a distance *s*, then the *work* done is given by

$$W = P.s \quad \ldots \quad (69,1)$$

If the direction of the force and that of the displacement are not the same, but they make an angle θ with each other, then the component of the force in the direction of displacement is effective in doing work. In this case the work done by the force is given by

$$W = P.\ s.\ \cos\theta \quad \ldots \quad (69,2)$$

It is evident from eqn. (69,1) that *if there is no displacement, no work is done.* Again as a force has no component in a direction at right angles to its own line of action, no work is done by or against a force in a direction perpendicular to its line of action.

70. Units of Work.—The unit of work is the work done when a unit force acts through a unit distance in its own direction. Evidently the unit of work depends on the units of force and distance. The **absolute unit of work** is work done by the *absolute unit of force* acting through unit distance.

In the F. P. S. system, the *absolute unit of work* is the work done by a force of 1 *poundal* in moving its point of application through a distance of 1 *foot* along the line of action of the force. This unit is termed a **foot-poundal.** In the C. G. S. system, the *absolute unit of work* is the work done by a force of 1 *dyne* in moving its point of application through a distance of 1 *cm.* in its own direction. The unit is termed **an erg.** As this unit is extremely small, the unit chosen for practical purposes is the **joule ;** 1 joule being 10^7 ergs.

Gravitational Unit of Work.—In practice, work is often expressed in the gravitational unit. The *gravitational unit of work* is the

work done in raising a unit mass vertically upwards against the force of gravity through a unit distance. In the F. P. S. system, the gravitational unit of work is the work done in raising a mass of *one pound* through a vertical distance of *one foot*. The unit is termed one **foot-pound** (ft. lb.). In the C. G. S. system, this unit is termed one **gramme-centimetre** and is equal to the work done in raising a mass of *one gram* through a vertical distance of *one centimetre*. As a gramme-centimetre is very small, the unit chosen by the engineers is the *kilogramme-metre* which is the work done in raising a mass of 1 *kgm.* through a vertical distance of 1 *metre*.

Relation between the two units of work.—It has been explained before that the gravitational unit of force is g times the absolute unit of force. Hence,

Gravitational unit of work $= g$ times absolute unit of work.
$\therefore$ 1 ft. lb $= 32\cdot2$ ft.-poundals and 1 gramme-centimetre $= 981$ ergs. As 1 poundal $= 13825$ dynes and 1 foot $= 30\cdot48$ cm.

1 ft. poundal $= 30\cdot48 \times 13825$ ergs $= 4\cdot214 \times 10^5$ ergs.
Also 1 foot-pound $= 32\cdot2 \times$ foot-poundals $= 32\cdot2 \times 4\cdot214 \times 10^5$ ergs
$= 1\cdot36 \times 10^7$ ergs.

Hence, **1 foot-pound $= 1\cdot36$ joules.**

Work is of the dimension of force × displacement. Now dimension of force is MLT^{-2} and that of displacement is L. Hence work has a dimension of ML^2T^{-2}.

71. **Power.** - *The power* or *activity* of an agent is the *rate of doing work* and is measured by the quantity of work done by the agent in unit time. The absolute unit of power in the F. P. S. system is the rate of doing one ft.-poundal of work per second and in the C. G. S. system it is the rate of doing one erg per second. The practical unit of power, as used by the engineers, is termed a *horse-power*.

The term was first used by James Watt, the inventor of steam engine. In order to establish a relation between the working capacity of a horse and of his newly invented engine, he performed an experiment in which on average a weight of 150 lbs. of coal was raised by a horse from a coal-pit through a height of 220 ft. in some minutes. The work done by the horse in one minute was thus equal to (150×220) or 33,000 ft.-lbs., which was therefore considered to be the average rate of working of a horse. Although the estimate is found to be too high, the term has been retained.

Hence, a **horse-power** (H. P.) is the power of an agent doing 33000 ft.-lbs. of work per minute or 550 ft.-lbs. per second.

There is another unit of power called the watt, so named after James Watt ; this is defined as 1 joule per sec. *i.e.*, 10^7 ergs per second.

Now 1 H. P. $=$ 550 ft.-lbs. per second
$= 550 \times 1\cdot36 \times 10^7$ ergs per second
$= 746$ joules per sec. $= 746$ watts

$\therefore$ 1 kilowatt $= 1000$ watts $= (1000 + 746) = 1\cdot34$ H. P.
$= (1\cdot34 \times 550)$ ft.-lbs. per sec.

$\therefore$ 1 kilowatt-hour $= (1{\cdot}34 \times 550 \times 60 \times 60) = 2653{,}200$ ft.-lbs. of work.

The power being work per unit time, its dimension would be $ML^2T^{-2} \div T$ or ML^2T^{-3}.

Examples :—

1. If clouds were one mile above the earth and rain fell sufficiently to cover 1 square mile at sea level, $\frac{1}{2}$ inch deep how much work was done in raising the water to the clouds? (C. U.—1920)

A. Volume of the rain water $=$ 1 sq. mile $\times \frac{1}{2}$ in. $= (1760 \times 3)^2 \times \frac{1}{2 \times 12}$ cu. ft.

Mass of this rain water $= (1760 \times 3)^2 \times \frac{1}{24} \times 62{\cdot}5$ lbs.

$\therefore$ Work done in raising this mass through 1 mile $= \frac{(5280)^2}{24} \times 62{\cdot}5 \times 1760 \times 3$ ft. lbs. $= 383328 \times 10^6$ ft. lbs.

2. A man weighing 140 lbs. takes his seat on a lift which weighs 2 tons. He is taken to the 3rd floor, which is at height of 75 ft. from the ground floor in 2 minutes. Calculate the work done and the power required in the process.

A. The total weight of man and the lift $= 140 + 2 \times 2240 = 4620$ lbs.

Work done in raising this mass through 75 ft. $= 4620 \times 75 = 346500$ ft. lbs.

As this work is done in 2 minutes,

Power required $= \frac{346500}{2 \times 60}$ ft. lbs. per sec. $= \frac{346500}{2 \times 60 \times 550}$ H. P. $= 5{\cdot}25$ H. P.

3. An engine of mass 5 tons drags a train of mass 23 tons up a slope of inclination 1 in 28 at 15 miles per hour. If the frictional resistance be equivalent to a force of 0·4 ton weight parallel to the rails, find the horse power of the engine.

A. The speed of the train $= \frac{15 \times 1760 \times 3}{60 \times 60} = 22$ ft. per second.

Total mass of the engine and train $= 28$ tons $= (28 \times 2240)$ lbs. $= 62720$ lbs.

$\therefore$ Effective force against motion along the inclination.

$= (62720 \times \frac{1}{28} + 0{\cdot}4 \times 2240)$ lbs. wt $= (2240 + 896)$ lbs. wt.

$\therefore$ Work done by the engine per second $= (2240 + 896) \times 22$ ft. lbs.

Hence power of the engine $= \frac{(2240 + 896) \times 22}{550} = 125{\cdot}44$ H. P.

4. Show that if a gas expands against a constant external pressure the work done by gas is equal to the product of the pressure and the increase in volume.

A. Let a mass of gas be contained in a cylinder with an air-tight piston of cross-sectional area A. Let p be the pressure exerted on the gas and let v_1 be its original volume.

Suppose now that the gas expands to a volume v_2 and as a consequence the piston is displaced through a small distance x. As the pressure is supposed to remain the same during the change, the force on the piston $= Ap$ and hence the external work done by the gas during expansion $= Ap \times x = p\ (v_2 - v_1)$.

The result holds good irrespective of the shape of the containing vessel.

72. Energy.—The *energy* of a body is defined as its capacity for doing work and is measured by the total quantity of work that a body or a system can perform under the circumstances in which it is placed. Hence the *units* and the *dimension* of energy are identical with those of work.

The energy of a body must not be confused with its power. *Energy* indicates the *total quantity of work* the body can do, while *power* indicates the *rate* at which the work is done. We shall consider here only *mechanical energy*. It is of two kinds *viz.*, kinetic and potential.

Kinetic Energy.—The *kinetic energy* of a body is the energy it possesses by virtue of its motion. It is measured by the amount of work that the body can do against an external impressed force, before coming to rest.

A ball rolling on a surface possesses kinetic energy, for on striking against a similar ball it can set the latter in motion against the frictional force of the ground. A rifle bullet moving with a very high velocity possesses considerable kinetic energy due to which the bullet penetrates some distance into the target which it strikes. A torpedo in motion can do work by penetrating into the side of a ship at which it is fired. Water in motion can turn a wheel, which in its turn can put other machines in motion. The energy of the wind acting against the sail of a boat may put the latter in motion.

Potential Energy.—The *potential energy* of a body or a system of bodies is the energy it possesses by virtue of its position or configuration. It is measured by the amount of work the body can do in passing from its present position or configuration to some standard position (usually called its *zero position*) or configuration. The surface of the earth is usually taken to be the standard position, so that when a body is on the surface of the earth its potential energy is zero. But when raised up above the ground, the body possesses potential energy owing to its elevated position.

In the case of pile driver, a large piece of iron called the *ram*, is connected to a rope passing over a pulley. The mass, raised up to a height by a number of men pulling at the other end of the rope, possesses potential energy. When released, the mass falls down upon the pile and drives it to some distance into the ground. The stretched spring, a compressed spring when fully stretched, the compressed spring in an air-gun, the spring of a clock or watch when wound, all possess potential energy by virtue of their altered configuration.

73. Measurement of Energy.—The energy of a body is always measured by the maximum amount of work that can be done by a body.

Kinetic Energy.—Let a body of mass m be moving with a velocity u at any instant. We are to find the amount of work it can do before coming to rest. Let a constant force P resist the motion of the body and thereby produce a retardation f, so that $P = mf$. Let the body thus retarded traverse a distance s before it comes to rest.

Then from eqn. in art. 19, $u^2 - 2fs = 0$ whence $u^2 = 2fs$...(1)

Kinetic energy of the body = work done against the force before coming to rest = force × distance = $P.s = m\,f.s$.

$\therefore$ from (1) kinetic energy = $\frac{1}{2}\,mu^2$ ... (73,1)

Thus the kinetic energy of a moving body at any instant is given by half the product of its mass and the square of its velocity at that instant, being measured in the same unit as work.

If a force P acting upon a mass m increases its velocity from u to v, while moving through a distance s along a straight path, we have,

$$v^2 - u^2 = 2fs \quad \text{or} \quad s = \frac{v^2 - u^2}{2f}$$

$$\therefore \quad \text{Work done by the force} = P.s = m\,f.\frac{v^2 - u^2}{2f} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$\therefore$ Work done = final K. E. − initial K. E.
= Increase of kinetic energy of body.

Thus, *the work done by a force on a body is equal to the increase of the kinetic energy of the body.*

Potential Energy.—We know that where the acceleration due to gravity is g, a body of mass m is attracted by the earth vertically downwards with a force mg, which is the weight of the body.

Hence if the body be raised through a vertical distance h, the work done *against* the force of gravity is mgh. The energy spent for this purpose is stored up in the body and constitutes the potential energy *gained* by the body by virtue of its position. Again if a body of mass m falls through a vertical height h, the work done by the force of gravity is mgh, which is the *loss* of potential energy of the body.

Hence potential energy (P. E.) = mgh ... (73,2)

$\therefore$ In F. P. S. system, P. E. = mgh ft. poundals = mh ft. lbs.

Also in the C. G. S. system, P.E. = mgh ergs = mh gm. cm.

Examples :—

1. An engine is used to pump 22,000 gallons of water per hour through a vertical height of 100 ft. Find the H. P. required. (1 gallon of water = 10 lb.)
(And. U.—1932)

A. Since 1 gallon of water weighs 10 lbs. 22,000 gallons weigh 22×10^4 lb.

In one hour work done = $22 \times 10^4 \times 100 = 22 \times 10^6$ ft. lb.

$\therefore$ Work done per second = $\frac{22 \times 10^6}{60 \times 60}$ ft. lb.

Since 550 ft. lb. per sec. = 1 H. P. The horse-power of the engine is

$$\frac{22 \times 10^6}{60 \times 60 \times 550} = 11{\cdot}1 \text{ H. P.}$$

2. Find the energy stored in a train weighing 250 tons and travelling at the rate of 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour?
(C. U.—1925)

A. Mass of the train $= 250 \times 20 \times 4 \times 28$ lbs. $= 560000$ lbs.

Speed of the train $= 60$ miles per hour $= \frac{60 \times 1760 \times 3}{60 \times 60}$ ft. $= 88$ ft./sec.

K. E. of the train $= \frac{1}{2} \times 560000 \times 88^2 = 216832 \times 10^4$ ft. poundals.

Also increased speed $= 65$ miles per hour $= \frac{65 \times 1760 \times 3}{60 \times 60} = \frac{286}{3}$ ft. per sec.

$\therefore$ K. E. at the time $= \frac{1}{2} \times 560000 \times \frac{286^2}{3^2} = 254476{\cdot}4 \times 10^4$ ft. poundals.

$\therefore$ Energy to be added $= (254476{\cdot}4 - 216832) \times 10^4 = 37644{\cdot}4 \times 10^4$ ft. poundals.

3. An engine increases the speed of a train of mass 200 tons from 30 miles per hour to 60 miles per hour, when it moves a distance of 3 miles. If the resisting force be 15 lbs. wt. per ton, find the work done by the engine and the power at which it is working.

A. Total resisting force $= 15 \times 200 = 3000$ lbs. wt.

This force acts over 3 miles *i.e.*, $(3 \times 1760 \times 3) = 15840$ ft.

$\therefore$ Work done against the force $= 3000 \times 15840 = 47{,}520{,}000$ ft. lbs.

Now mass of the train $= 200$ tons $= 448{,}000$ lbs.

30 miles per hour $= 44$ ft. per sec.; 60 miles per hour $= 88$ ft./sec.

$\therefore$ Increase of energy $= \frac{1}{2} \times 448000 \times (88^2 - 44^2)$ ft. poundals $= 40{,}656{,}000$ ft. lbs.

Hence total work done by the engine $= (47520 \times 10^3 + 40656 \times 10^3)$ ft. lbs.
$= 88176 \times 10^3$ ft. lbs.

Average speed of the train $= \frac{1}{2}(30 + 60) = 45$ miles per hour.

Time to cover 3 miles $= (3 \div 45)$ hour $= 240$ seconds.

Horse-power of the engine $= \frac{88176 \times 10^3}{240 \times 550}$ H. P. $= 668$ H. P.

Various forms of Energy.—The energy possessed by a body appears in a variety of *forms*. The different forms in which energy appear are :—

(1) Mechanical energy, (2) Heat, (3) Light, (4) Sound, (5) Electrical and Magnetic energies, (6) Chemical energy.

Transformation of Energy.—The two kinds of energy, kinetic and potential, are mutually interchangeable. Again energy does not appear in various forms but can *pass from one form into another.* This is known as the **transformation of energy.** Thus, when a body is raised to a height, the energy possessed by it is wholly potential and when allowed to fall, the potential energy is gradually changed into kinetic energy. When the body is just on the point of striking the ground, the energy is wholly kinetic and when it strikes the ground, the kinetic energy is transformed into sound, heat and mechanical energies.

Again when we wind a clock or watch, the energy spent is stored up in the spring as potential energy. As the spring unwinds and keeps the clock or watch going the potential energy is continuously transformed into kinetic energy. The potential energy of the compressed

spring in an air-gun is transformed into kinetic energy when the shot is released.

The oscillations of a pendulum furnish another example. When at the uppermost position, the energy of the bob is wholly potential. As the bob moves down, the potential energy gradually decreases and kinetic energy is gained and when at the lowest position, the energy is wholly kinetic. Again when it moves up from the lowest position towards the other side, it gradually loses kinetic energy and gains potential energy, till at the highest position of the other side the energy is again wholly potential.

In a *hydro electric plant*, the water at the higher level possesses potential energy. The water in moving down acts against the turbine pits and rotates the wheels when the potential energy is transformed into kinetic. The motion of the turbine is utilised by suitable arrangement to revolve the armature in a dynamo. The mechanical work thus done is converted into electrical energy. This electrical energy may be converted into heat in an electric oven, heat and light in an electric lamp, mechanical energy in driving a tram car or revolving a fan and magnetic energy in an electro-magnet. The recently constructed Tilaiya dam across the river Barakar near Hazaribagh has provided an opportunity of erecting a hydro-electric power station having an installed capacity of 4000 kilowatts of power for the present, with a provision for future expansion of an additional 2000 kilowatts. The steam turbine station at Bokaro situated at the Damodar Valley zone is already producing 150,000 k. w. of power.

When coal burns, the carbon of the coal combines with the oxygen of the air and the chemical energy is converted into heat and light energies. Vast quantities of water are evaporated from the seas and the oceans by the heat of the sun. The heat gives sufficient energy to the molecules of water to enable them to evaporate. The vapour so formed condenses into clouds which melt down into rain which in its turn fills the running streams that flow down to the seas again. The energy of the running stream may therefore be ultimately traced to the heat energy of the sun. It is to be noted that the earth is receiving energy from the sun at the rate of about 232×10^{18} horse-power.

In the steam-engine, the chemical energy of combustion is changed into heat energy ; the heat energy is utilised in producing steam and is thus converted into mechanical energy in the motion of the piston and of the wheels. When the engine drives a dynamo, the mechanical energy is changed into electrical energy.

When two insulated conductors with opposite kinds of electricity are placed at a distance from each other, the system consisting of the two charged bodies and the space between them possesses some amount of electrical energy. If the two bodies are brought near each other, a spark passes between them, when the electrical energy is converted into heat, light and sound energies.

Again when a body changes from solid to liquid or vapour even at the same temperature, some amount of heat is absorbed. This heat energy is transformed into molecular potential energy and is utilised in bringing about the change of state. On the other hand when water freezes, heat energy is given out. Thus when the water in a lake freezes, sufficient heat is given out, which keeps temperature of the vicinity greater than that of inland localities. All these cases show transformation of energy from one kind to another.

75. Illustrations of Cases of Transformation of Energy.—The following are further examples of transformation of energy.

(1) Mechanical.—(*a*) *Kinetic to potential*—The motion of a pendulum bob from its position of rest to the extremity of the swing. (*b*) *Potential to kinetic*—A body falling from a height ; the bob of a pendulum moving down to its mean position from the highest position on either side. (*c*) ***Kinetic to heat energy.***—The wheel of a moving engine stopped by putting brakes ; a piece of stone being struck against another giving out sparks. (*d*) ***Kinetic to sound.***—Sound by vibration of bodies. (*e*) *Kinetic to electrical energy.*—Action of a dynamo.

(2) Heat.—(*a*) *Heat to Mechanical.*—Action of heat engines. (*b*) *Heat to light energy.*—A white hot body, as the filament in an electrical lamp (*c*) *Heat to sound energy.*—Singing flame. (*d*) *Heat to electrical energy.*—Thermo-electric phenomena. (*e*) *Heat to chemical energy.*—Formation of chemical compounds by the application of heat. *e.g.*, oxygen and hydrogen combine with each other when heated. (*f*) *Heat to molecular.*—Change of state of a substance.

(3) Light.- (*a*) *Light to electric energy.*—The working of a photo-voltaic cell in a photophone. (*b*) *Light to chemical energy.*—The action of light in photographic plates ; combination of hydrogen and chlorine in light.

(4) Sound.—(*a*) *Sound to mechanical energy* —Forced vibration and resonance. (*b*) *Sound to electrical energy.*—Telephone.

(5) Magnetic.—(*a*) *Magnetic to heat energy.*—Rapid magnetisation and demagnetisation generate heat in steel bars. (*b*) *Magnetic to mechanical energy.*—Electro-magnetic induction.

(6) Electrical.—(*a*) *Electrical to mechanical energy.*—Electric motors of tram cars, electric fans, lifts, etc. (*b*) *Electric to heat energy.*—Heating of conductors by the passage of electric current, as in electric furnace, electric iron, etc. (*c*) *Electrical to light energy.*—Electrical lamps. (*d*) *Electrical to sound energy* —Electric bells, telephones, microphones ; sound is produced when discharge takes place in electric machines or induction coils. (*e*) *Electrical to magnetic energy.*—Magnetic field produced by electric current ; electro-magnet. (*f*) *Electrical to chemical energy.*—Electrolysis.

(7) Chemical.—(*a*) *Chemical to heat energy.*—Combination, as in the burning of coal or when lime is slaked ; various other chemical

reactions. (*b*) *Chemical to light energy.*—Phosphorous combines with oxygen producing a dull glow ; magnesium in combining with oxygen produces bright light. (*c*) *Chemical to mechanical energy* —The chemical energy of a loaded cartridge is changed into the kinetic energy of the flying bullet. (*d*) *Chemical to electrical energy.*—Voltaic cells (*e*) *Chemical to heat, light, sound and mechanical energy.*—Explosion of gun-powder and other explosives.

76. Conservation of Energy.—It is thus found that in all cases of transformation, energy in one form disappears and reappears in other forms. But in no case the energy is destroyed altogether or any form of energy appears of itself without the loss of some other. It has been found also that the energy lost in any form by one system is equal to the gain of the same amount by some other system, so that the total energy of the combined system remains constant. This is known as the **Principle of Conservation of Energy.** There is no general proof of the principle but it has been verified in various cases, and so it is accepted as true. This very important principle forms the foundation of the whole of Physical Sciences. It may be stated thus :—

The total amount of energy possessed by a material system can neither be created nor destroyed by any action between the parts of the system, though it may be transformed from one form into another.

The principle may be illustrated by the following simple example. Suppose a body of mass m is raised to a vertical height h at A from earth's surface S (Fig. 94). The energy possessed by the body is wholly potential and is equal to mgh.

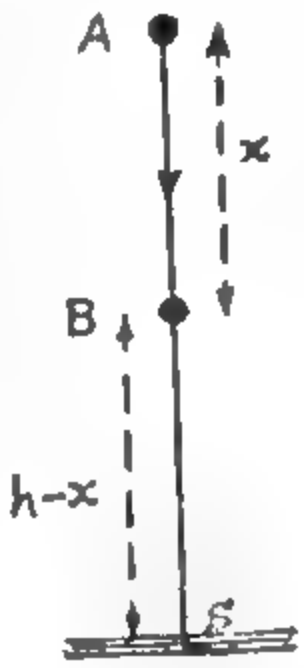


Fig. 94

Let the body fall freely through a height x as at B and let the velocity just at the end of this fall be v ; then $v^2 = 2gx$.

Kinetic energy of the body at the instant at B $= \frac{1}{2}mv^2 = mgx$

Potential energy of the body at the instant at B $= mg(h-x)$

$\therefore$ Total energy of the body at the time $= mgx + mg(h-x)$
$= mgh$ which is the energy it had initially at A.

Thus, we find that as the body falls down more and more, the potential energy decreases and the kinetic energy increases but the sum of the potential and kinetic energies at any instant remains constant. When the body strikes the ground, both its kinetic and potential energies disappear. But even then the principle is not violated, for the energy reappears as heat, sound and mechanical energies.

The swinging of a pendulum bob presents another example. At the highest position of the bob its energy is wholly potential. As it begins to descend the energy of the bob changes gradually from potential to kinetic. At the lowest position its energy is wholly kinetic. It can be

shown that at any point of its path, the sum of the two kinds of energy remains the same.

The above principle denies the possibility of the existence of *a perpetual motion machine*, that is, a machine from which work can be continually obtained without the expenditure of an equivalent amount of energy. Even if the machine were perfectly free from friction, the amount for work taken out of the machine would be *equal* to the amount put in, but it would never be greater. There are, however, always some frictional losses ; so even if the machine does no useful work, the energy initially supplied will gradually be used up to overcome frictional resistances and the machine will ultimately come to a stop.

Example :—

The bob of a pendulum has its centre 1 metre below the support. It is pulled aside till the string makes an angle of 30° with the vertical. If it is now released, calculate its speed when it moves past the rest position. (Mys. U.—1951)

A. When the bob of the pendulum is suspended with a string to a support and is *hanging vertically* it must have been above the ground level. So it has some initial potential energy. Call it E. When the string is pulled aside at an angle of 30° to the vertical, the centre of gravity of the bob is raised up from its original level through a vertical distance of 100 (1 — cos 30°) = 13·4 cm.

Taking the mass of the bob to be m and acceleration due to gravity to be g the potential energy of the bob increase from E by an amount 13·4 mg. ergs. If the string is now released, the bob moves down over the arc in which this excess potential energy is gradually converted into kinetic energy. At its lowermost position, all its excess potential energy is converted into kinetic energy, assuming that there is no other transformation of energy.

Thus $13{\cdot}4\ mg. = \frac{1}{2}\ mv^2$ or, $v^2 = 26{\cdot}8\ g$

Taking $g = 980$ cm./sec.^{2}, $v = 162$ cm./sec.

77. Dissipation of Energy.—According to the principle of conservation of energy, the disappearance of energy in one form is always accompanied by the appearance of an equivalent amount in some other forms. But in every case of transformation, some amount of the energy may appear in a form in which we cannot utilize it further for doing useful work. So far as practical purpose is concerned, this amount may be said to be lost to us. This is known as the *dissipation of energy*. When a stone falls to the ground from a great height, its kinetic energy disappears completely. But the stone as well as the ground where it strikes becomes a little warmer and as already mentioned, the energy is wholly converted into heat, sound and mechanical energies. But we cannot get back any of these into practical work and so to us the energy of the stone is lost.

In a railway engine, a considerable portion of the mechanical energy derived out of heat energy is used up in overcoming frictional resistances, which again appear as heat in the rails and the axle bearings of carriages. Similarly in all machines a part of the energy supplied is used up in overcoming friction and is so changed into heat. This heat cannot be used to do work by any process hitherto known

to us. Similarly, the heat radiated by a source in space cannot be received back and so may be taken to be lost to us. The total amount of energy in the universe being supposed constant, the quantity available for effective work is continually diminishing. It may happen that at sometime there will be no energy available for practical work and the whole universe will become an inert mass of uniform temperature and "no longer a fit abode for living beings"

NICOLAS LEONARD SADI CARNOT (1796—1832)

Carnot was born on June 1, 1796 at the Luxembourg Palace, Paris. His father took a very active part in the French Revolution and had to flee to Germany with his family and remain there for some years. In 1812, Sadi was sent to the Polytechnic School at Paris. When the allied army invaded France in 1814, he entered the army and took part in the defence of capital. He was then promoted to the rank of the military lieutenant. In 1828 he was forced to resign from the military department on account of ill-will of his superiors.

His resignation was a boon in disguise, for now he was able to devote most of his time in pursuit of scientific studies. He was most interested in the laws of heat and their application to the steam engine and all prime movers in which heat is used. His famous book "Reflexions on the motive Force of Fire and on the Machines proper to develop this Force" came out in 1822. He remarked that in all heat machine, capable of doing work, there is always a fall of temperature of the working substance as it moves from the boiler to the condenser. This fall of temperature of the substance was responsible for the output of work from the machine. He then formed his famous idea of perfect engine in which the heat transformed into work within the engine may be given back as the same amount of heat when it is worked in the reverse way. Thus he established the principle of the conservation of energy. He died prematurely of cholera on August, 24, 1832.

Summary

Work is said to be done by a force when the point of application of the force is displaced. Work = Force × Distance.

Units of Work.—*The absolute unit of work* in the F. P. S. System is termed a **foot-poundal** and that in the C. G. S. system is an erg. The *gravitational unit of work* in the F. P. S. system is termed **a foot-pound** and that in the C. G. S. system is a **gramme-centimetre.**

Power.—The *power* of an agent is the rate of its doing work.

Units of Power.—The *absolute unit of power* in the F. P. S. system is the rate of doing one foot-poundal of work per second and that in the C. G. S. system, is the rate of doing one erg per second. The practical unit of power in Great Britain is one **Horse Power (H. P.)** : **1 H. P.** = 550 ft. lbs. of work per second and 1 kilowatt = **1·34 H. P.**

Energy is the capacity for doing work. Energy may appear in different forms and may be *transformed* from one form into another.

Two kinds of energy.—(*i*) Kinetic and (*ii*) Potential.

(i) Kinetic Energy is the energy due to motion. K. E. $= \frac{1}{2}mu^2$

(ii) Potential Energy is the energy due to position or configuration. P. E. $= mgh$.

Conservation of Energy.—*The total amount of energy possessed by a material system can neither be created nor destroyed by any action between the parts of the system, though it may be transformed from one form into another.*

Exercises on Chapter V

Reference

Arts. 69, 72, 74

1. Define work, energy and power. Give some examples of transformation of energy. (Dac. U.—1933; Pat. U.—1932; *Cf.* C U.—1954; Mad. U.—1951; *Cf.* Gau. U.—1952; *Cf.* Del. U.—1951; Del. H. S.—1951; P. U.—1951, '52; Nag. U.—1952).

Art. 71

2. Show that if a piston is moved along a cylinder against a constant pressure, the work done in a stroke is equal to the product of the pressure into the volume swept out by the piston. Explain clearly the units in which the work will be given by this calculation. (C. U.—1941)

Art. 71

3. A company sells electrical energy for lighting purposes at 4½d. per kilowatt hour unit and for motors etc. etc. at 1½d. per unit. What is the weekly bill for a factory in which ten 5 horse-power motors run for forty hours per week and fifty 100 watt lamps are alight for forty four hours per week? (1 H. P. = ¾ kilowatt).

A. £13. 10s.

Art. 71

4. Define the terms 'Joule' and 'kilowatt-hour.' (Mys. U.—1952)

Art. 71

5. A train of mass 200000 kgm. travelling at 54 km. per hour is brought to rest in 500 metres by the application of a constant force. Find the value of the force and the time necessary for the purpose.

A. 45×10^5 dynes; 1 min. 6 $\frac{2}{3}$ sec.

Art. 71

6. Calculate the horse power of an engine drawing a train of mass 150 tons up an incline of 1 in 55 against a frictional resistance of 12 lb. per ton at the rate of 15 miles/hour. If the steam is shut off, how far will the train move before it comes to rest? ($g = 32$ ft./sec.^{2}) (Mad. U.—1951)

A. 215·5 H. P.; 416' nearly.

Art. 73

7. A man weighing 130 lbs. lifts a weight of 80 lbs. to the top of a building 50 ft. high in one minute. Find the work done by him and his horse-power. (Del. H. S.—1951)

A. 6399 ft. lbs.

Art. 78

8. Water is pumped from a well through a height of 30 ft. by means of a 5 H. P. motor. If the efficiency of the pump be 85%, find in gallons the quantity of water pumped up per minute. (1 gallon of water weighs 10 lb.). (C. U.—1954)

A. 467·5 gals.

Art. 78

9. What is the work done when a weight of 500 kilograms falls through a height of 50 metres and is then stopped? Assume the normal value of gravity. (Dac. U.—1933)

A. 24525×10^5 ergs.

Reference

10. A bullet of mass 25 gm. travelling with a velocity of 500 metres per second strikes a target which it penetrates, the velocity on emergence being 100 metres per second. What energy was expended in passing through the object? Art. 73

A. 3,000 joules.

11. A man climbs a tower 500 ft. high in 10 minutes. If the man weighs 165 lb. find the rate at which he works. (P. U.—1952) Art. 73

A. 137·5 ft. lb./sec.

12. An elevator is designed to lift a load of 1,000 lb. through 5 floors of a building averaging 11 ft. per floor in 5 seconds. Calculate the power of the elevator. (Anna. U.—1951) Art. 73

A. 20 H. P.

13. Define kinetic and potential energy. Give some examples. (Cf. Mys. U.—1951; P. U.—1950) Arts. 72 & 75

14. A solid mass of 100 grams is allowed to drop from height of 10 metres. Calculate the amount of kinetic energy gained by the body, g being 980 cm. per sec.^{2}. (Dac. U.—1930) Art. 73

A. $9{\cdot}8 \times 10^7$ ergs.

15. Find an expression for the kinetic energy of a body of mass m moving with a velocity v. (P. U.—1950; Mys. U.—1951) Art. 73

16. The hydro-electric station at Konar dam project is to generate a power of 40,000 kilowatts, and the reserves of coal in the Damodar valley coal fields are estimated to be 25,950 million tons. Supposing that the coal is deposited at a mean depth of 300 ft. below the surface level and the power of the station is incessantly utilised in raising coal, calculate the time required to empty all the deposits of coal of that area. ($g = 32$ F. P. S.). Art. 78

A. 500·13 years.

17. A simple pendulum is formed by suspending a metal bob from a long string. If it swings through a small angle of 6° on either side of its mean position so that the vertical descent of the centre of the bob from the extreme to the lowest position is 5 mm., find the velocity and acceleration of the centre of the bob at the ends of the swing and when crossing the mean position, given $g = 980$ cm./sec.^{2} (Raj. U.—1951) Art. 76

A. 0,102·4 cm./sec.^{2}; 31·3 cm./sec. 0.

18. A body is falling freely under gravity; show that the sum total of energy is always constant. (C U.—1932, '36; Pat. U.—1952) Arts. 72 & 76

19. State the law of conservation of energy and prove it in the case of a body falling freely under gravity. (Mys. U.—1952; Anna. U.—1951; *Cf.* P. U.—1951) Art. 76

20. Trace the transformations of energy which occur when a pendulum bob is displaced from its position of rest and is let free to oscillate till it comes to rest again. (C. U.—1943) Arts. 74 & 76

21. A body falls under gravity and strikes the ground. Explain how the phenomenon supplies an illustration of the transformation of energy. Does it also illustrate the principle of conservation of energy? (Pat. U.—1952) Arts. 74 & 76

22. State and explain the principle of conservation of energy with suitable illustration. (Pat. U.—1952; Dac. U.—1931; C. U.—1928) Art. 76

CHAPTER VI

SIMPLE MACHINES

78. Machine.—A machine is a contrivance by which 'work done upon it' is converted into 'work done by it' in a desired manner. To do work on the machine some amount of force is to be applied which is called the **effort** or **power** and the resisting force overcome by the machine is called the **resistance** or **load.** The effort is usually denoted by P and the resistance by W.

The **purposes** served by a machine may be roughly grouped into three divisions :—

(*i*) To apply the force at a more convenient point or in a more convenient manner, for example, a poker.

(*ii*) To overcome greater resistance and to raise heavier load, for example, a crow-bar.

(*iii*) To convert a slow motion at one point into a more rapid motion at some other point ; *e.g.*, a bicycle, a sewing machine.

Principle of Work.—According to the principle of conservation of energy the work that we may derive out of a machine can never be more than that done upon it. It may be possible that a small force applied through a machine may lift a heavy weight ; but if there is no loss of energy in form of friction within the machine, the work done by the effort on the machine is always equal to the work done by the machine against external resistance. In simple language 'in put' energy is equal to 'out put' energy. This is known as the *principle of work.*

Efficiency of a Machine.—A part of the energy supplied to a machine is used in overcoming internal resistance in the form of friction and this part is always dissipated in form of heat energy. So the useful work done by a machine is always less than the energy supplied.

The ratio which indicates what fraction of the total energy supplied to a machine is utilised by it in doing useful work is termed the efficiency or *modulus* of the machine. Hence,

$$Efficiency\ (E) = \frac{Energy\ utilised\ or\ work\ usefully\ performed}{Total\ Energy\ supplied} \qquad ...(78,1)$$

The efficiency of a machine is therefore always less than unity and is often expressed as a percentage by being multiplied by 100. For example, suppose that a machine takes up 400 ft.-pounds of work in a certain interval and delivers 320 ft.-pounds of work during the same interval. Then the efficiency of the machine is $320 \div 400 = .8$. In terms of percentage, it is 80%.

A *perfect machine* is that which would utilize the entire energy supplied and so *the efficiency of perfect machine is unity*. Thus if P denotes the effort applied and W is the resistance overcome, and the distance through which the effort and the resistance move be d_1 and d_2 respectively, then in a *perfect machine*.

$$P \times d_1 = W \times d_2$$

If P is much smaller than W, d_2 is much smaller than d_1 so *what is gained in power is lost in speed*.

Velocity Ratio.—The ratio of the distance d_1, through which the effort works to the distance d_2, through which the resistance is overcome, is called the *velocity ratio* of the machine.

Thus, velocity ratio,—

$$= \frac{\text{Distance through which applied force moves}}{\text{Distance through which resistance is overcome}} = \frac{d_1}{d_2} \quad \ldots(78,2)$$

Mechanical Advantage.—The *mechanical advantage* or the *force ratio* of a machine is the number which expresses the ratio of the resistance overcome to the effort applied to the machine to produce equilibrium.

$$\therefore \text{ Mechanical advantage} = \frac{\text{Resistance overcome}}{\text{Effort applied}} = \frac{W}{P} \quad \ldots (78,3)$$

Machines are usually so constructed that the mechanical advantage is greater than unity.

$$\text{Again from above, Efficiency } E = \frac{Wd_2}{Pd_1} = \frac{W/P}{d_1\ d_2}$$

$$\text{Therefore, Efficiency } E = \frac{\text{Mechanical advantage}}{\text{Velocity Ratio}} \quad \ldots(78,4)$$

As in a *perfect machine*, $E = 1$

so Mechanical Advantage = Velocity Ratio.

79. Lever.—It is simply a rigid bar straight or bent, capable of turning about a fixed point of support, which is called the **fulcrum.** The effort and the resistance are applied at any two points either on the same side or on opposite sides of the fulcrum. The perpendicular

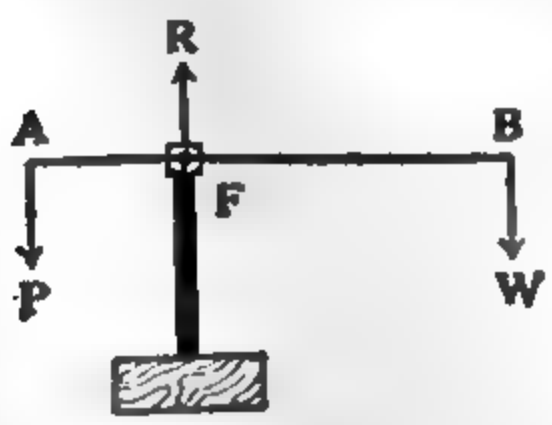


Fig. 95

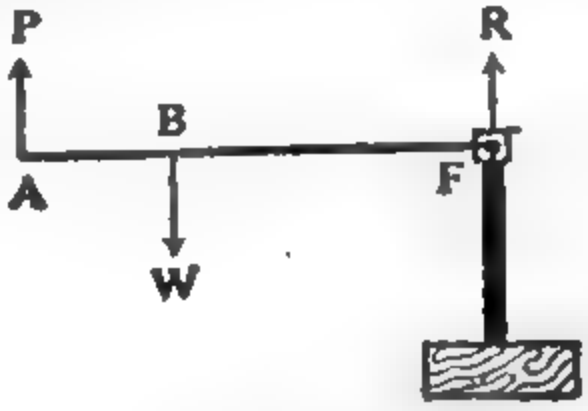


Fig. 96

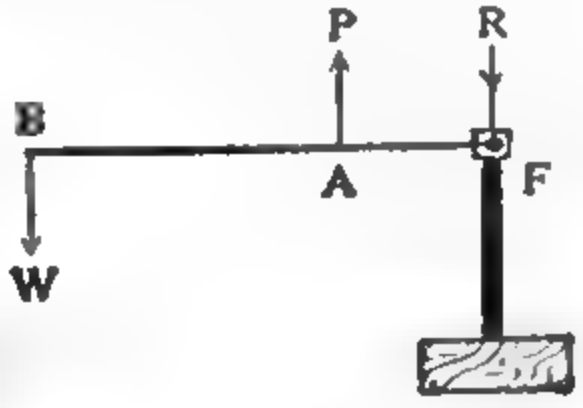


Fig. 97

distance between the fulcrum and the line of action of effort is called the *power arm*, and that between the fulcrum and line of action of the resistance is called the *load arm*. When the lever is straight, both

the effort and the resistance act perpendicularly to the lever. Since levers are used to overcome resistances, soft and flexible substances should not be used as the material of the levers, as otherwise they might get bent by the forces and a part of the energy supplied may be spent in bending levers.

Different Classes of Lever.—According to the different positions of the fulcrum relative to the point of application of the effort and the resistance, levers are usually divided into *three classes*.

Class I.—Here the fulcrum lies between the points of application of the effort and the resistance. Thus in fig. 95, P represents the effort acting at A, W is the load acting at B and F is the fulcrum. placed between A and B. The effort P, applied to overcome the load W, tends to rotate the lever about F in a counterclockwise direction while W tends to rotate it in a clockwise direction. Hence for equilibrium,—

The moment of P about F = the moment of W about F

i.e. $$P \times AF = W \times BF \qquad \ldots (79,1)$$

$$\therefore \quad \textit{Mechanical advantage} = \frac{W}{P} = \frac{AF}{BF} = \frac{a}{b}$$

where a and b are the lengths of the power arm and the load arm.

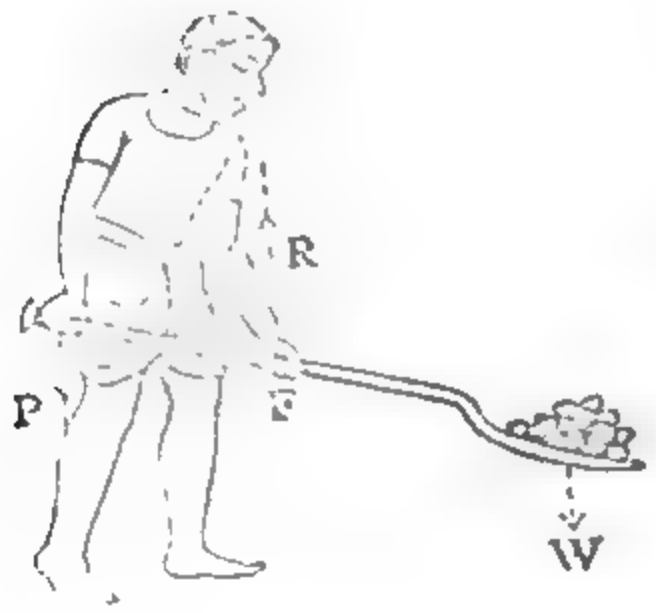


Fig. 98

The reaction R at F is always equal and opposite to the resultant of P and W. As P and W are parallel and both act downwards, the *resultant reaction* at F is given by R = P + W, which acts upwards. Hence with this lever, the mechanical advantage may be greater than, equal to or less than unity, according as the power arm a is greater than, equal to or less than the resistance arm b.

Examples of single lever of Class I.—A crow-bar, used to raise a weight; a poker used to raise coal in a grate; a spade in digging the earth; the handle of a common pump; the beam of a common balance; a boat's rudder; a see-saw. To use a poker, for example, one of the hands griping at the middle of the handle, acts as the fulcrum (Fig. 98). The other hand pressing down at one end with a force P acts as the power and raises the load W at the other end.

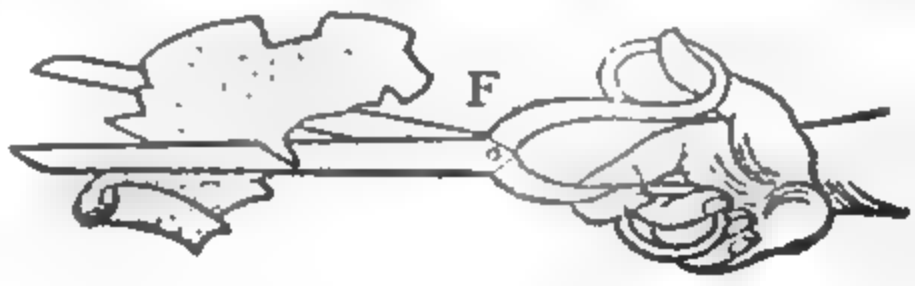


Fig. 99

Double levers of this class—A pair of pincers or scissors (Fig. 99).

Class II (Fig. 96).—Here fulcrum F lies at one end. The effort P and the load W act at A and B respectively, on the same side of F

but in opposite directions, the former acting at a greater distance than the latter from the fulcrum.

Now the Mechanical advantage

$$= \frac{W}{P} = \frac{AF}{BF}$$

Since AF is greater than BF, *the mechanical advantage is greater than unity.*

As P and W act in opposite direction and W > P, the resultant reaction at F acts upwards and is given by R = W − P.

Examples of single Lever of Class II.—A wheel barrow with the fulcrum at the axle and the power applied at the handle ; a punching machine, an oar when used in rowing (the end of oar in this case acts as the fulcrum, which is supposed to be fixed), a crow-bar with one end in contact with the ground ; a cork-squeezer.

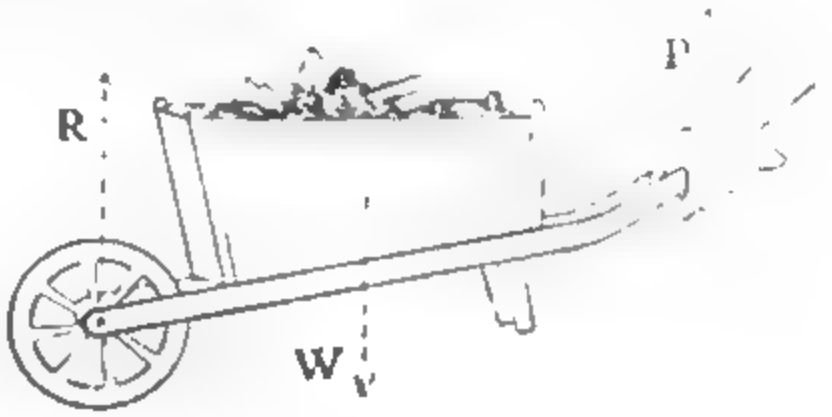


Fig. 100

A push cart for carrying luggages or board sweepings is an example. Here the man pushing the cart applies the effort P at the ends of the handles and the weight W of the load acts at some intermediate point (Fig. 100).

Double lever of this Class.—A pair of nut-crackers.

Class III.—(Fig. 97)—Here the fulcrum F is at one end ; effort P and the resistance W act at A and B respectively, on the same side of F but in opposite direction, the former acting nearer to the fulcrum than the latter. As before,

$$\text{The Mechanical Advantage} = \frac{W}{P} = \frac{AF}{BF}$$

Since AF is less than BF, *the mechanical advantage is always less than unity.* Hence a larger effort is to be applied to overcome a small resistance ; in other words, the lever has *mechanical disadvantage.* This class is generally used to convert a slow motion into a more rapid one. As P and W act in opposite directions and P > W, the resultant reaction at F acts downwards and is given by R = P − W.

Examples of single Lever of Class III.—The treadle of a lathe or a sewing machine ; the pedals of a piano ; the human force-arm (when used to raise or support a load placed on the palm ; the elbow-joint acts as the fulcrum and the tension of muscles and tendons as the effort).

Double lever of this class.—A pair of coal tongs ; a pair of forceps in a weight-box ; the upper and the lower jaws of the mouth.

Lever system of a Type-writer.—In a type-writer the motion is conveyed from the key board to a type through a system of three inter-

connected levers. K represents one stud of the key board and KDM is one of the levers made of thin steel bar with fulcrum at M. (Fig. 101) There is a light spring below the bend D of this lever which keeps it ordinarily pressed upwards at the position shown. With finger pressure on K, it is depressed below as shown by dotted line. At some intermediate point of this lever, another lever FNG is connected with a short thick wire and this has got its fulcrum at N. The second lever is connected with the third lever HOT with a light rod. The third lever has its fulcrum at O, and carries a type at the end T. In the normal con-

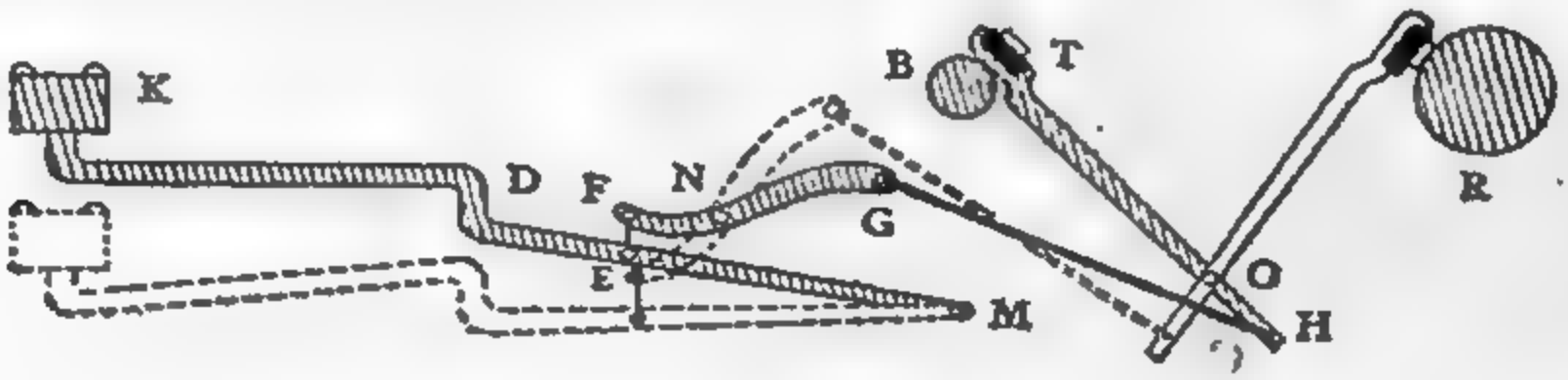


Fig. 101

dition this lever rests in an inclined position with its back against a rubber band B. But when the key is depressed, all the levers take the positions as shown by dotted lines and the type impinges against the roller R with a ticking sound. Here the lever KDM belongs to the second system and the other two to the first system.

Example :—

1. A pair of nutcrackers is 5 inches long and when a nut is placed ½ inch from the hinge, a force of 8½ lb. wt. applied at the end will crack the nut. What weight if simply placed at the top of the nut would crack it? (Utk. U.—1952)

A. This is a class of lever in which fulcrum is at one end, the effort is applied at the other end and the resistance overcome is at the intermediate position. Evidently it belongs to class II. The effort P = 8·5 lb. wt. Arm of the effort = 5 in. Arm of the resistance = ½ inch. Hence for equilibrium,

$8{\cdot}5 \times 5 = W \times \frac{1}{2}$ whence $W = 20$ lb. wt.

Thus a force of 20 lb. wt. placed directly on the nut would just crack it.

30. **Pulley.**—A pulley consists of a small circular disc or wheel of wood or metal L, called the *sheave* the circumference of which is cut into a groove (Fig. 102). The disc can revolve freely about an axle passing through its centre at right angles to the plane of the disc, the axle being supported in a frame-work B, called the *block*. The pulley can be fixed to any rigid support with its ring R.

Fig. 102—Pulley

If the block, be fixed, as in fig. 103, the pulley is said to be *fixed*, while if the block can ascend or descend, as in fig. 104, the pulley is termed *movable*. In a *single* fixed pulley, the weight W is attached to one end of the string and the effort P is applied by pulling at the other end. If the pulley is smooth,

and the string is considered weightless and perfectly flexible, the tension of the string is the same throughout. Also the distance through which the load ascends is equal to the distance through which the power descends. Hence from the principle of work W = P

$$\therefore \textit{Mechanical advantage} = \frac{W}{P} = 1 \qquad \ldots (80,1)$$

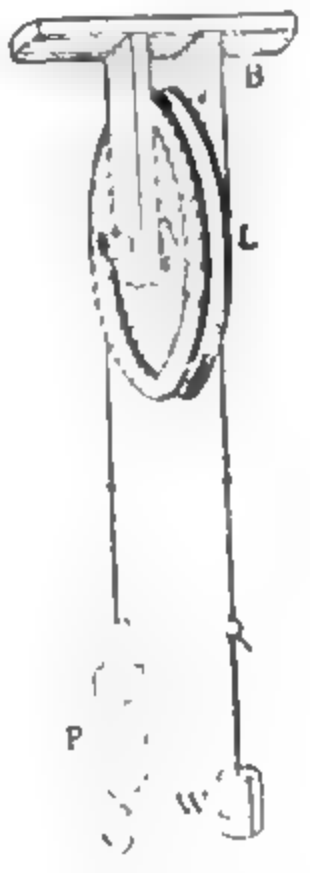


Fig. 103

In practice, however, due to friction at the pulley W is always less than P and so the mechanical advantage is slightly less than 1. This pulley is used only *in changing the direction of the force*, a downward pull in this case being changed into an upward one. It is used for raising weights, drawing curtains, etc.

movable pulley (Fig. 104) the pulley is supported by a string, one end of which is attached to a fixed support, while effort P is applied at the other end. The weight W to be raised is attached to the pulley block. As the pulley is smooth, the tension T in any part of the string is P. Neglecting the weight of the movable pulley, the weight W is supported half by the tension of the fixed portion of the string and half by the tension of the moving portion. Hence W = 2P,

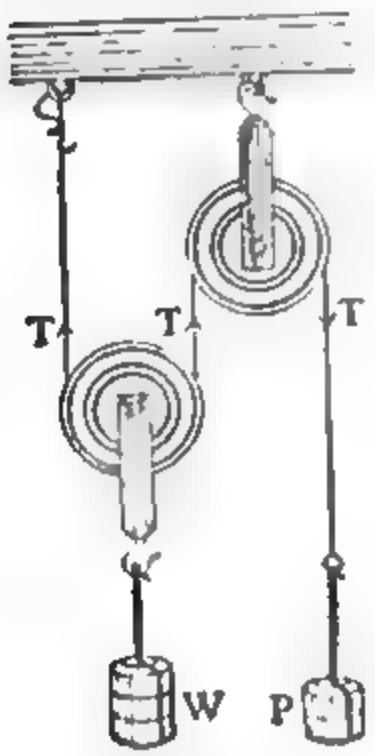


Fig. 104

$$\therefore \textit{Mechanical advantage} = \frac{W}{P} = 2 \qquad \ldots (80,2)$$

Hence by a given effort twice the weight can be raised. In practice, as the pulley is not weightless the mechanical advantage is a little less than 2.

Combination of Pulleys.—Pulleys can be put into different combinations; the different *systems* having different mechanical advantages are used for different purposes. In the combination, shown in fig. 105 & 106, the pulleys are arranged in two blocks, the upper one being fixed to the support and the lower movable, to which the weight W is attached. The same string passes round all the pulleys, one end of which is attached to a block while the effort P is applied at its free end. Fig. 105 shows a *co-axial* system and fig. 106 a *collinear* system of pulley blocks.

The tension of the string is the same throughout and is equal to the effort P in each portion. Let weight to be raised be W and the weight of the lower block *w*.

Let the number of portions of string at lower block = *n*

Then $nP = W + w$

If the weight of the lower block is neglected. The *Mechanical advantage*

$$= \frac{W}{P} = n \qquad \ldots (80,3)$$

Remembering that the efficiency of a machine is the mechanical advantage divided by the velocity ratio, it is to be noticed that in a block consisting of two pulleys, although the mechanical advantage is 4 neglecting weight of the block, but drawing of 8 feet length of the rope, would raise the load only by 2 feet. Hence velocity ratio is $8 \div 2 = 4$. Thus the efficiency comes out to be unity or 100% theoretically. But due to the weight of the lower block and frictional resistances at various parts, the effective efficiency is less than the theoretical limit.

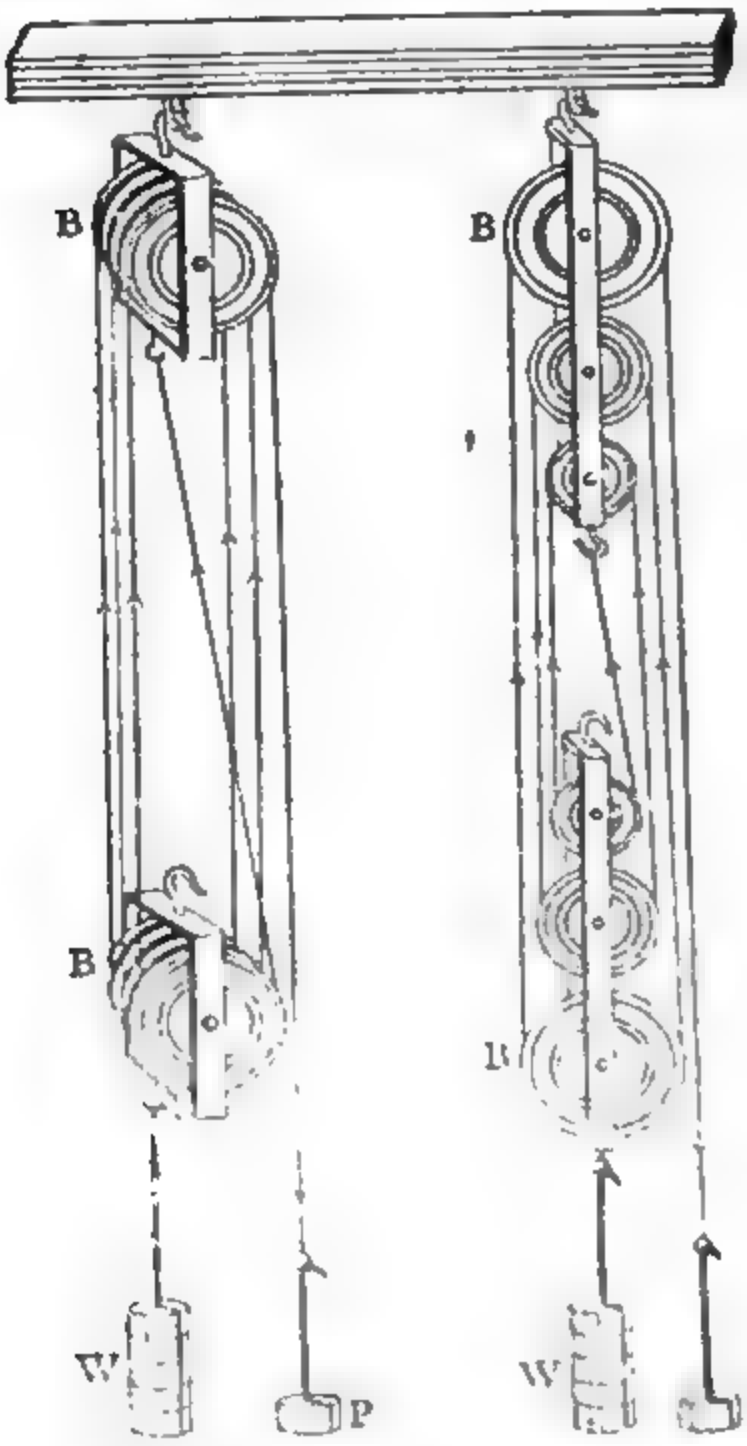


Fig. 105 Fig. 106

81. Wheel and Axle.—It consists of two cylinders arranged so as to turn on the same horizontal axis ; one of these is a grooved wheel AB (Fig. 107) of larger radius called the **wheel**, while the other is a coaxial cylinder of much smaller radius called the **axle** ; which can turn freely on fixed supports. A rope is wound round the axle, one end of which is fixed to the axle, while the free end supports the weight W. Another rope is coiled round the wheel in *opposite* direction, one end of which is attached to the wheel, while the effort P is applied downwards at its free end. When P moves down, the rope round the wheel unwinds and that round the axle coils up causing W to move upwards.

Let the radii of the wheel and the axle be a and b respectively. When they make one complete revolution, a length $2\pi a$ of the rope attached to the wheel unwinds i.e., P moves down through $2\pi a$, while a length $2\pi b$ of that attached to the axle coils round it i.e., W moves up through $2\pi b$. Now according to the principal of work in a frictionless machine, the work done by the effort is equal to that done on the resistance.

Hence $P \times 2\pi a = W \times 2\pi b$

or $P \times a = W \times b$.

$$\therefore \quad \textit{Mechanical Advantage} = \frac{W}{P} = \frac{a}{b}$$

$$= \frac{\text{radius of wheel}}{\text{radius of axle}} \qquad \ldots (81,1)$$

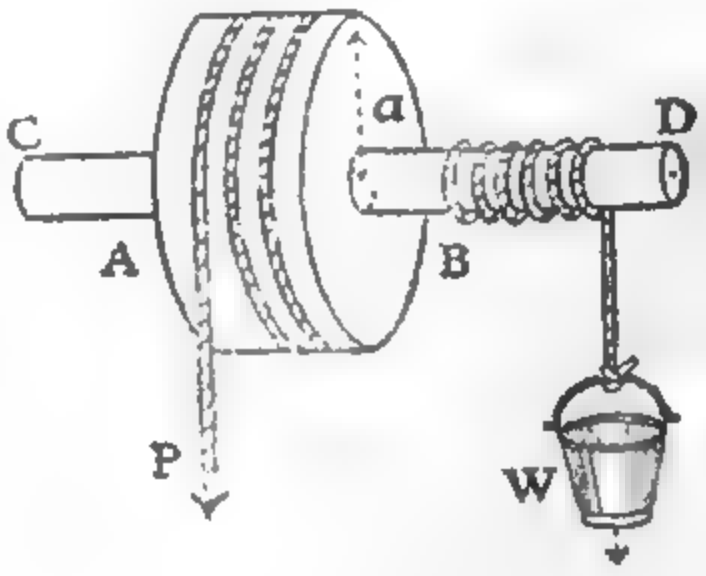


Fig. 107

By making a larger and b smaller within practical limits, the mechanical advantage can be increased. A modification of the wheel and the axle is the windlass which is a device used to draw water from wells and consists of a horizontal roller with handle attached at one end. Another modification is the *capstan* which is used on board a ship to raise anchors by coiling the rope round a vertical axis.

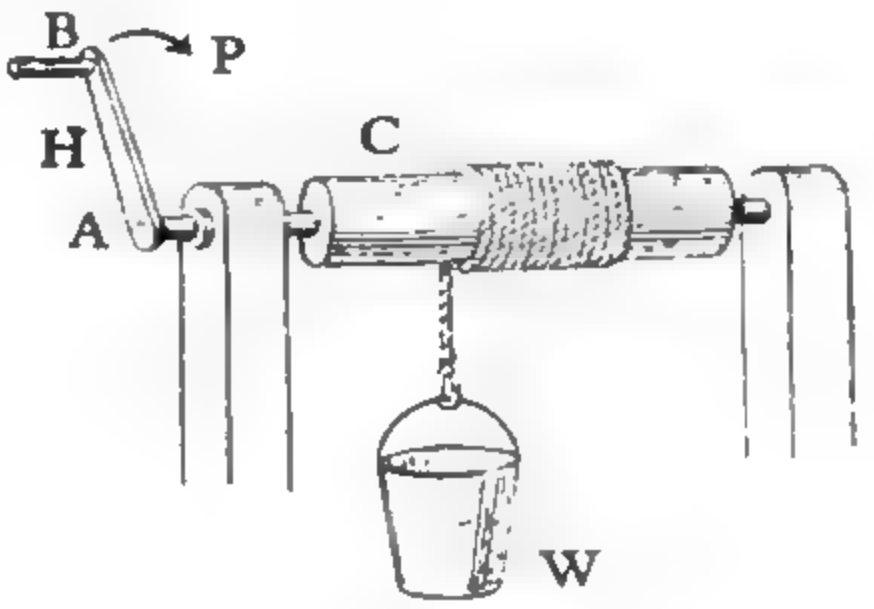


Fig. 108

82. Toothed Wheels.—In machinery, the speed of revolution of different parts is often required to be adjusted to suitable needs, although the main shaft possesses a constant revolution. When the main shaft is at a considerable distance, it is usual to fix a flywheel with a broad rim at each shaft and to connect these two wheels with a belt of leather or rope running over the rims. The driven shaft can be geared to its flywheel whenever necessary. Such a belting arrangement is shown in the diagram of the steam engine in Chap. X of the part on Heat. When the 'load' on the driven shaft circuit is high or the friction between the rim of the wheel and belting not sufficient, there may be a slip between the two and the desired rotation may not be obtained. Consequently sufficient precaution must be taken to secure a good grip between the belt and the flywheel. Nevertheless belting has one advantage that it can be run with a high speed.

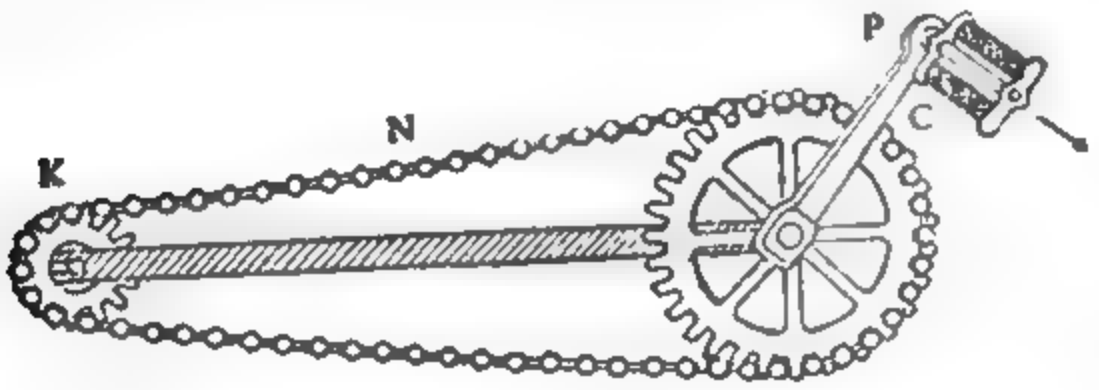


Fig. 109

To secure the best possible grip on the flywheels without any chance of slip, the rim of each wheel is cut into a number of identical teeth and a grooved chain fitting into such teeth, may be made to pass over the two. Such an arrangement is called *chain gearing* or *chain driving*. The pedal of a bicycle is connected to the axle of the rear wheel by means of chain drive. (Fig. 109). The disadvantages of this arrangement at increasing speed is a high friction between the chain and the axle.

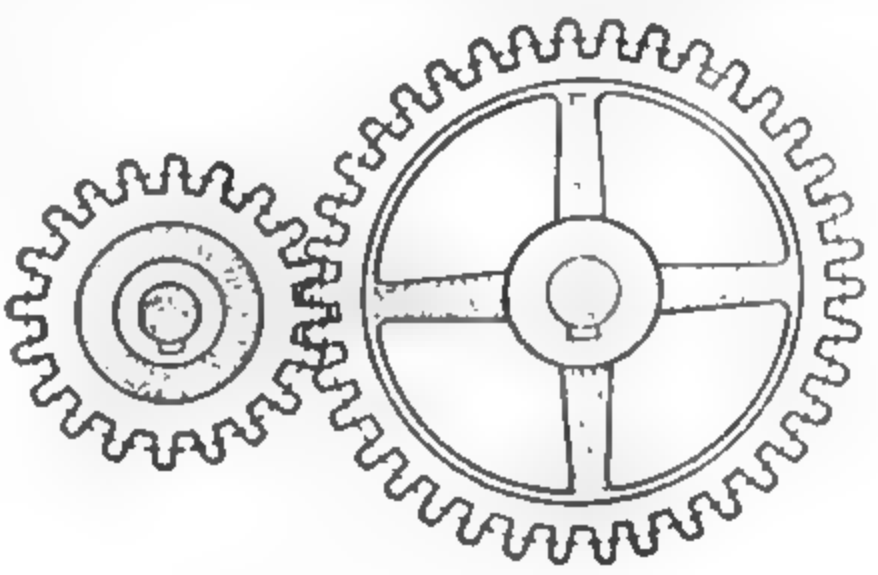


Fig. 110

When the shafts are quite close together, the rims of the flywheels are provided with teeth

which engage with each other as shown in Fig. 110. The teeth of both the wheels are made exactly identical. The radius of a wheel is taken to be the distance from the axis of rotation to a point at half the height of any tooth. The effective rims of the wheels are represented by dotted circles. The distance along this dotted line between midpoints of any two consecutive teeth is called the pitch of the tooth. Since the pitch of the tooth is equal on both the wheels, the number of teeth multiplied by the pitch is equal to the circumference of a wheel. Thus, if n_1 and n_2 be the numbers of teeth on the wheels, p the pitch and r_1 and r_2 be their respective radii,

Then, $n_1 p = 2\pi r_1$ and $n_2 p = 2\pi r_2$, or $\frac{n_1}{n_2} = \frac{r_1}{r_2}$. ... (82,1)

If N_1 be the number of revolutions of the first wheel per second, then across any point, $N_1 n_1$ teeth would pass by during the period. The same number of teeth of the second wheel would pass during one second across that point. Call this number $N_2 n_2$, where N_2 represents the number of revolution per second of the second wheel.

Thus, $\frac{N_2}{N_1} = \frac{n_1}{n_2} = \frac{r_1}{r_2}$ ... (82,2)

Therefore the number of revolution is inversely proportional to the radius of the flywheel. If it is desired to make the speed of the driven shaft double that of the driver, the secondary toothed wheel would have to be chosen of half the radius of the primary wheel.

83. **Common or Roman Steelyard.**—The Common Steelyard is a machine for quickly weighing heavy bodies. It forms a lever of class I. It consists of a long uniform rod CD (Fig. 111), movable about a fixed fulcrum O, which is situated a little away from the centre of gravity of the rod. A hook is attached near one end say D, of the rod, from which bodies to be weighed are hung. A movable standard weight P slides along the arm OC. The weight of a body is determined by finding the division at which the weight P is to be shifted, so as to make the rod horizontal. The instrument is previously calibrated and the divisions are engraved on the rod. Thus we see that this steelyard is a balance in which weighing is done by adjusting the power arm and not by changing weights as in common balance

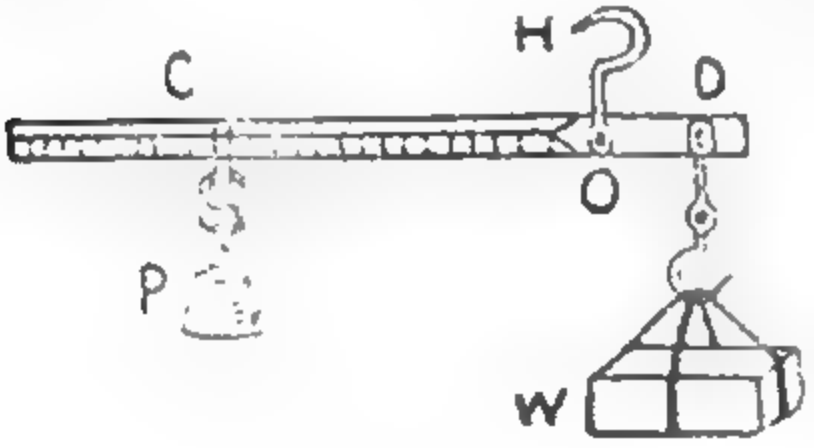


Fig. 111

Platform Balance.—This is used to find the mass of a heavy load. When used to weigh luggages and heavy parcels in railway stations, it is called a *platform balance*. It works on the principle of a common steelyard as is indicated in fig. 112. It consists of three levers fF_3g, F_1de and cb F_2, having their fulcrums respectively at F_3, F_1 and F_2. The platform P, upon which the load is to be

placed, rests on two knife edges *a* and *b*, fixed on the levers *ab* and *bc* close to the respective fulcrums. Any pressure exerted on the platform may be regarded as being distributed at the points *a* and *b*. The pressure is communicted to the end *e* of the lever *ae* and a very small force is required to balance it, as the arm F_1e is much longer than F_1a. Then *e* is again attached by a vertical rod to the point *f* of the lever *fg* which has its fulcrum F_3 very near to *f*. Thus a small effort acting at the end *g* of this can balance the force at *f*. Standard weights suspended from *g* constitute the effort. Very small fractions of weights are measured by sliding a small weight along F_3g, which has been previously graduated for the purpose.

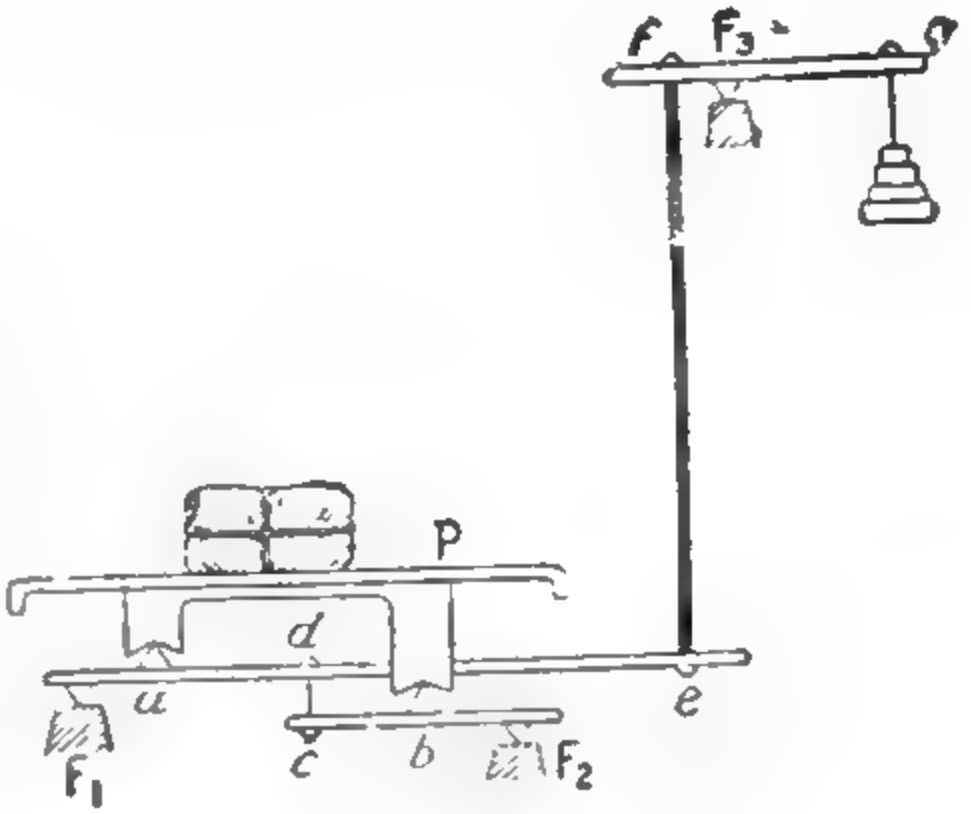


Fig. 112

It is evident that considerable mechanical advantage is gained here, as quite a small weight balances a very large one. The *weigh bridge* used for weighing trucks of coal, etc., or any loaded cart with its contents, also consists of platform resting on a system of levers in which a system of small weights balances the load, so that there is a considerable mechanical advantage.

84. Inclined Plane.—Sometimes a heavy load may be conveniently lifted upwards with the help of an inclined plane Suppose AC to be a rigid plane inclined at an angle θ with the horizontal line AB (Fig. 113). A load W is placed on the plane and further suppose that there is little or no friction between them.

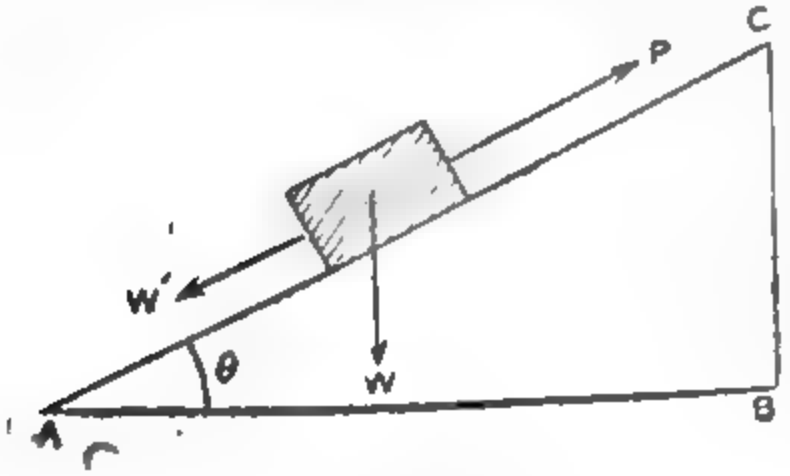


Fig. 113

The weight of the load acts vertically downwards. The component of this force parallel to the plane is W′ which is $W \times \sin\theta$. Hence if an effort P equal to this component be applied in opposite direction, the load may be kept stationary at any position on the plane. Therefore for equilibrium. $P = W \sin\theta$

$$\text{or } \frac{W}{P} = \frac{1}{\sin\theta} = \frac{l}{h} \quad \ldots (84,1)$$

where *l* and *h* are respectively AC and CB. Thus the mechanical advantage of an inclined plane is the ratio of the length of the plane to the height of the plane.

85. Screw.—The screw is a modified form of an inclined plane. The formation of a screw is understood from the following illustration. Imagine a vertical cylinder and a piece of paper cut in the form of a right-angled triangle. Place the triangular piece of paper with its base at right angles to the axis of the cylinder L and gradually warp it round

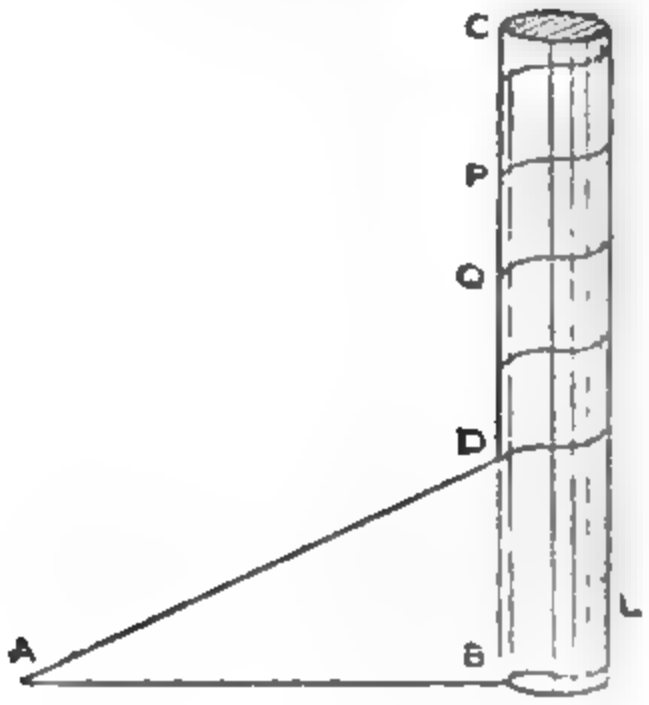


Fig. 114

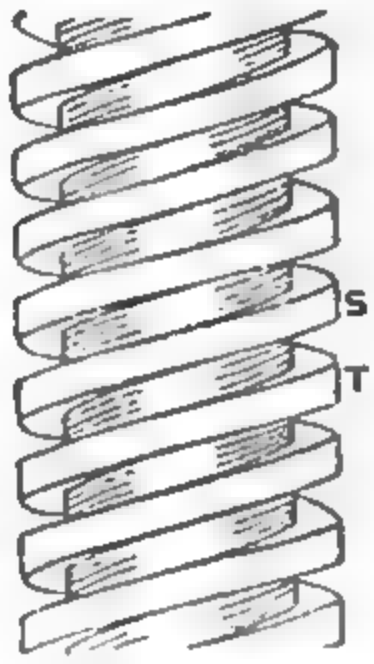


Fig. 115

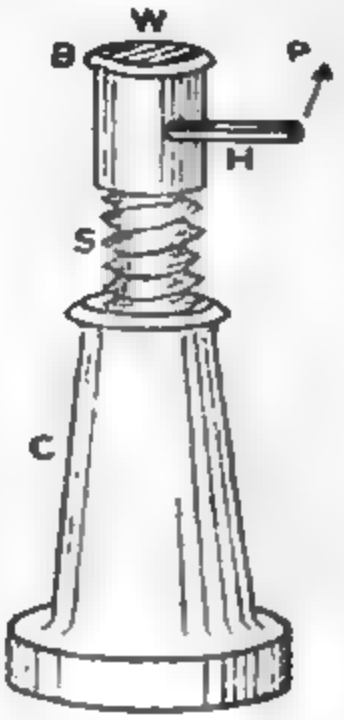


Fig. 116

the cylinder (Fig. 114). The hypotenuse will trace a spiral path over the cylinder, which is called the thread of the screw. A screw is generally provided with an arm by which it can be rotated. Sometimes a groove is cut at the head of the screw to facilitate its rotation by means of a screw driver. When a screw is fixed up in a piece of wood or in a nut, then by the application of the power applied, it turns round and moves bodily forward in the direction of the length. For each complete turn, the screw moves forward a distance which is equal to the difference between consecutive threads measured parallel to the axis of the screw such as a length PQ. This distance is called the *pitch* of the screw. There are various forms in which a thread of a screw may be cut. Fig. 115 represents a screw with a square thread. Fig. 116 shows a screw jack which is used to raise a heavy load, such as a motor car, through a small height. The jack is placed on a firm base in such a way that its head B very nearly touches the underside of the load W to be raised. The screw S is then rotated by the handle H, whereby it rises through a stout nut fixed within the cylinder C acting as its base.

Suppose that a screw is rotated with a force P applied at right angle to the end of the arm of length l. Then for one complete rotation the work done by the effort is $2\pi lP$. If the pitch of the screw is h and the resistance W, then work against the resistance is hW. For an ideal screw there is no loss of energy in friction,

$$\therefore \quad 2\pi lP = hW, \quad \text{whence } \frac{W}{P} = \frac{2\pi l}{h} \qquad \ldots \ (85,1)$$

But for an actual screw, there is some loss of energy due to friction at various parts and so the force of resistance W falls short of the ideal

value. Thus the ratio W/P becomes less than $2\pi l/h$. In the practical case then, the efficiency of the screw takes the form,

$$\text{Efficiency} = \frac{\text{Actual mechanical Advantage}}{\text{velocity ratio}} = \frac{W}{\frac{2\pi l}{h}P} = \frac{Wh}{2\pi l P} \quad \ldots (85,2)$$

Examples :—

1. Ten men raise a marble block weighing 1 ton with the help of two triple pulley blocks, each weighing 100 lbs. If half the energy is wasted in friction, find (*i*) the average pull per man and (*ii*) the average horse-power per man if the block rises 10 ft. in every minute. (P. U.—1929)

A. The total weight to be raised = wt. of marble block + wt. of lower pulley block = 1 ton + 100 lbs. = 2340 lbs.

As half the work is used in overcoming friction, total resistance to be overcome.

W = 4680 lbs. wt.

As the number of segments of strings on the lower block is 6. W = 6 P.

∴ Total effort to be applied by 10 men = 4680 ÷ 6 = 780 lbs. wt.

∴ Hence average pull per men = 780 ÷ 10 = 78 lbs. wt.

∴ Total work done in one *minute* = 4680 × 10 = 46800 ft. lbs.

∴ Work done per man per minute = (46800 ÷ 10) = 4680 ft. lbs.

Hence the average horse-power per man = 4680 ÷ 33000 = 0·142 approx.

2. A uniform bar AB, 2 ft. long and weighing 3 lbs. is used as steelyard being supported from a point 4″ from A. Find the greatest weight that can be weighed with a movable wt. of 2 lbs. and find also the point from which the graduations are measured. (P. U.—1928)

A. When the movable weight is at the extreme end B, the weight that can be weighed is the greatest. Let this weight be W which acts at A. The distance of the fulcrum from A = 4″.

The weight of the bar axis through the middle point of AB, whose distance from the fulcrum is thus equal to 8″.

Hence taking moments about the fulcrum.

$W \times 4 = 3 \times 8 + 2 \times 20$ whence W=16 lbs.

As the weight of the steelyard acts through a point on the right of the fulcrum, the movable weight must be at some point on the left of the fulcrum to counterpoise the weight of the steelyard. Let the distance of this point from the fulcrum be x inches.

Then $2 \times x = 3 \times 8$ when $x = 12$ inches.

Hence the point does not lie in the bar, but is 8 inches to the left of A.

3. A screw jack with a pitch of 0·25 inch has a handle 30 inches long. A force of 20 lb. must be applied when a load of 6600 lb. is being lifted. Calculate the velocity ratio, the actual mechanical advantage and the efficiency. (E. P. U.—1937)

A. Actual mechanical advantage $= \frac{W}{P} = \frac{6600}{20} = 330$

$$\text{Velocity Ratio} = \frac{2 \times 3{\cdot}14 \times 30}{{\cdot}25} = 753{\cdot}6$$

$$\therefore \text{Efficiency} = \frac{\text{Actual mechanical advantage}}{\text{velocity ratio}} = \frac{330}{753{\cdot}6} = 43{\cdot}8\%$$

CARL GUSTAV PATRIK DE LAVAL (1845—1913)

De Laval was born at Blasenborg, Sweden in 1845. He was graduated from the University of Upsala. He received the degree of the Doctor of Philosophy from the University of Stockholm in 1872. After finishing his University career he was engaged for a few years in the construction of several industrial plants. After 1877 he directed his attention to inventions of various types of machines such as a Bessemer converted for making steel, a centrifugal cream separator, etc. The principle of a centrifugal separator has been utilised today to varieties of purposes.

In 1870, he devised a machine in which the force of steam through a nozzle was utilised in blasting sand. The nozzle was capable of rotation and was bent at the exit. He observed that the nozzle rotated at high speed as the steam pressure was increased. Thus he conceived the idea of making a wheel rotate by impinging steam on its blades. In 1883 he made his first practical application of a steam machine ;—the smaller turbines acquiring a speed of revolution as high as 30,000 per minute. Modern high speed machines are all run with steam turbines.

Laval also invented steam boiler capable of withstanding a pressure of 1500 lbs. to the square inch. In 1892 he received the gold medal of the Swedish Academy of Sciences. In 1904 he was awarded the Grashof medal from Germany. He died in February, 1913.

Summary

A **machine** is any contrivance by which a force applied at a given point in a given direction is made to appear at some other point in a different direction and with different intensity, to overcome some resistance there.

$$\text{Mechanical Advantage of a machine} = \frac{\text{resistance}}{\text{effort}} = \frac{W}{P}$$

$$\text{Efficiency of a machine} = \frac{\text{Useful work done}}{\text{Total energy supplied}}$$

In any **lever,** effort $\times$ perpendicular distance from fulcrum.
$=$ resistance $\times$ perpendicular distance from fulcrum.

In **lever of class I,** *fulcrum* is in the middle; $W \div P$ may be $> = \text{or} < 1$

In **lever of class II,** *resistance* is in the middle; $W \div P$ is always > 1

In **lever of class III,** *effort* is in the middle; $W \div P$ is always < 1.

In a single pulley the mechanical advantage is unity, whereas in a **differential pulley system,** the mechanical advantage is increased a number of times.

In a **wheel and axle,** the ratio of the radius of wheel and that of the axle gives the mechanical advantage.

For a firm grip **toothed wheels** and **pinions** are used in machinery.

Roman Steelyard is an appliance belonging to a lever class for weighing objects conveniently. A **platform balance** is an improved lever system for weighing purposes.

An **inclined plane** is an appliance for raising heavy objects. A screw is an improved form of an inclined plane.

EXERCISES ON CHAPTER VI

Reference

Art. 78

1. Define the terms "velocity ratio" "mechanical advantage" and "efficiency" as applied to machines. (Mys. U.—1952; P. U.—1952: E. P. U.—1951, '53).

Art. 79

2. Describe a lever of class III. Calculate its mechanical advantage and show how the principle of work is satisfied there. (Utkal. U.—1952)

Art. 79

3. What are levers? Give examples of different classes of lever. (P. U.—1921)

Art. 80

4. Give a very brief description of the second system of pulleys; and deduce the mechanical advantage. (E. P. U.—1952)

Art. 81

5. Describe a simple pulley and explain its action. A pulley system consists of a fixed block and a moving block each containing three pulleys. Explain what the mechanical advantage is. (Del. H. S.—1950)

Art. 81

6. Deduce the mechanical advantage of a wheel and axle from the general principle of the conservation of energy. (P. U.—1929, '31)

Art. 88

7. Sketch a common steelyard. Explain its underlying principle and show how it is graduated. (E. P. U.—1951)

Arts. 83

8. Explain the construction and action of a railway platform balance used for weighing heavy parcels or luggages. (U. P. B.—1942, All. U.—1941)

Arts. 83 & 84

9. What are the two ways in which effort can be applied in raising a body up an inclined plane? Which is more advantageous and why? (Mys. U.—1952)

Art. 85

10. Derive an expression for the mechanical advantage of a screw. (P. U.—1951)

Art. 85

11. The arm of a screw jack is 15 inches long and the screw has 10 threads per inch. Find the force in seers that will be required to lift one side of a lorry weighing 60 mds. (P. U.—1952)

A. 2·55 seers.

Art. 85

12. Find the mechanical advantage and efficiency of a screw jack if an effort of 1 kgm. weight is applied at the end of the arm 21 cm. long to lift a mass of 250 kgm., the pitch of the screw being 5 mm. (E. P. U.—1950)

A. 250; 94·7 %.

CHAPTER VII

PROPERTIES OF MATTER

86. Physical States of Matter.—Matter can exist in three different states namely *solid*, *liquid* and *gas*. A **solid** preserves a *definite shape and volume* and it opposes any force which tends to change its shape. A given mass of **liquid** has a *definite volume* but *no fixed shape*; and when at rest it takes the shape of the containing vessel which it fills, maintains a free horizontal surface and offers no resistance to any external force tending to change its shape. A **gas** has neither a *definite shape* nor *volume*; it entirely fills up any space in which it is enclosed and always tends to expand. Liquids and gases are both included under the common term **fluid**, for if un-opposed, they can flow from one place to another.

A few solid like pitch, sealing-wax, etc., do not maintain a definite shape while liquids like treacle and tar do not readily assume a free horizontal surface; such substances are said to be *viscous*. On being heated, a solid may be converted into a liquid and a liquid into gas; while on cooling, the process may be reversed. Thus ice, water and steam are the three different states of a substance of the same chemical composition.

87. Constitution of Matter.—A portion of matter, whether solid, liquid or gas, may be divided and subdivided into smaller and smaller parts, each of which possesses properties identical with those of the original matter. A **molecule** is the smallest particle of any kind of matter, which can exist in a free state still possessing the properties of that matter.

Intermolecular Spaces.—When a body is compressed or when it is cooled, its volume contracts; on the other hand, its volume increases, when it is pulled out or heated. Again if sugar is dissolved in water, the volume of the solution is less than the total volume of sugar and water taken separately. These facts prove that the molecules constituting a body are not in intimate contact with one another but there are spaces between the molecules; these are known as **intermolecular spaces.** Any alteration in these spaces by any physical process produces a change in volume.

Intermolecular Forces.—The molecules of a body are held together by means of forces of attraction, called the *intermolecular forces*. This force is very strong when the distance between them is small compared to their dimensions, but it becomes vanishingly small, when the distance exceeds a certain critical value. The molecules are supposed to be in a state of *rapid motion* within the intermolecular spaces due to which they tend to separate from one another but due to mutual attraction they cannot leave each other permanently. Thus,

the molecules are under the simultaneous action of two opposing tendencies and a change in their relative proportion gives rise to different physical states of matter.

In a *solid*, the intermolecular attraction is far stronger than the molecular tendency to separate, so that the molecules are held together with a strong force. It is for this reason that a solid has a definite *shape* and volume and it offers a great resistance to any change in its shape or volume. In a *liquid*, the molecular attraction is rather feeble, so that any portion can move relatively to the other. A liquid has therefore no definite shape and yields to any external force tending to change the shape. In a *gas*, the molecules are widely separated from one another, so that the molecular attraction is negligibly small or practically absent and hence molecules can move about independent of each other. A gas has therefore no definite shape or volume but completely fills the containing vessel and offers much less resistance to any tendency of change in volume.

Physical and Chemical Changes.—Water, when sufficiently cooled, changes into ice and when heated, passes into vapour. But in each state the constituent molecules retain the same composition. The changes in which the constituent molecules and consequently the substance as a whole retain the same composition, are called *physical changes*. If now electric current is passed through a mass of water (acidulated) two kinds of gases are produced. The properties of these gases are quite different from each other and also different from those of water. Again if electric sparks be passed through a requisite proportion of hydrogen and oxygen, the gases explosively combine to form water. These changes in which the constituent parts lose their identities and give rise to a substance having different properties, are called *chemical changes*.

88. Properties of Matter.—Properties possessed by bodies in all the physical states,—solid, liquid or gaseous, are termed *general properties*. While those properties which are found in a particular state or states but not in all the three states, are called *special properties*.

Density.—Every body whether solid, liquid or gaseous, contains some quantity of matter and so possesses some inertia. If the body be homogeneous, the mass contained in unit volume of the body is called the *density* of the substance of which the body is made. Hence density is a general property.

Elasticity.—It is the property by virtue of which a body offers resistance to external forces tending to change its volume or shape or both. Whenever deformed upto a certain limit, it will tend to regain the original volume or shape. Hence elasticity is also a general property.

A body is said to be *perfectly rigid*, when an external force however large, cannot produce any relative displacement of its parts. No body is perfectly rigid, but substances such as glass and steel,

possess rigidity to a very high degree. Again a body is said to be *perfectly elastic* when it completely recovers its original volume or shape after the deforming force has ceased to act.

Elastic Limit.—No body is perfectly elastic. It is found however that a body will behave as a perfectly elastic body *i.e.*, it will completely regain its original condition, when the deforming force does not exceed a certain limit, depending on the nature of the body. This is called the *limit of elasticity* or **elastic limit** of the substance. If the force is increased still further, the body acquires a *permanent set* and is said to be overstrained.

Elastic Fatigue.—If a comparatively large force even within the elastic limit acts upon a body for a time, the body does not *readily* return to its original state, even when the deforming force has ceased to act. The body is then said to be in a state *elastic fatigue.* If the force is taken away and the body is allowed to rest for some time, it gradually recovers and comes back to its former state.

Hardness.—It is the property due to which one solid offers resistance to being scratched by another. There is no absolute value for the hardness of a substance but it is a relative property and is determined by referring the particular substance to a scale of hardness, usually *Mohr's Scale of Hardness*, in which the substances are arranged in such a way that each of them can be scratched by those following and not by any that precedes it. Steel is harder than many substances and diamond is the hardest substance known. Hardness is special property of a solid state.

Cohesion.—It has already been mentioned that the molecules of a body attract each other with a force which tend to bind the molecules to one another. The binding force is called the *cohesive force* and the phenomenon is known as *cohesion*. In case of cohesion, the forces of attraction bind molecules of the *same* kind. Cohesion is very high in solids, so that the molecules are bound to one another with great force, and thus give a *definite shape* to a solid. It is, however, different in different solids. In a chalk or lead pencil, the particles are held together by cohesion. Cohesion is also exhibited to a slight extent in liquids and is practically absent in gases. The formation of drops by liquids when in small masses *e.g.*, dew or rain drops, is due to cohesion.

Adhesion.—The force of attraction that bind molecules of *different* kinds are called *adhesion*. Thus due to adhesion water adheres to glass. If a glass plate is made to float on water, great force will be necessary to take off the plate. Sticking wood by glue, joining bricks by mortar and the process of soldering or nickel plating are instances of adhesion. Properties such as surface tension, viscosity, diffusion etc. which are characteristic of fluids are dealt with in Chapters XII & XIII of General Physics.

89. Stresses and Strains.—When a force or system of forces acting upon a body causes a relative displacement of its various parts,

a change of length, volume or shape takes place. The body is then said to be *strained* by the force and the change in the length, volume or shape of the body to the original quantity is called a *strain*. The strain is therefore a pure *ratio* and has got no unit.

When a body is put to a strain by external forces, internal forces of reaction are called into play due to relative displacement of its parts, which tends to restore the original shape of the body. The reactions, set up within the body, are equal and opposite to the forces applied, so long as there is no permanent distortion. The restoring force generated per unit area of the body is called a *stress*. The internal forces of reaction called into play during a state of deformation of the body are, by Newton's Third law of Motion, equal and opposite to the external applied forces. Hence we measure a stress by examining the external forces per unit area causing the deformation of the body.

(*i*) **Volume Stress and Strain.**—When the body is subjected to uniform pressure acting perpendicularly at every point on the surface, there is a *change in volume only* without any change in shape. Suppose that a body of the shape of a cube represented by ABCDKLMN of fig. 117 is acted upon by uniform force P perpendicular to its faces. This system of forces will tend to squeeze the body uniformly from all directions and consequently the body would contract in volume its shape remaining unaltered. The amount of contraction is proportional to the magnitude of the pressure applied at each face provided certain limit called the elastic limit, dependent on the material of the body, is not surpassed. Fig. 118 shows the nature of contraction, in which the face ABCD reduces to EFGH. Correspondingly other faces contract by the same amount. If V be the original volume of the body and v, the change produced in this volume, then

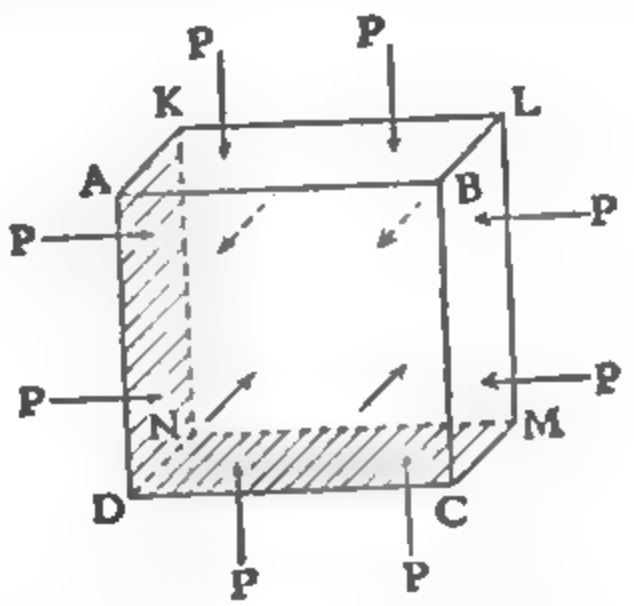


Fig. 117

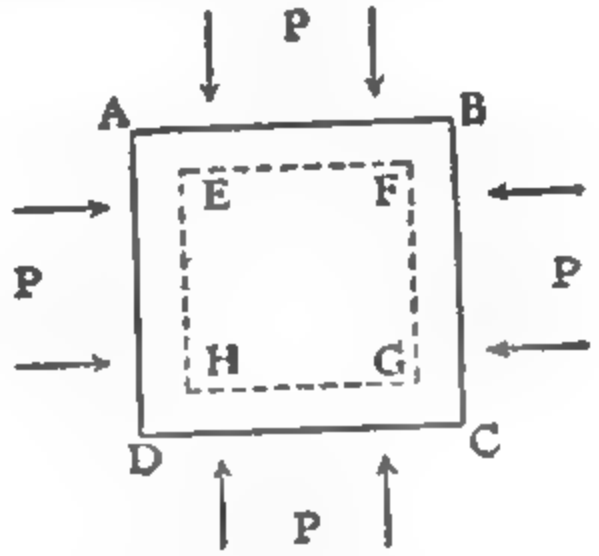


Fig. 118

$$\text{Volume strain} = v/V \qquad \ldots (89,1)$$

Bodies in all the three states may suffer **volume strain.** In the case of *solids* and *liquids*, the strain produced is very small even for a large force, while in case of gases, even a small force produces a measurable change in volume, so that the strain is appreciable.

When the body is squeezed in volume by the application of external forces perpendicular to the surface of the body, it tends to regain the original volume due to elasticity. Hence the stress developed within the body is equal to the externally applied force per unit area or

pressure. If P be the force applied on any face of the cube and if the face area be α, then the *normal stress* is P/α.

(*ii*) **Shearing Stress and Strain.**—When a solid is strained in such a way that it undergoes a change only in shape or form but not a change in volume, it is said to suffer a *shearing strain*.

Let ABCDEFGH represent a rectangular block (Fig. 119) whose face DCGH is kept fixed to a horizontal bed. Let a force P act uniformly and tangentially over the faces DG and BE, so that the place BE is displaced through a small distance relatively to the face DG and the block assumes a form CDLKGHIJ of a rhombus. The material of the block between the faces DG and LJ undergoes a change in form only but no change in volume. The block therefore suffers a shearing strain, which is measured by the angle ADL (Fig. 120) called the *angle of deformation* or **angle of shear.**

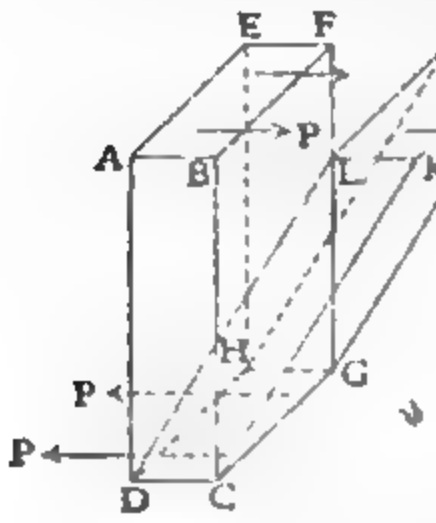


Fig. 119

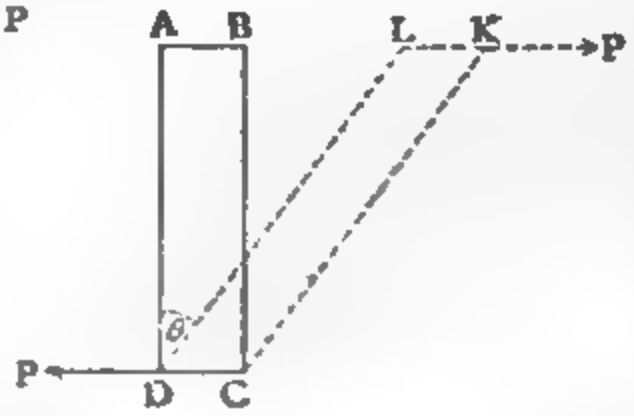


Fig. 120

Let $\angle ADL = \theta$, $AD = b$, $AL = x$; as θ is small, $\theta = \tan\theta$.

$$\therefore \quad \textit{Shearing Stress} = \theta = \tan\theta = \frac{x}{b} \qquad \ldots(89,2)$$

The relative displacement for planes at unit distance apart is known as the *displacement gradient*. The corresponding stress is called the **shearing stress.** If the area of the upper or lower face is s, on which the tangential stress is acting, then

Shearing Stress $= P/s = p$, say.

The elasticity in this case is called the *elasticity of shape* or **simple rigidity.** As only solids possess a definite shape of their own, this kind of elasticity can occur in solids only.

(*iii*) **Longitudinal Stress and Strain.**—If a body which is very long in comparison to other dimensions, for example, a thin rod or a wire, is acted upon by stretching forces PP at both ends in the direction of its length, then the deformation takes place along the length only (Fig. 121); the strain in this case is termed the **longitudinal strain** or **tensile strain.**

Fig. 121

If L be the original length of the body and l the change produced n this length, then

$$\textit{Longitudinal Strain} = l/L \qquad \ldots(89,3)$$

The corresponding stress is sometimes called the **tensile stress** and the elasticity in this case is termed *longitudinal elasticity*. If P be the force acting at each end of the specimen over an area of cross section $\varkappa$, then the tensile stress is $P/\varkappa$.

90. Hooke's Law : Modulus of Elasticity.—In all cases of strains, there should be some relation between the strain or deformation produced and the deforming force. Robert Hooke experimentally found that in all cases of stresses and strains *within elastic limit, the stress developed is proportional to the strain.* This is known as **Hooke's law.**

$$\text{Hence, stress} \propto \text{strain} \quad \therefore \quad \frac{\text{Stress}}{\text{Strain}} = \text{a constant} \qquad \ldots \quad (90,1)$$

This constant is termed the modulus of elasticity or *co-efficient of elasticity* and depends only upon the nature of the material and kind of the deforming force used to produce the strain. The modulus may also be defined as the *stress developed to produce unit strain*

Different moduli of Elasticity.—The modulus of elasticity becomes different according to the nature of the strain produced. As strain is a pure ratio, each modulus is expressed in the same unit as the corresponding stress.

(*i*) **Bulk Modulus.**—Let a force of p dynes per unit area act normally all over the surface of a body of volume V and thereby produce a diminution of volume v. Then,

Stress = p = force in dynes per unit area and strain = v V

Modulus of volume elasticity or **bulk modulus** k is given by

$$k = \frac{p}{v/\text{V}} \text{ dynes per sq. cm.} \qquad \ldots \quad (90,2)$$

If the force is expressed in poundals and area in square inches, the unit of bulk modulus is then expressed in poundals per sq. inch. As a body possesses some volume in every state of matter, the bulk modulus is a general property. A liquid possesses very high volume elasticity. The reciprocal of bulk modulus is called *compressibility.*

(*ii*) **Modulus of Rigidity.**—Let a body undergo a change only of shape or form, as in fig. 119, without any change in volume, due to a tangential force p per unit area ; then

Shearing stress = p and Shearing strain = $\theta = x/b$

$$\therefore \quad \textit{Modulus of Rigidity } n = p/\frac{x}{b} = \frac{pb}{x} \qquad \ldots \quad (90,3)$$

The unit of rigidity modulus is of the same nature as that of bulk modulus.

The shape of a solid may be changed in various ways, *viz.*, by *flexure* or bending, by *torsion* or twisting and by *tension* or stretching. The *Elasticity of Flexure* is the property in virtue of which a straight rod or plank bent by the application of a force, will recover itself when

the force is taken away. This is found in clock and watch springs, bows, carriage springs, etc.

The *Elasticity of Torsion* is brought into existence when one end of a wire is kept fixed, while the other end is twisted. This is shown by suspending a ball from a long wire at the upper end. The ball is rotated and when set free, it will keep on revolving in both the directions and will come to rest after a time.

(*iii*) **Young's Modulus** (*modulus of longitudinal elasticity*).—Suppose a wire of length L cm. and of cross-section a sq. cm. is stretched by a force P dynes along its length and thereby suffer an increase l cm. in the length. (For Expt. vide J. Chatterjee's Intermediate Practical Physics).

Then Tensile stress $= P/a$ and Tensile strain $= l/L$

$$\therefore \quad \text{Young's Modulus (Y)} = \frac{P}{a}\Big/\frac{l}{L} = \frac{PL}{al} \text{ dynes/sq. cm.} \quad \ldots \quad (90,4)$$

(*iv*) **Poisson's Ratio.**—When a rod is subjected to a longitudinal extension by a tensile stress, it is accompanied by a lateral contraction. The ratio of the lateral contraction and longitudinal strain is called the Poisson's ratio. If b be the amount of contraction of a rod of width B, then lateral contraction is b/B. The longitudinal strain is l/L. Thus the Poisson's ratio, usually denoted by σ, is given by,

$$\sigma = \frac{b/B}{l/L} = \frac{bL}{lB} \quad \ldots \quad (90,5)$$

It has got no dimension. For a long and thin wire, this ratio is negligibly small and may be neglected.

Work done in a Strain.—When any type of strain is produced on a system by some external forces, the strain gives rise to a corresponding type of stress. So long as the external force is not applied, the strain and so the stress are not produced. If now, the external force, applied on the system, be continuously increased then so long as the elastic limit is not exceeded, the deformation of the system increases proportionately. At every stage, the stress developed within the body is equal and opposite to the external force applied.

When the external force is gradually increased, there would be a gradual deformation of the system. Consequently the external force originally applied at *some region of the system* would shift its place along with the displacement of that region due to strain. Hence the external force in moving through some distance *does some work* on the system. The greater is the strain, the larger is this amount of work done. Suppose that an external force in rising from a zero value to F makes a total displacement S of its applied region. We may simplify the idea by assuming that the average value of the external force is $\frac{1}{2}$F. Hence the total work done in bringing about the deformation is $\frac{1}{2}$ FS.

For a *unit cube* of a medium, the force F applied at any face, *is the stress* since face area is unity. The displacement S of any face area is the strain, since the distance between opposite faces is unity. Hence the work done *per unit volume* of a medium due to elastic deformation is $\frac{1}{2} \times$ stress $\times$ strain. This work goes to increase the potential energy of the medium. When the external force is withdrawn, this amount of work is given out by the system.

91. Experimental Determination of Young's Modulus.—Two exactly similar wires A and B (Fig. 122) of the material under investigation are hung up close to each other from the same fixed support. The wire A has a fixed load W attached to its lower end to keep it tight and a short scale S, graduated in mm. is fixed to this wire towards the lower end. The wire B is the experimental wire, to which is attached a weight hanger at the lower end, on which weights are placed to stretch the wire. This wire carries a vernier V which slides along the scale S.

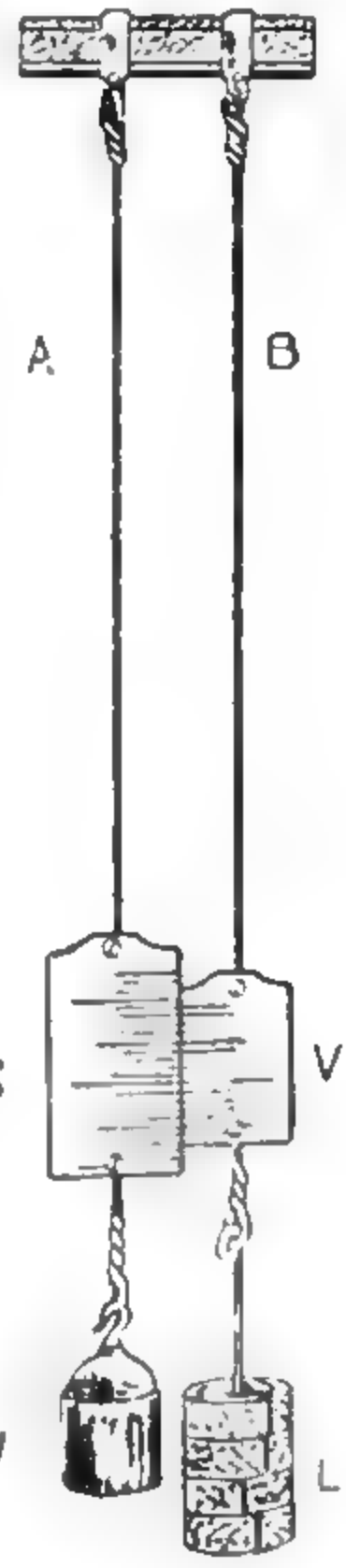


Fig. 122

At the beginning, both the wires are stretched by suitable weights to make them free from kinks. The length of the wire B from the point of suspension to the point where the vernier is attached is measured with an ordinary metre scale. The diameter of the wire is measured at several places by means of a screw-gauge and at each place two readings should be taken at right angles to each other. The mean of all such readings is the average diameter of the wire and thus the radius is found.

Now remove any extra load from the experimental wire keeping only that amount just sufficient to make it free from kink. Take the reading of the vernier. Then increase loads by equal instalments of 1 kg., and take reading of the vernier at each step. The difference of readings gives the corresponding elongation in each case. The total load must not stretch the wire beyond the limit of elasticity. The weights are then removed one by one in the same instalments as before and the readings of the vernier are again noted. The two sets of readings should agree fairly well and from these sets the mean elongation for a given load may be calculated.

Let the length of the wire be L cm., mean radius of the wire be r cm., mean elongation of the wire be l cm., corresponding to the load W gms,

Then *Young's Modulus* $(Y) = \frac{Wg}{\pi r^2} \frac{l}{L}$

$$= \frac{WgL}{\pi r^2 . l} \text{ dynes per sq. cm.} \quad \ldots \quad \ldots \quad (91,1)$$

In practice, a graph (Fig. 123) a drawn with *load* as *ordinate* and the corresponding *elongation* as *abscissa*. Within the limit of elasticity this graph is a *straight line*, showing that Hooke's law is obeyed, Any suitable elongation and the corresponding load may be taken from the graph and the calculation made as before.

The two wires are fixed to the same support since any yielding of the support due to a load on any wire, would equally affect the depression of both the wires. They are also of the same material so that any change of temperature affects both of them equally without altering the reading.

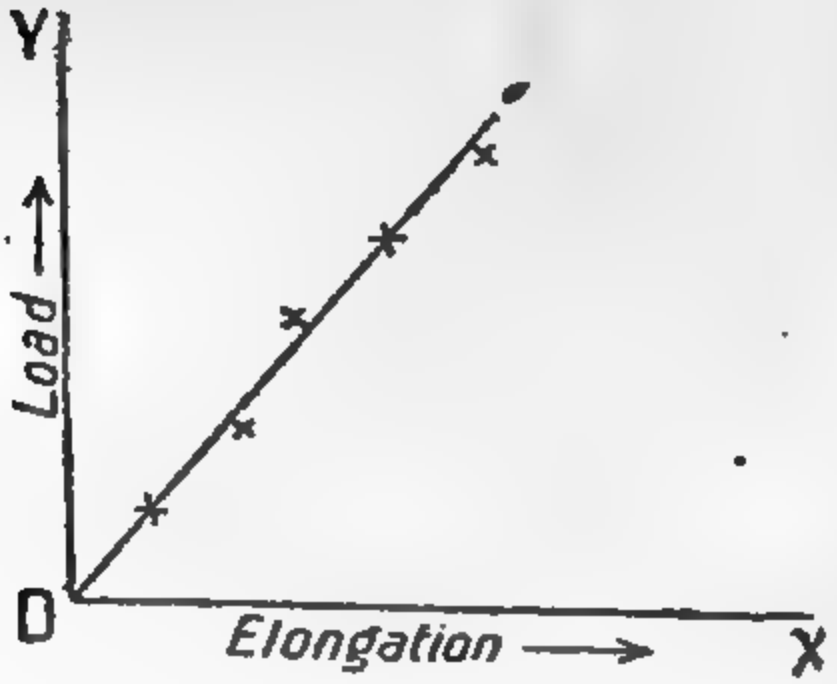


Fig. 123

When the load is zero, the strain is also zero, so that the graph passes through the origin. If the load is too much increased, the stress goes beyond the *elastic limit* and the wire has a chance of breaking for a certain load. This load is called the **breaking weight** for the wire. The breaking weight for a wire of unit cross-section is called the **breaking stress** for the material of the wire and to find the breaking weight for a certain wire, we are to multiply the corresponding breaking stress by the area of cross-section of the wire.

The following Table supplies the various types of elastic constants for common substances :—

Elasticities of Metals

Metal	Young's Modulus dynes/cm^2 $\times 10^{11}$	Rigidity dynes/cm^2 $\times 10^{11}$	Poisson's Ratio	Volume Elasticity dynes/cm^2 $\times 10^{11}$
Aluminium	7·0	1·67	·34	7·46
Copper	12·3	4·55	·337	13·1
Steel	20·9	8·12	·29	16·4
Silver	7·9	2·87	·34	10·9
Constantan	16·2	6·1	·325	15·5
Manganin	12·4	4·65	·33	12·1
Brass	9·7 – 10·2	3·5	·34 – ·4	10·65
German Silver	11·6	4·3 – 4·7	·37	...

Examples—

1. A mass of 20 kgm., is suspended from a vertical wire 600 cm. long and 1 sq. mm. in cross-section. When the load is removed, the length of the wire is found to be 599·5 cm. Find the Young's modulus for the material of the wire.

A. Original length of the wire (L) = 599·5 cm.

Elongation in the wire (l) = 600 – 599·5 = 0·5 cm.

Deforming force per unit area $= \frac{20 \times 10^3 \times 981}{0{\cdot}01}$ dynes.

$\therefore$ from eqn. (91,1) $Y = \frac{20 \times 10^3 \times 981}{0{\cdot}01} \times \frac{599{\cdot}5}{0{\cdot}5}$

$= 2{\cdot}35 \times 10^{12}$ dynes/cm^2.

2. A uniform steel wire of density 7·8 gm. per c. c. weighs 16 gm. and is 250 cm. long. It lengthens by 1·2 mm when stretched by a force of 8 kgm weight. Calculate the value of Young's modulus for steel. (Gau. U.—1953)

A. Let α be the area of cross-section of the wire in sq. cm. Then the volume of the wire $= 250\ \alpha$ c. c.

Since mass = volume $\times$ density, 16 gm. $= 7{\cdot}8 \times 250\ \alpha$

whence $\alpha = \frac{250 \times 7{\cdot}8}{16}$ sq. cm.

Now stress $= \frac{\text{force}}{\text{cross-section}} = \frac{8 \times 1000 \times 981 \times 250 \times 7{\cdot}8}{16}$ dynes/sq. cm

and strain = elongation per unit length $= \frac{\cdot 12}{250}$

$\therefore\ Y = \frac{\text{stress}}{\text{strain}} = \frac{8 \times 1000 \times 981 \times 250 \times 7{\cdot}8 \times 250}{16 \times \cdot 12}$ dynes/sq. cm.

Using a log-table for calculations,

log 8	= 0·9031	log 16 = 1·2041	
log 1000	= 3·0000	log ·12 = $\bar{1}$·0792	antilog 12·2994
log 981	= 2·9917		= 1·993 × 10^{12}
log 250	= 2·3979	·2833	correct to the order of 10^{12}
log 7·8	= 0·8921	12·5827	
log 250	= 2·3979	0·2833	
	12·5827	12·2884	

$\therefore$ Young's modulus for steel = 1·993 × 10^{12} dynes/sq. cm.

3. A stress of 1 kgm. per square millimetre is applied to a wire of which the Young's modulus is 10^{12} dynes per sq. cm. Find (*a*) the percentage increase in length and (*b*) the work done per unit volume of the wire during the stretching. (Raj. U.—1951)

A. The word 'stress' in the problem is rather popularly used to mean the external force. Thus, the external force being 1 kgm. per sq. mm., stress on it is,

(*a*) $\frac{\text{Force}}{\text{area}} = \frac{1000 \times 981}{\cdot 01} = 981 \times 10^5$ dynes/cm^2.

Now, $Y = \frac{\text{Force per unit area}}{\text{elongation per unit length}}$

whence, elongation per unit length $= \frac{981 \times 10^5}{10^{12}}$

$\therefore$ percentage increase in length $= \frac{100 \times 981 \times 10^5}{10^{12}} = \cdot 00981\%$.

(b) Stress $= 981 \times 10^5$ dynes/cm.^{2} and strain $= 981 \times 10^{-7}$

$\therefore$ Work done per unit volume $= \frac{1}{2}$ stress $\times$ strain.

$= \frac{1}{2} \times 981 \times 10^5 \times 981 \times 10^{-7} = 4812$ ergs approximately.

4. Find the change in volume of a solid iron sphere of radius 5 cm. subjected to a uniform hydrostatic pressure of 0·1 ton weight per square inch, the bulk modulus of iron being 14×10^{11} dynes per sq. cm.

A. Volume of the sphere $= V = \frac{4}{3}\pi r^3 = \frac{4 \times 3{\cdot}14 \times 5^3}{3}$ c.c.

Normal stress $=$ ·1 ton/sq. in. $= \frac{{\cdot}1 \times 20 \times 4 \times 28 \times 32{\cdot}2 \times 13825}{2{\cdot}54 \times 2{\cdot}54}$ dynes/cm.^{2}

Since $k = \frac{pV}{v}$

$\therefore v = \frac{pV}{k} = \frac{4 \times 3{\cdot}14 \times 5^3 \times 8 \times 28 \times 32{\cdot}2 \times 13825}{3 \times 2{\cdot}54 \times 2{\cdot}54 \times 14 \times 10^{11}} = 5{\cdot}8 \times 10^{-3}$ c.c.

92. Friction.—The surface of a solid is never perfectly smooth; consequently whenever one body slides over another, there is a sort of resistance to its motion. Hence if two bodies be in contact with each other and if we try to drag one of them over the other, a force is set up at the surface of contact, tending to resist the motion. This is called the force of **friction** between the surfaces in contact. The friction is the outcome of the *roughness* of the material surfaces in contact. So if the surfaces be perfectly smooth, there is no force of friction to oppose the motion. The force of friction always acts parallel to the surfaces in contact and opposite to the direction of motion. Friction is a special property of solids. When a liquid or a gaseous mass flows, there is something like frictional resistance between various moving layers This peculiar type of friction within a fluid media is called its *viscosity*.

Let a rectangular solid body B (Fig 124) remain at rest on a plane horizontal surface A. The forces acting on B are (*i*) its weight W acting vertically downwards and (*ii*) the reaction R, acting vertically upwards. In a state of rest the upward reaction R balances the weight W and no friction is brought into play. If now a small force P be applied to B, parallel to the surface, a resistance say F, is offered to the motion. If this body is still at rest, it is in equilibrium under the action of the forces, R, W, F and P. As R is equal and opposite to W, the force F in this case must be equal and opposite to P.

As P is increased, F also increases; it is found that so long P does not exceed a certain limit, there is no motion. F being thus alway equal to P. The resistance F which is thus brought into play by the external force P, in a direction opposite to that of the latter, is a *self-adjusting force* and so long as the body is at rest, the force is equal to the pulling force. The force F is called the *frictional force* between the two bodies in contact.

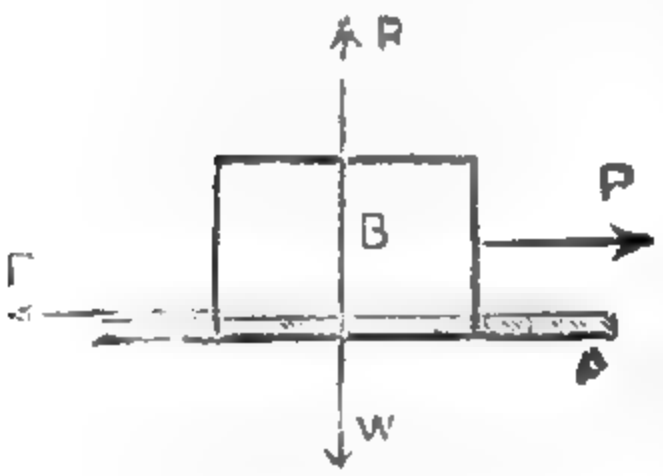


Fig. 124

Limiting Friction.—Although friction is a self-adjusting force, it does not however increase indefinitely with the external force. Thus, if the external force P is gradually increased, the force of friction reaches a maximum or

limiting value which depends on the nature of the surfaces in contact and the magnitude of the pressure (*i.e.*, the normal force) between them. The body is now on the point of sliding and the friction then exerted is called **limiting friction** between the two surfaces under the given normal force. If the external force be increased further, the equilibrium will be lost and the body begins to move It is to be noted that under given conditions the *limiting friction* is the *maximum* between the two bodies in contact.

The friction is said to be *sliding* or *rolling* according as one body slides or rolls over another. The dragging of a heavy load over the ground, the rubbing of two hands, etc furnish examples of sliding friction while the rolling of a cart-wheel on the road offers an example of the latter.

When motion has once set in, the force to be applied to maintain a steady motion is found to be slightly less than that necessary to start the motion. Hence *sliding friction is slightly less than the limiting friction*. If, when the equilibrium is limiting, the normal reaction and the frictional force be compounded into a resultant single force, the angle which this resultant makes with the normal to the surface, is called the **angle of friction** and the single force is called the *resultant reaction*.

Friction plays an important part in our everyday life. Without friction on the ground, we would not have been able to walk ; without friction we cannot fix nails, tie knots, climb trees, etc Again it becomes sometimes necessary to *increase* the friction when sand is thrown on uphill railway lines after rains, violin bow is rubbed with resin etc. When a brake is applied, it comes in contact with the moving wheel and owing to the friction produced, the car is stopped.

Also in many cases friction in minimised. Thus the moving parts of different machines are freed from friction as far as possible by lubricating oil, graphite, vaseline, etc.; Fig. 125 shows the cross section of the *collar bearing*, in which the axle S of the revolving part is loosely fit in the socket so as to be able to rotate.

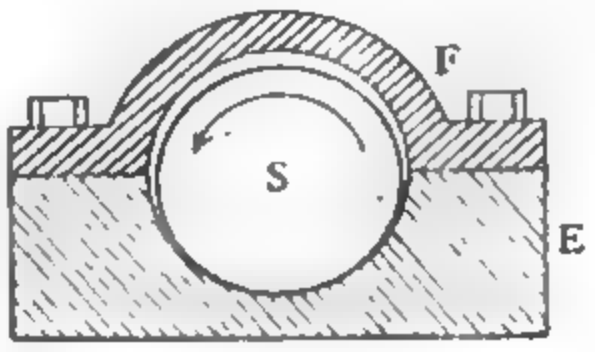


Fig. 125

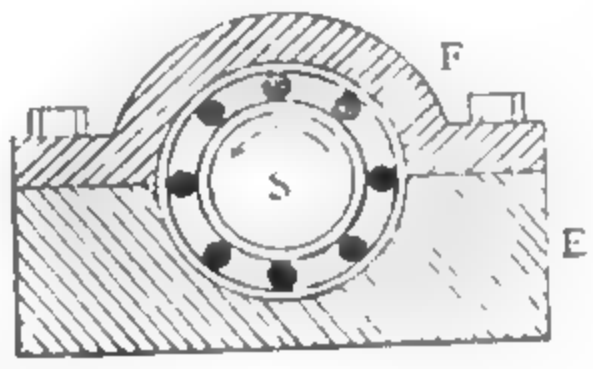


Fig. 126

The space between the two is well lubricated. Roads are covered with tar macadam so as to present an even surface, while the rims of wheels are provided with rubber tyres and motor cars with pneumatic tyres. To diminish friction to the greatest possible extent, agate knife-edges are used in balances and jewel centres in watches. Again a rolling friction is less than sliding friction, heavy pieces of furniture are provided with wheels at the base, which can rotate in different vertical

planes. In bicycles, etc., the sliding friction is replaced by rolling friction with *ball bearing* arrangement, in which a number of hard steel balls are placed loosely in a metal case round the axle. Fig. 126 shows a ball bearing arrangement in which the axle S is very free to move.

93. Laws of Friction.—When one body is at rest in contact with another, the friction between them is said to be **static.** When it is just on the point of sliding over the other, the friction is said to be *limiting* ; and when one body is actually sliding over the other, the friction is termed *kinetic* or **dynamic.**

Laws of Limiting Friction :—

(1) The *direction* of the limiting friction is opposite to the direction in which the point of contact tends to move.

(2) The *magnitude* of the limiting friction always bears a constant ratio to the normal reaction.

(3) The *constant ratio* depends only on the material and the nature of the surfaces in contact and not on their extent or shape, so long as the normal reaction remains the same.

Co-efficient of Friction.—The constant ratio of the limiting friction to the normal reaction acting across two surfaces in contact, is called the co-efficient of friction and is usually denoted by μ. Thus, if F be the limiting friction and R the normal reaction, then

$$\mu = \frac{F}{R} \quad \text{whence } F = \mu R \qquad \ldots (93,1)$$

The value of the coefficient of friction is different for different pairs of substances but it is always less than unity for any pair. When one of the bodies is sliding over the other, the forces to maintain the motion is slightly less than that at start. Let the force be F_1. Then the co-efficient of dynamic friction μ_1 is denoted by the ratio,

$$\mu_1 = \frac{F_1}{R} \qquad F_1 = \mu_1 R \qquad \ldots (93,2)$$

Again, if the resultant reaction between the two surfaces is denoted by Q and the angle of friction by α, then

$$F = Q \sin \alpha \qquad \text{and} \qquad R = Q \cos \alpha$$

$$\text{Whence } \frac{F}{R} = \tan \alpha \qquad \text{or} \qquad \tan \alpha = \mu \qquad \ldots (93,3)$$

i.e., the coefficient of friction is equal to the tangent of the angle of friction.

Angle of Repose.—Let a body of weight W be placed on a rough inclined plane BAC (Fig. 127). The weight W acts vertically downwords and the reaction R acts normally to the plane AB. Under the action of these two forces the body would tend to move down the inclined plane but the motion is opposed by the frictional force

between the plane and the body acting opposite to the direction of motion. If the inclination of the plane to the horizontal is small, the force of friction may be equal to the resultant force tending to slide the body down the inclined plane and the body will remain at rest. If the angle of inclination is gradually increased, the sliding force increases and so also the force of friction. But ultimately the friction reaches its limiting value F, say. The angle, which the plane now makes with the horizontal, is called the angle of repose.

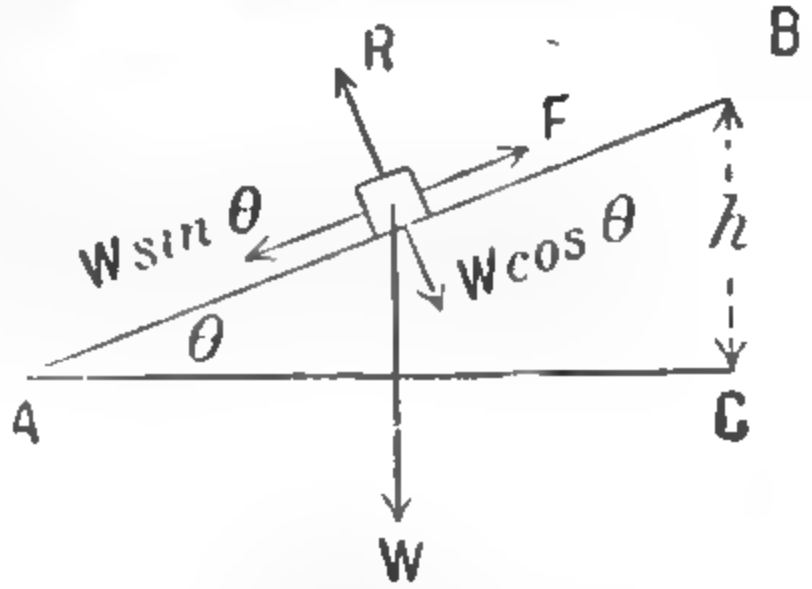


Fig. 127

Let the force of friction = F, and the angle of repose = θ.
Then the component of W parallel to the plane = W sin θ
And the component of W normal to the plane = W cos θ

∴ For equilibrium, W sin θ = F and W cos θ = R

$$\text{or } \tan\theta = \frac{F}{R} = \mu = \tan\alpha \qquad \text{whence} \qquad \theta = \alpha$$

$$\text{Also } \mu = \tan\theta = \frac{BC}{AC} = \frac{\text{height of the plane}}{\text{length of base}} \qquad \ldots (93,4)$$

Hence the angle of friction is equal to the angle of repose, which is therefore also called the *angle* of friction. The co-efficient of friction μ is obtained by dividing the height by the base of the inclined plane at the limiting angle.

Example :—

If the angle of friction is 30°, calculate the co-efficient of friction.
(P. U.—1951)

A. Since the co-efficient of friction is equal to the tangent of the angle of friction,

$$\mu = \tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{1}{\sqrt{3}} = 0{\cdot}5774$$

94. Motion of a body on a Rough Inclined Plane.—Let a body of mass m be placed upon a rough inclined plane of inclination θ to the horizon, when the equilibrium is limiting. Let μ be the co-efficient of friction between the body and the plane and let R be the normal reaction.

(i) Let the body be on the point of moving *down* the plane, so that the frictional force F acts *up* the plane.

Then R = $mg \cos\theta$ and $F = \mu R = \mu mg \cos\theta$

∴ The resultant force on the body down the plane

$$= mg \sin\theta - \mu mg \cos\theta \qquad \ldots (94,1)$$

∴ Acceleration of the body

$$= g \sin\theta - \mu g \cos\theta$$
$$= g \sin\theta - g \cos\theta \tan\alpha$$
$$= \frac{g \sin(\theta - \alpha)}{\cos\alpha} \qquad \ldots (94,2)$$

(*ii*) Let the force P, acting on the body parallel to the plane and along the line of the greatest slope, cause it move *up* the plane. Then the resultant force acting on the body *up* the plane

$$= P - mg \sin\theta - \mu mg \cos\theta \qquad \ldots(94,3)$$

$$= P - mg(\sin\theta + \tan\alpha \cos\theta)$$

$$= P - mg\frac{\sin(\theta+\alpha)}{\cos\alpha}$$

$$\therefore \text{ Acceleration} = \frac{P}{m} - g\frac{\sin(\theta+\alpha)}{\cos\alpha}$$

when the body is just in equilibrium, $P - mg \sin\theta - \mu\, mg \cos\theta = 0$

whence $P = mg(\sin\theta + \mu \cos\theta)$ ... (94,5)

95. Determination of the Co-efficient of Friction.—The co-efficient of friction between two solid surfaces can be determined by the following method :—

(1) A horizontal table is made of one material and a block of the other is taken (Fig. 128). The block is placed on the table. One end of a light string is attached to a hook fixed to the block while the other end passes over a smooth pulley fixed at the end of the table and carries a scale pan. A known weight is now placed on the block ; weights are then placed on the scale pan, till the block is just on the point of motion. This condition can be ascertained by tapping the plane gently. Let the total weight of the slab and the load be equal to W and the total weight on the scale pan + its own weight = w, Then force of limiting friction = w and the normal reaction, R = W.

Hence the co-efficient of friction $\mu = \frac{F}{R} = \frac{w}{W}$

The experiment is repeated with different weights on the slab and the mean value of μ is taken.

(2) An inclined plane is made of one material and a rectangular slab of the other is placed on it. The angle of inclination of the plane is gradually increased, till the slab is just on the point of sliding down the plane.

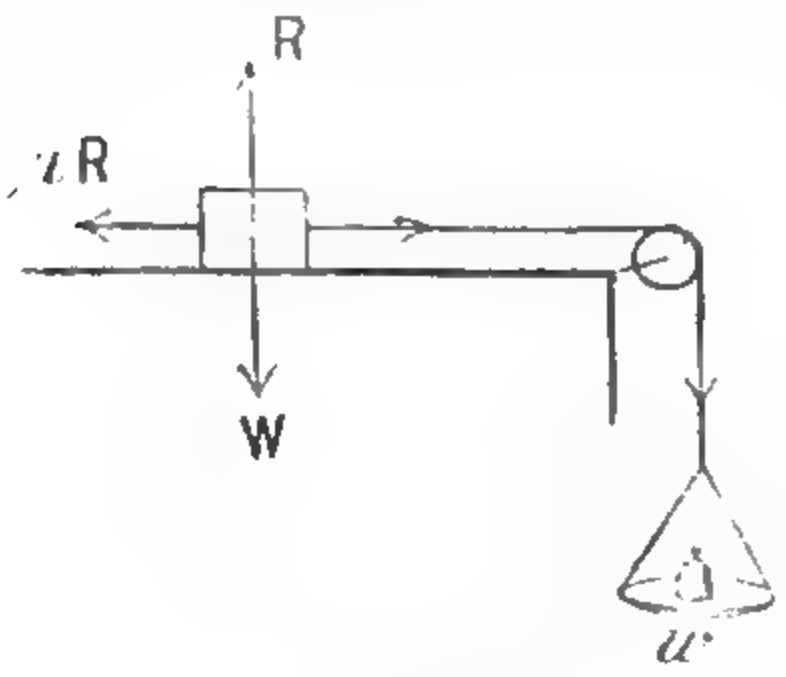


Fig. 128

Since the slab is in limiting equilibrium, the tangent of the angle of inclination of the plane to the horizontal is equal to the co-efficient of friction. Keeping the plane fixed at the position, the height and the length of the base are measured. The tangent of the angle of inclination is thus found whence the co-efficient of friction is obtained. (Vide J. Chatterjee's Intermediate Practical Physics).

The following Table supplies the Coefficients of Limiting Frictions between common substances :—

Co-efficients of Limiting Friction

Substances	Co-efficients	Substances	Co-efficients
Wood on wood	0·3 – 0·5	Stone on stone	0·7
Metal on wood	0·1 – 0·2	Metal on Leather	0·5
Metal on metal	0·2 – 0·5	Metal on stone	0·55 – 0·65

Examples :—

1. A metal block of mass 2000 gm. slides from rest 3 ft. down a rough inclined plane, the slope being 1 in 5. Find the resultant force down the plane, the co-efficient of friction being 0·15 ($g = 980$ cm. per sec. per sec.)

A. Resultant force down the plane $= mg \sin \alpha - F = mg \sin \alpha - \mu R$

$= mg \sin \alpha - \mu mg \cos \alpha = mg (\sin \alpha - \mu \cos \alpha)$

Here $\sin \alpha = \frac{1}{5}$, $\cos \alpha = \frac{2\sqrt{6}}{5}$, $\mu = 0{\cdot}15$ and $m = 2000$ gm.

$\therefore$ Resultant force down the plane $= 2000 \times 980 \times \left[\frac{1}{5} - 0{\cdot}15 \times \frac{2\sqrt{6}}{5}\right]$

$= 104468$ dynes approx.

2. A body of mass 30 kgm. is just pulled up a rough inclined plane of slope 1 in 3 by a rope lying in the vertical plane through the greatest slope and parallel to the plane. If the co-efficient of friction is 0·4, find the force.

Here $\sin \alpha = \frac{1}{3}$, $\cos \alpha = \frac{2\sqrt{2}}{3}$, $\mu = 0{\cdot}4$ and $W = 30000$ gm.

$\therefore$ From eqn. (94,5) the required force,

$P = 30000 \times 981 \times \left(\frac{1}{3} + 0{\cdot}4 \times \frac{2\sqrt{2}}{3}\right) = 2075680$ dynes approx.

3. A rough inclined plane makes, an angle of 30° with the horizontal. A load of weight 50 gm. rests on the inclined plane and is attached to a string which passes over a frictionless pulley, and supports a scale pan in which weights are gradually added. It is found that, when the weight in 50 gm. the weight just begins to move up the plane. Calculate the co-efficient of friction. (The weight of the scale pan being neglected). (Duc. U.—1928)

A. Let the required co-efficient of friction be μ. As weight in the pan is 50 gm., the body is pulled up along the plane with a force of $50g$ dynes.

Here $W = 50g$ and $R = W \cos \theta = 50g \cos 30°$

and $F = \mu R = \mu \times 50g \cos 30°$

$\therefore$ From eqn. (94,5) $50g = 50g \sin 30° + 50\mu g \cos 30°$

or $1 = \frac{1}{2} + \mu\frac{\sqrt{3}}{2}$ whence $\mu = \frac{1}{\sqrt{3}}$

ROBERT HOOKE (1635—1703)

Hooke was born on July 18, 1635 at Freshwater, Isle of Wight. Due to his ill health, his father did not send him to school until he was thirteen years of age. At this time his father died living his boy in poverty. He received his first education at Westminster School. From his boyhood he showed remarkable talent at mechanics and drawing. His wonderful genius was exhibited at the school when he read and mastered six books of Euclid in course of a week only. He learnt Latin and Greek at the school. At eighteen he entered Christ Church College, Oxford. After graduation from the Oxford Univertity, he was engaged for some years as a research assistant under Professor Boyle, during which period he did much work on the air pump and properties of gases.

Hooke was the first to conceive the idea of a flying machine. He also attempted to propound a theory of universal gravitation. He made the first balance wheel of the watch. He wrote original papers on light, architecture and music. He made improvements on telescopes and microscopes and made important advances in astronomy.

In 1662 he was appointed "curator of experiments" to the newly formed Royal Society and in 1677 he became the Secretary of the Society. He made extensive researches on the elasticity of solids and in 1678 he enunciated the famous law which goes by his name. He also made original investigations on thermometers and propounded a theory of heat. He died on March, 3, 1703.

Summary

Three states of matter.—(*i*) Solid. (*ii*) Liquid. (*iii*) Gas. These are due to relative magnitude of *molecular attraction* and grouping.

Hooke's Law.—*Within the elastic limit the stress developed is proportional to the strain.* Strain is the deformation produced per unit form of the body; stress is the internal force called into play within the body when undergoing strain due to external force.

$$\text{Modulus of Elasticity} = \frac{\text{stress}}{\text{strain}} = \text{a constant, within elastic limit.}$$

Different Moduli of Elasticity.— (*i*) *Bulk modulus* or modulus of elasticity of volume (*ii*) *Modulus of Rigidity* or modulus of elasticity of shape (*iii*) *Young's modulus* or modulus of longitudinal elasticity.

Friction is the property due to which force is set up at the surface of contact of two bodies preventing one from sliding over the other. The opposing force set up is called the *force of friction*. Friction is a *self-adjusting* force.

Laws of Friction.—These are laws of friction.—

The *direction* of the frictional force between two bodies in contact is opposite to the direction in which the point of contact moves or tends to move.

The *magnitude* of the limiting friction always bears a constant *ratio* to the normal reaction, which depends only on the materials and the nature of surfaces in contact. When motion ensues, this ratio is slightly less than that when the bodies are just in limiting equilibrium.

Co-efficient of friction is the ratio of the limiting friction to the normal force acting across two surfaces in contact and is usually denoted by μ. Thus, $F = \mu R$. Again the co-efficient of friction is equal to the tangent of the angle of friction. The value of μ can be determined experimentally.

EXERCISES ON CHAPTER VII

Reference

1. What is Hooke's law? Explain the terms 'stress and 'strain'. In what units are they expressed? (Del. U.—1952) — Arts. 88

2. A wire of 0·4 cm. diameter is loaded with 25 kgm. weight. A length of 100 cm. is found to be extended to 102 cm. Calculate the Young's modulus of the material of the wire (C. U.—1953) — Art. 88

A. $\cdot 9 \times 10^{10}$ dynes/cm.^{2}

3. Define 'Young's modulus of Elasticity'. How do you determine Young's modulus for a steel wire? (C. U.—1938; And. U.—1951; U. P. B.—1942; Raj. U.—1951; Dac. U.—1942; Del. H. S.—1950; Del. U.—1949; '52) — Arts. 89 & 90

4. A steel wire 4 metres in length and ·5 mm. in diameter is stretched by a weight of 5 kgm. find the elongation of the wire. Young's modulus of the wire $= 2 \cdot 04 \times 10$ kgm. weight per sq. cm. (All. U.—1936) — Art. 89

A. 0·5 cm.

5. An India-rubber cube of side 7 cm. has one side fixed while a tangential force equal to the wt. of 200 kgm. is applied to the opposite side. Find the shearing strain produced and the distance through which the strained side moves. (Mod. of rigidity for India-rubber $= 2 \times 10^7$ dynes per sq. cm.; $g = 980$). — Art. 89

A. 2×10^{-3} radian; 1·4 cm.

6. What force is required to stretch a steel wire of 1 sq. cm. in cross-section to double its length? Young's modulus of steel $= 2 \times 10^{12}$ dynes/cm^2. (U. P. B.—1942) — Art. 89

A. 2×10^{12} dynes.

7. What is meant by terms 'stress' and 'strain' as applied to a wire which is stretched by forces in the direction of its length? (P. U.—1952; U. P. B.—1943) — Art. 89

8. A piece of steel wire of diameter 0·075 cm. and of length 503 cm. is elongated by 0·7 cm. by a weight of 12·7 kgm. Find the value of Young's modulus of elasticity for steel. ($g = 981$ cm. per sec.^{2}). — Art. 89

A. $2 \cdot 03 \times 10^{12}$ dynes per sq. cm.

9. Calculate the shearing stress and strain produced in a pin of diameter 1 inch subjected to a shearing force of 2 tons weight; $n = 2100$ tons weight per sq. in. — Art. 89

A. 2·555 tons weight; $1 \cdot 2 \times 10^{-3}$ radian.

Reference

Arts. 91 & 95

10. Explain the terms friction, limiting friction, co-efficient of friction and angle of friction. (P. U.—1951)

11. Show that once a body is just ready to slide down on inclined plane, the tangent of the angle of inclination of the plane is equal to the co-efficient of friction. (Del. H. S. 1952; *cf.* Nag. U.—1952; Dac. U.—1942).

Art. 91

12. State what do you mean by "Limiting Friction" and the angle of friction. (Dac. U.—1928)

Art. 94

13. A block of stone of mass 4 kgm. is just moving up a slope 1 in 20 by a force of 2×10^6 dynes applied parallel to the inclined plane. What is the value of the co-efficient of friction between the surfaces concerned?

A. 0·46.

Art. 94

14. A body of mass 2 kgm. slides down a rough inclined plane, the gradient being 1 in 8 and of co-efficient of friction equal to 0·05. Find the resultant force down the plane. What will be the velocity of the body at the bottom of the plane, if its length be 1200 cm.?

A. 147000 dynes; 420 cm. per second.

Art. 94

15. A body of mass 5 kilograms is being pulled up along a rough inclined plane with an acceleration of 5 cm/sec.^{2} by means of a weightless string running parallel to the plane. If the inclination of the plane be 45° to the horizontal and the co-efficient of friction between the body and the plane be 0·3, find the pull in the string. (Pat. U.—1952)

A. $45{\cdot}285 \times 10^5$ dynes.

Art. 91

16. A body of mass 4 lbs rests in limiting equilibrium on a rough plane whose slope is 30°. The plane being raised to a slope of 60°, find the forces required along the plane when the body (i) is on the point of sliding down and (ii) is on the point of moving up the plane. (Nag. U.—1952)

A. (*i*) 2·31 lbs. wt. (*ii*) 4·62 lbs. wt.

Art. 93

17. Explain the laws of limiting friction and describe experiments to verify them. (Pat. U.—1952)

Art. 93

18. Explain fully the terms 'Friction' and Limiting Friction.' Justify the statement that friction is a necessity and an evil. Give some familiar methods for reducing and increasing friction. (P.U.—1952)

Art. 93

19. A ladder 30 ft. long rests with one end against a smooth vertical wall and with the other on the ground, which is rough, the co-efficient of friction being 0·5. Find how high a man whose weight is four times that of the ladder can ascend before it begins to slip, the foot of the ladder being 6 ft. from the wall.

A. 88 ft. hence the man may ascend the full length without a chance of slip of the ladder.

HYDROSTATICS

CHAPTER VIII

PRESSURE IN LIQUIDS

96. Hydrostatics.—It is that part of Physics which deals with fluids at rest under the action of external forces and pressures they exert either within their own masses or on the sides of the containing vessel.

Fluid.—A fluid is a substance having no shape of its own and which takes up the shape of the containing vessel. Liquids and gases fall under the category of fluids. A given mass of liquid has got a particular volume, but a gas does not possess a definite volume. Maintenance of a definite shape, as is the characteristic with solids, is due to rigidity or elasticity due to shear Every solid substance possesses elasticity of shear But from the fact that a liquid or a gas cannot maintain a definite shape, we understand that it possesses no shearing elasticity ; although the *compressibility* of a liquid is extremely small, while that of a gas is considerable.

97. Fluid Pressure.—If a number of holes be made at the sides and the bottom of a vessel containing a liquid, the liquid is found to come out through all the openings The outflow of the liquid can however be stopped by covering the holes with plates, when it is felt that some force is to be applied from outside on each plate to keep it in position. This shows that the liquid inside exerts some force on any part of the containing vessel, with which it is in contact. It is also found that this force always acts at right angles to the surface of the vessel at any point. The force thus exerted by a liquid on any surface is called the *thrust* of the liquid (Fig. 129) It is due to this thrust that the liquid forces its way through an opening of the containing vessel. If the walls of the containing vessel are strong enough to withstand this thrust from point to point, water remains steady within it. Hence the wall exerts a reaction to fluid pressure.

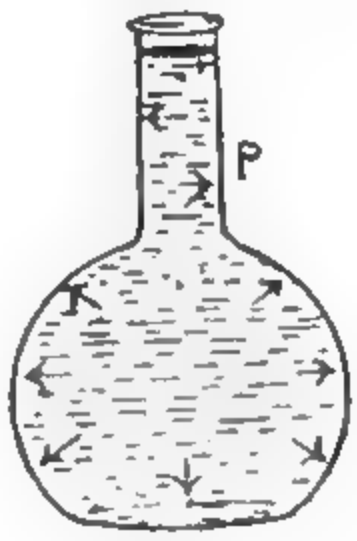


Fig. 129

The **pressure** of a fluid at rest on any surface is the normal force or thrust exerted by the fluid on unit area of that surface. The pressure is said to be *uniform* when the thrust on any two equal areas of the surface is equal. If P be the total thrust exerted by the fluid uniformly and normally on a surface of area A, then the pressure p on it is given by

$$p = \frac{P}{A} \qquad \ldots (97,1)$$

The pressure exerted by a liquid on the wall of a vessel may be uniform or may vary from point to point. If the pressure is not uni-

from, the above expression gives the *average pressure* over the given surface.

Pressure at a Point.—The *pressure at a point* due to a fluid is defined to be the normal force exerted by the fluid on unit area containing that point. In the C.G.S. system, the pressure is expressed in dynes or in grammes weight per square centimetre, while in the F.P.S. system, it is expressed in poundals per square foot or pounds weight per square inch. Suppose, for instance, that a hole of 6 sq. cm. in area is made at the side of a vessel containing a liquid. A plate just fitting the hole is pressed against to stop the outflow of water. If a force of 48 gm. wt. is *just* required to check the out-flow, then the average pressure is 8 gm. wt. per sq. cm. or 8 g dynes per sq. cm.

98. Pressure in a Liquid.—That a liquid exerts a pressure on the sides of the containing vessel may be shown by making a hole anywhere on its wall when the liquid will be forced out. There is also some pressure within the mass of a liquid. This can be shown by the following experiment.

Take a thistle funnel F and stretch a thin sheet of rubber over its mouth. Connect the funnel to a glass tube G (Fig. 130) of a narrow bore, by means of a length of rubber tubing T. A drop I of a coloured liquid, introduced into the glass tube, acts as an index. The glass tube is supported horizontally and

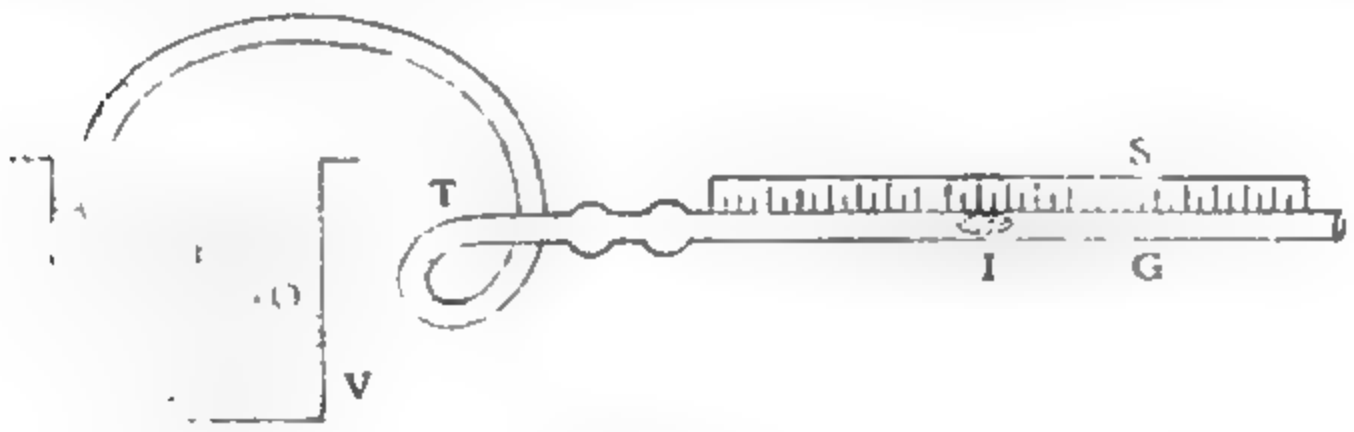


Fig. 130

scale S is placed along its side. If the rubber membrane is now pressed with finger, the index moves forward and the larger is the pressure, the more does the index move away ; when the pressure is withdrawn, the drop comes back to the initial position.

Now take a deep vessel V containing water and introduce the funnel into the liquid ; the index is seen to move outward. Move the funnel to greater and greater depth and note that the index is displaced more and more from its initial position. Hence it proves that pressure continuously increases with depth of the liquid. Now keep the funnel at a certain depth say at O and mark the possition of the index on the scale. Turn the mouth of the funnel to different directions about this point. Note that the index does not move with the rotation of the funnel. Therefore the amount of pressure around a certain point within the liquid is equal. Now displace the funnel to some other point in the same horizontal plane and turn it to different directions as before ; note that the position of the index remains unchanged. Hence liquid pressure at the same horizontal level is the same.

Thus the experiment proves the following facts.

(*i*) *A liquid exerts pressure at every point within its mass and in all directions.*

(*ii*) *The pressure at any point in the liquid or at different points on the same horizontal plane is the same in all directions.*

(*iii*) *The pressure at a point within a liquid is directly proportional to its depth.*

99. Vertical Upward Pressure.—If an empty test tube is pushed down into water with the closed end downwards, the tube on being released springs up due to the upward force exerted by the liquid at its bottom. The following experiment further demonstrates the presence of vertical upward pressure.

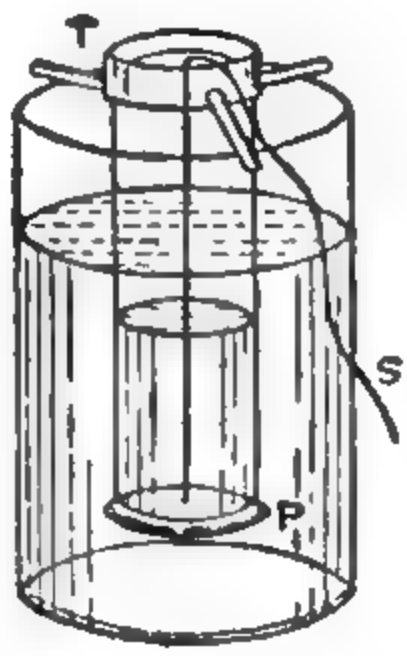


Fig. 131

Take a wide glass tube T with one end ground flat. Hold a thin plate P of glass or metal against the ground end with a string attached to it (Fig. 131). Immerse the whole thing vertically to some depth in a vessel of water and release the string. The plate is found to remain in position as if pushed from below. This is due to the *upward thrust* on the lower surface of the plate. Now slowly pour down water, say coloured, into the tube. If the plate is very thin it will drop down when the level of the water inside comes almost to the same height as that of the water outside. This shows that the upward pressure on the plate is equal to the downward pressure on it, which is equal to the weight of the column of water standing on the plate.

100. Lateral Pressure.—A cylindrical vessel of thin tin sheet fitted with a stop-cock on one side near the bottom is filled with water and is arranged to float on water being placed on a suitable piece of cork (Fig. 132). When the stop-cock is opened, water flows out and the cylinder is seen to float away slowly in a direction opposite to that of the issuing water.

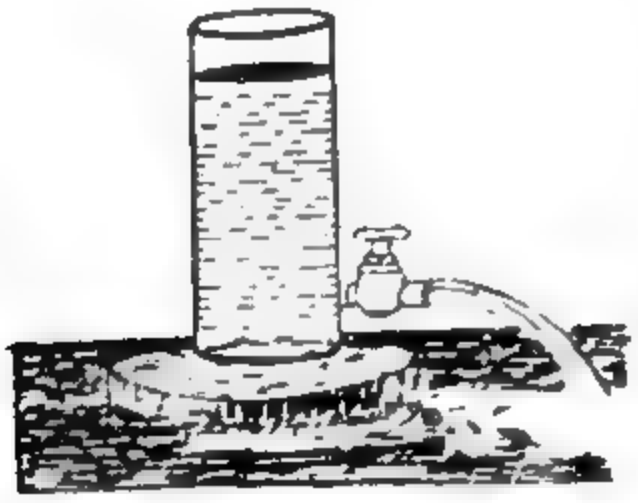


Fig. 132

The explanation lies in the fact that when the stop-cock remains closed, the lateral pressure exerted by water on the stop-cock is equal and opposite to the reactive pressure exerted by the stop-cock on water and consequently the vessel shows no tendency of motion. When the tap is opened, the pressure exerted by the stop-cock is released due to which water flows in a stream and the back push of the stream makes the vessel move in that direction.

The existence of lateral pressure can be demonstrated by the *hydraulic Tourniquet* or Barker's Mill apparatus. It consists of a vessel of water free to rotate about a vertical axis, having near the bottom horizontal discharging tubes, all bent at the ends in the same way.

As water rushes out of the apertures of the bent tube, the reaction of the stream at the bends of the tubes opposite the openings cause the whole apparatus to rotate in a direction opposite to that of the issuing water.

101. Magnitude of the Pressure within a Liquid.—Let a surface of area A (Fig. 133) be immersed in a liquid at a depth h below the free surface of the liquid and kept there in a horizontal position. Imagine vertical lines to be drawn from every point of the boundary line of this surface till they reach the surface of the liquid. Such vertical lines enclose a column which is filled with the liquid. This column of liquid is clearly standing on the surface of area A which serves as its base. The weight of this liquid gives the thrust on the base area. If the density of the liquid be ρ and the acceleration due to gravity at the place be g, then the volume V of the water column on the area is $A \times h$ and its weight is equal to $Ah\rho g$. Therefore the total downward thrust T upon the surface is given by,

$$T = Ah\rho g \qquad \ldots (101,1)$$

The total pressure on a surface immersed *horizontally* in a liquid is equal to the weight of the column of the liquid, whose base is the given surface and whose height is equal to the depth of the surface below the free surface of the liquid.

If P denotes the downward pressure at any point on the surface then remembering that pressure is the force or thrust per unit area,

$$P = \frac{Ah\rho g}{A} = h\rho g \qquad \ldots (101,2)$$

This is also the magnitude of the pressure in the upward as well as in any other direction at the point. The above relation shows that

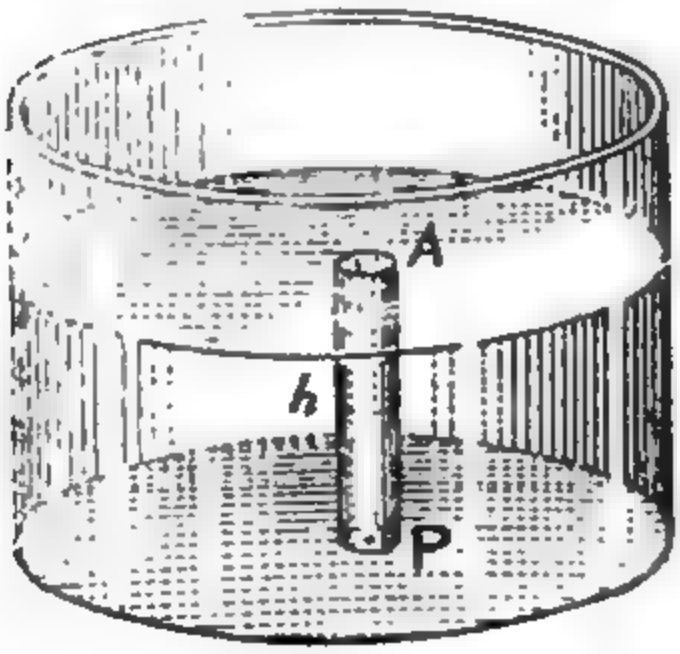


Fig. 183

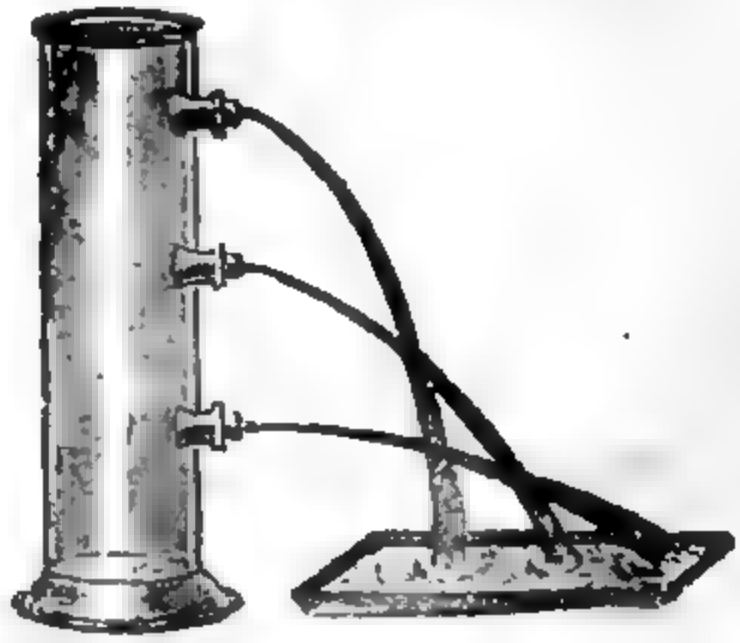


Fig. 184

the pressure at any point within the liquid is proportional to (*i*) the depth of the point (*ii*) the density of the liquid and (*iii*) the acceleration due to gravity at the place of observation. The air above the free surface of the liquid also exerts its weight on the liquid surface. The weight of air acting per unit area is called the atmospheric pressure, and if this pressure per unit area be π, the total

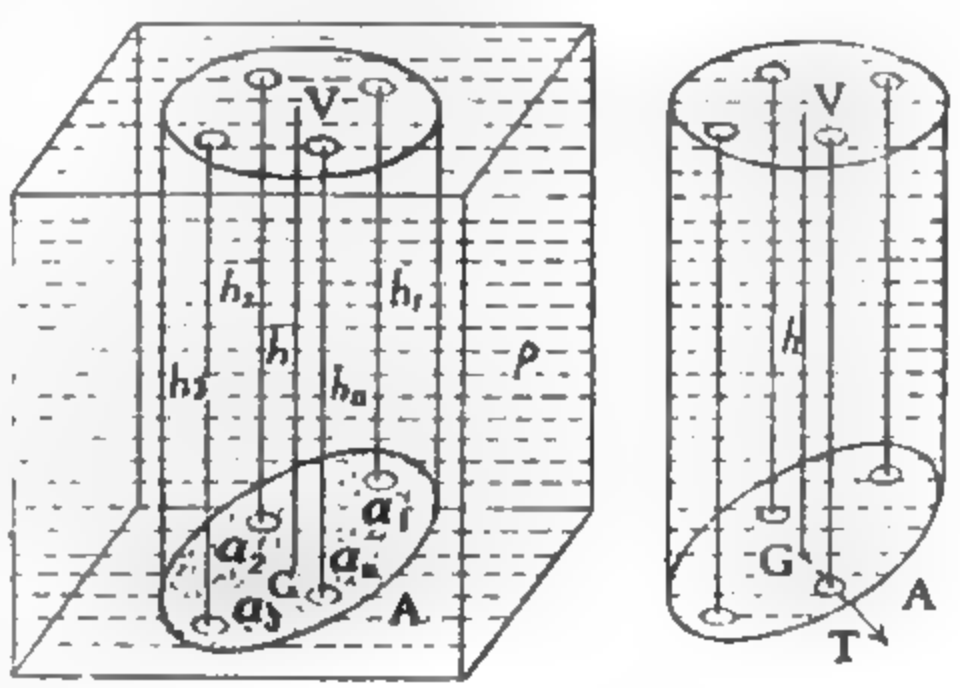
pressure at a point at a depth h below the free surface of the liquid is equal to $\pi + h\rho g$.

A tall jar with holes at different heights on the side is taken (Fig. 134). When the jar is filled with water, the water is seen to come out in streams from different holes but the throw of a stream varies as the depth of a hole from the free surface of water in the jar. This indicates that pressure increases with depth within the liquid. Since pressure is force per unit area, the dimension of pressure is force ÷ area or $MLT^{-2} \div L^2 = ML^{-1}T^{-2}$.

***102. Whole Pressure or Resultant Thrust.**—It has already been stated (Art. 97) that the *effective* pressure exerted by a fluid on the walls of the containing vessel is always at right angles to the surface of contact. This is also true if there be any partition wall within the liquid. In the experiment to demonstrate the existence of pressure in all directions around a point within a liquid (Art. 98), the thistle funnel covered with a piece of rubber membrane on one side, was taken down within the liquid to a certain depth and was slowly rotated in different directions. The position of the index was found to be fixed showing that the force exerted by water on the rubber membrane was the same at a given depth being always *at right angles to the surface* of the membrane. This force is called the *whole pressure* on the membrane, which may be defined thus.—

If, for every small element of area of a material surface immersed in a fluid, the force perpendicular to the surface of this small element be found, the sum of all such forces taken over the whole area of the surface is called the whole pressure or resultant thrust of the fluid upon the given surface.

Consider any plane surface of area A immersed in a liquid in any position, horizontal or inclined (Fig. 135*a*). Suppose that the surface is composed of small elementary such as $a_1, a_2, a_3 \ldots a$ Then vertical columns of liquid such as shown in the fig. 135*b* are standing upon the elementary areas, each contributing to the whole pressure on the surface. Let the depths of these surfaces below the free surface of the liquid be $h_1, h_2, h_3 \ldots\ldots h_n$ respectively. Then the thrust of the liquid column standing upon a_1 at a depth h_1 perpendicular to the element of the surface is $a_1 h_1 \rho g$, where ρ is the density of the liquid. In a similar way, it may be shown that the thrusts on other elements are $a_2 h_2 \rho g$, $a_3 h_3 \rho g$ etc. Hence the resultant thrust is the sum of the individual thrusts. Thus,



(*a*) Fig. 135 (*b*)

$$\text{Whole Pressure} = a_1 h_1 \rho g + a_2 h_2 \rho g + \ldots\ldots + a_n h_n \rho g$$
$$= \rho g [a_1 h_1 + a_2 h_2 + \ldots\ldots a_n h_n]$$

If h denotes the depth of the centre of gravity G of the surface A below the free surface of the liquid, then it can be shown from the principle of Statics that,

$$Ah = a_1 h_1 + a_2 h_2 + \ldots\ldots + a_n h_n$$

Hence, Whole Pressure $= T = Ah\rho g$

In the preceding article, it has been shown that the thrust on a horizontal surface of the same area at a depth h below the surface of a liquid has the same value. Hence the magnitude of the whole pressure is independent of the inclination of the surface provided that the depth of the centre of gravity of the surface does not change.

Centre of Pressure.—If a surface be immersed in a liquid, the point at which the resultant thrust on the surface acts, is termed the *centre of pressure* of the surface. When the surface is kept in a horizontal position within a liquid, then the point of the surface, where a vertical line dropped from the centre of gravity of the liquid column standing upon the surface meets, is the centre of pressure.

103. **Hydrostatic Paradox.**—Since the total pressure on an area A immersed horizontally at a depth h in a liquid of density ρ is given by $Ah\rho g$, the thrust at the bottom of a vessel due to the liquid contained depends only on the area of the base and the height of the liquid column but not on the total quantity of the liquid contained or the shape of the vessel. This can be demonstrated by **Masson's experiment,** as given below.

A, B, C and D are glass vessels, known as *Pascal's vases*, all being open at both ends. They are of different shape and size but the base of each is of equal sectional area so that each can be screwed to the same socket on the platform F (Fig. 136). A disc R, attached to one arm of the lever L, can press water tight against the bottom of each vessel which can be screwed to the platform and at the other end of the lever is attached a scale pan S on which weights can be placed. A pointer P slides along a vertical stand attached to the platform and can be fixed at any height by a screw.

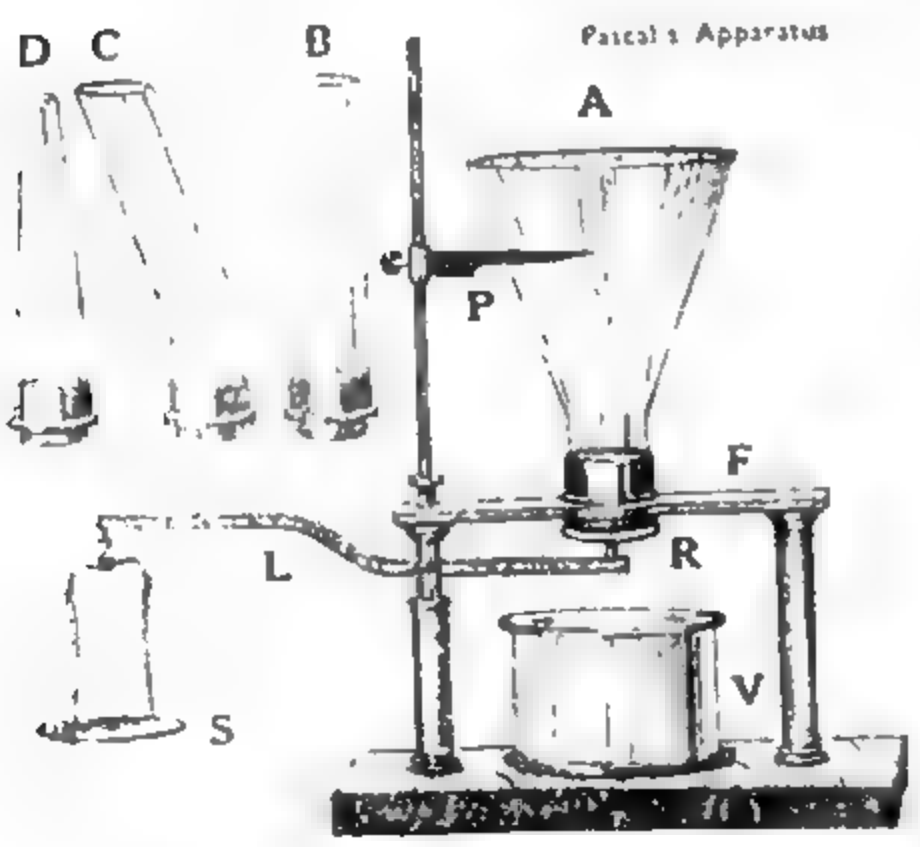


Fig. 136—Thrust on the base

Screw one of the vessels, say A, to the platform and put suitable weights on the scale pan. Now

slowly pour water into the vessel until the disc just gets loose and allows some water to escape. Mark the height of the water by the pointer P. Remove the vessel A and repeat the experiment with another vessel. It will be found that the water in it just begins to trickle down when the height of water in this vessel is the same as before.

Thus it shows that *the thrust on the base depends on its area and on the height of the liquid column* and not upon the shape of the vessel. This seems to be inconsistent at first sight for which reason the experiment is known as the **hydrostatic paradox.**

Pascal demonstrated hydrostatic paradox by a very simple experiment. A long narrow tube about 30 ft. in length was fixed vertically to the top of a stout cask, completely filled with water. Water was gradually poured into the tube till the cask burst. Although the quantity of water added was very small, the thrust at the bottom of the cask became equal to the weight of a column of water whose base was that of the cask and whose height was equal to that of the water column from the bottom of the cask to the level of water in the tube.

***104. Thrust at the Base.**—The resultant thrust on any surface immersed within a liquid depends upon the surface area, the depth of the centre of gravity of the surface below the free surface of the liquid, the density of the liquid and the acceleration due to gravity at the place of observation. Hence, if the position, shape and area of the base of vessel containing a liquid are known, then the whole pressure at its base may be found from an observation of the height of the liquid level within the vessel. For a *horizontal base*, the whole pressure upon it is

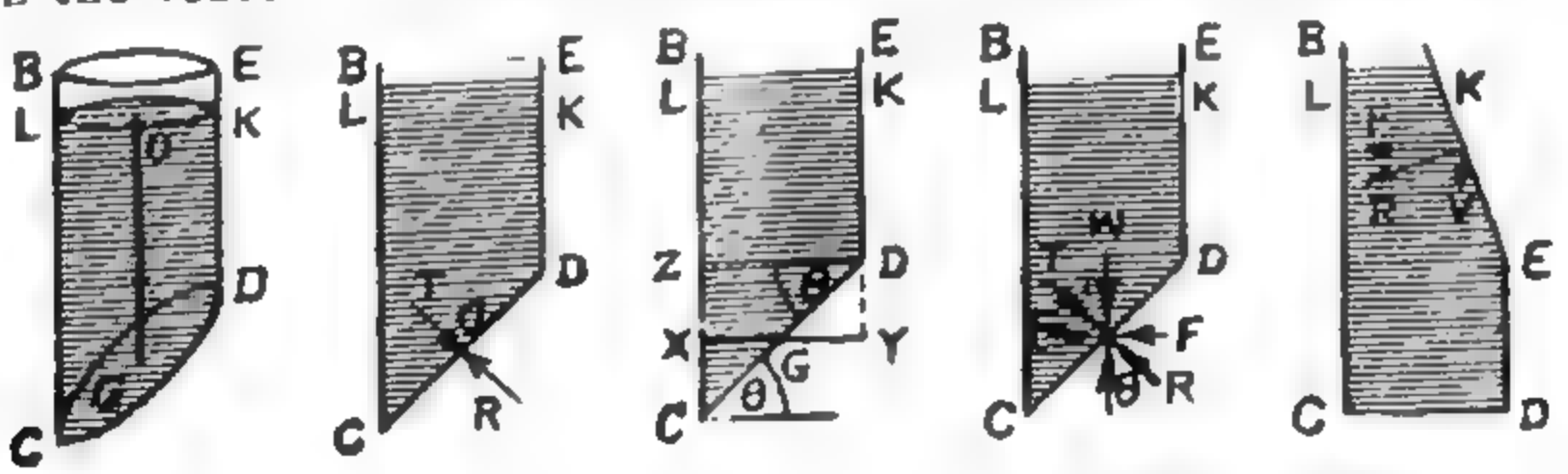


Fig. 137 Fig. 138 Fig. 139 Fig. 140 Fig. 141

$AH\rho g$, where H denotes the vertical height of the liquid surface standing upon it. Consequently if vessels of different shapes but of equal base area be placed so that their bases are horizontal and if all be filled up with a liquid to an *equal level*, the resultant thrust at the bases of all vessels would be equal. This has been practically verified in the experiment with Pascal's vases in the preceding article.

Now consider a cylindrical vessel BCDE. kept in a vertical position and having an inclined base (Fig. 137). Let the vessel be filled with a liquid of density ρ upto a level LK. Let G be the position of the centre of gravity of the base and height of the free surface of the liquid above G be H If the area of the base be A, then the whole pressure or

thrust T of the liquid column is $AH\rho g$ acting perpendicularly to the surface area G, as shown in the sectional diagram (Fig. 138). If the liquid within the vessel be steady, then the base offers a reaction R which is equal and opposite to T.

Imagine that a horizontal surface XY (Fig. 139) is made to pass through the point G of the base of the vessel extending upto the walls. Then the volume of the liquid within the vessel is equal to the volume KLXY, since the volume XGC = volume YGD. Surface XY = a parallel surface $ZD = A \cos\theta$. Therefore volume of liquid $= HA \cos\theta$ and hence its weight W acting vertically downwards is $AH\rho g \cos\theta$.

Again the base reaction R may be resolved into a vertical component $R \cos\theta$ and a horizontal component $R \sin\theta$ (Fig. 140). But since R is equal to $AH\rho g$.

$$R \cos\theta = AH\rho g \cos\theta \text{ and } R \sin\theta = HA\rho g \sin\theta$$

$$\therefore \quad R \cos\theta = W, \text{ and } R \sin\theta = F, \text{ say.}$$

Thus the vertical component of the base reaction supports the weight of the liquid while the horizontal component prevents the base from moving aside. Similarly the weight W of the liquid together with the lateral thrust F of the liquid on the base of the vessel produce the resultant thrust T.

If the shape of the vessel BCDEK be as represented in the fig. 141, the base CD being horizontal and if the liquid is filled to a height H, then the reaction R of the inclined part EK has got a vertical component, say V. The resultant thrust at the base of the vessel is the sum of the weight of the liquid and the vertical component V. In this case also the thrust at the base exceeds the weight of the liquid.

It is now easy to understand why in Pascal's vases, although masses of the liquid in different vessels are different, an equal upward

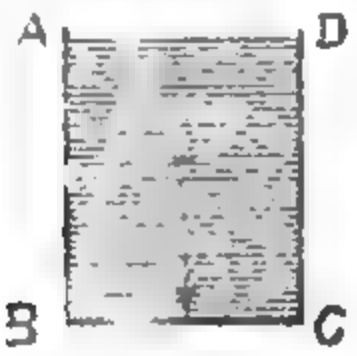


Fig. 142

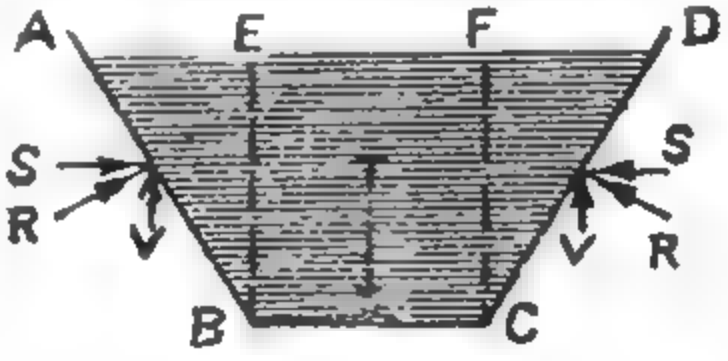


Fig. 143

Fig. 144

thrust maintains the same liquid height in different vessels. Consider the uniform cylinder ABCD with vertical walls (Fig 142). The reaction of the walls is everywhere horizontal having no vertical component. Hence the thrust at the base is only due to the weight of the liquid being proportional to the height of the column AB. When the vessel is wider at the open end (Fig. 143), consider a vertical column of liquid EBCF standing upon the base BC. The thrust at the base is due to the weight of this column only. The two other columns ABE and DCF are being supported by the vertical component of the wall reactions and have nothing to do with the thrust on the base. For

the vessel with a tapering end (Fig. 144), consider the column AEFD. Clearly the thrust on the part EF of the base is due to the column of water standing upon it. The thrusts on the parts EB and FC are partly due to the weight of water standing upon them and partly due to the vertical components of the wall reaction. The parts AB or CD in this figure are inverse of those of the preceding figure and consequently the vertical component of the wall reaction AB or CD is equal to weight of the column ABE or DCF. Hence the thrust at BC is due to a uniform column of liquid of base BC. If now base areas be equal, the thrusts at the base of all vessels due to liquids standing at equal height are equal although the actual quantity of liquid in each differs.

Examples :—

1. The pressure at the bottom of a well is 2 times that at a depth of 2 ft.; what is the depth of water in the well, if the pressure of the atmosphere is equivalent to 30 ft. of water? (Utkal U.—1951)

A. Let h be the depth of water in ft. The pressure at the bottom of the well $= (h + 30)\ \rho g$ where $\rho =$ density of water in F. P. S. units. The pressure at a depth of 2 ft. $= (30 + 2)\ \rho g$. According to the problem,

$$2\ (30 + 2)\ \rho g = (h + 30)\ \rho g \quad \text{whence } h = 34 \text{ ft.}$$

2. A cube, each of whose sides is 20 cm. is suspended in a liquid of density 1·2 with its sides vertical and its upper surface at a depth of 30 cm. below the free surface of a liquid. Find the thrust in grams weight on each of its faces. (Neglect atmospheric pressure).

A. The upper surface of the cube is at a depth of 30 cm. and so its lower surface is at a depth of 50 cm. and the centre of gravity of each of the vertical faces is 40 cm. below the surface of the liquid. Therefore,

Thrust on the upper surface $= 20 \times 20 \times 30 \times 1{\cdot}2 = 14400$ gm. wt.

Thrust on the lower surface $= 20 \times 20 \times 50 \times 1{\cdot}2 = 24000$ gm. wt.

Thrust on each vertical face $= 20 \times 20 \times 40 \times 1{\cdot}2 = 19200$ gm. wt.

3. A rectangle of length l and breadth b is immersed vertically in water, with side b on the surface of water. Calculate the thrust and the centre of pressure on the rectangle. (Mad. U.—1950)

A. The area of the rectangle $= l \times b$; if the density of water be ρ according to the system of units in which l and b are measured, and if g be the acceleration due to gravity. Then,

Resultant Thrust due to liquid $= l \times b \times h \times g$, where h is the depth of C. G. below the free surface of liquid.

Now, as the rectangle is suspended vertically with the side b on the surface $h = l/2$. Thus whole pressure due to liquid $= l^2bg/2$. The centre of pressure on the lamina is at the point where the two diagonals of the rectangle meet.

If the atmospheric pressure is taken into account, and if P be the air pressure per unit area, the total air pressure $= Plb$ whirh is to be added to $l^2bg/2$.

105. Transmission of Fluid Pressure :— A change of pressure at any point of a liquid or a gas causes an equal change in pressure at all other points. The transmission of fluid pressure, as given by Pascal's Law, may be stated as follows :—

Pressure exerted anywhere in a mass of a confined fluid is transmitted by the fluid in all directions, so as to act with undiminished force per unit area and at right angles to the surface exposed to the fluid.

To establish Pascal's law in case of a liquid we take a vessel with a number of openings of different cross-sectional areas a,b,c,d, etc. (Fig. 145), all of which are fitted with water tight movable pistons. Let the vessel be completely filled with water and the pistons be kept in position. If any one of the pistons, say at a, be now pressed inwards, all other pistons will be found to move outwards, which shows that the *pressure is transmitted in all directions.*

Again if a force F_1 be applied on a, then the pressure p per unit area, is F_1/a. It will be found that in order to keep the pistons at b,c,d, in positions additional forces F_2, F_3, F_4, etc. are to be respectively applied to them, so that F_2/b, $F_3/c, F_4/d$ are all equal ; in other words the pressure on unit area of each is the same. This shows that the pressure is transmitted *undiminished* in all directions.

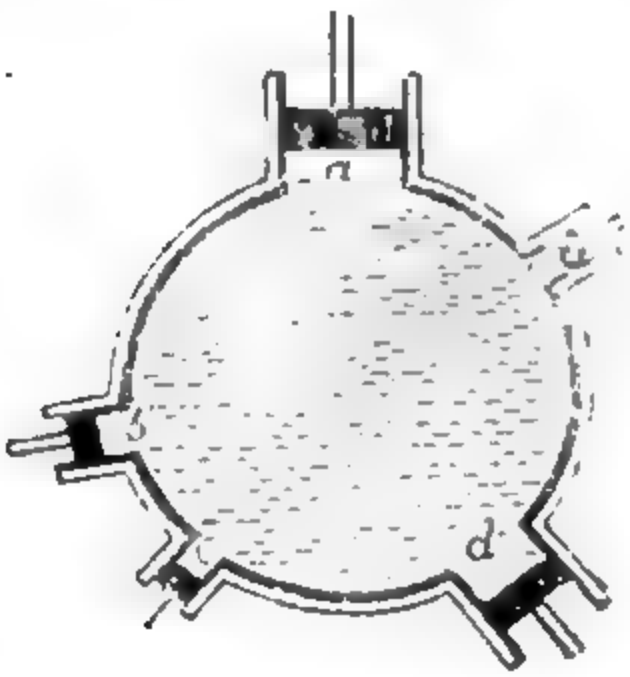


Fig. 145

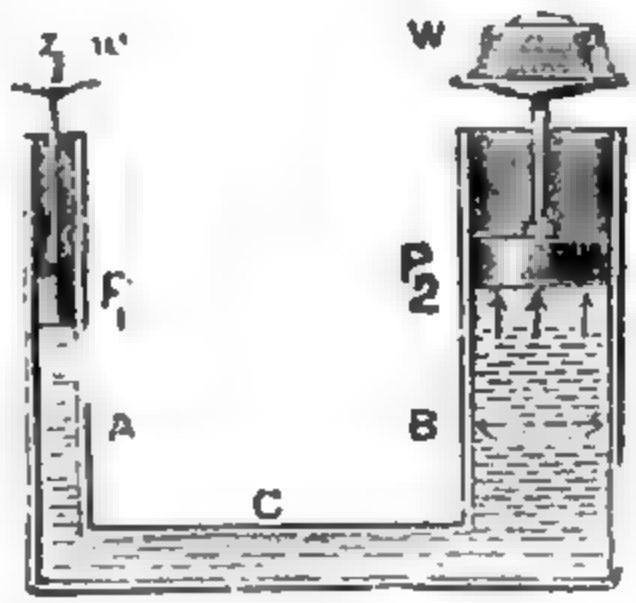


Fig. 146

Another simpler apparatus may by used to verify Pascal's law. A and B are two cylinders, of sectional areas a and b respectively, connected with a pipe C, the cross-section of B being larger than that of A (Fig. 146). They are filled with water to some height and fitted with water tight pistons P_1 and P_2 of negligible masses.

Let a weight w be placed on the smaller piston P_1, then the increase of pressure on this piston is given by $p = w/a$. This pressure is transmitted to the liquid within the cylinder B and the piston P_2 is pushed with an upward force. It is found that to maintain equilibrium, *i.e.*, to keep P_2 in position, a weight W is to be placed on this piston,

$$\text{where } W = \frac{w}{a}b. \quad \text{Hence} \quad \frac{W}{w} = \frac{b}{a} \text{ or } \frac{W}{b} = \frac{w}{a} \qquad \ldots \quad (105,1)$$

which shows that the force per unit area remains the same.

For example, if the cross-section of A and B be 1 sq. cm. and 50 sq. cm. respectively and if a mass of 1 kgm. be placed on the piston P_1 then a mass of 50 kgm. is to be placed on P_2 for equilibrium.

This illustrates the principle of *multiplication of force* by transmission of fluid pressure.

This experiment also illustrates that on increasing pressure at one end of a column of liquid, it can be made to move towards the other end where the pressure is less. Thus a fluid moves from a region of higher pressure to a region of lower pressure.

Examples :—

1. A bottle full of water with a bottom of 30 sq. cm. area, has a cork fitted to it. The cork has 1 sq. cm. as its area of cross-section and is pressed in with a force of 40 gm. wt. What is the total thrust on the bottom ? (Del. H. S.—1952)

A. The total thrust at the bottom of the bottle is the sum of the thrust due to water within it and the thrust due to the cork. The thrust due to atmospheric pressure is cancelled because it acts at the bottom of the vessel vertically downwards as also upwards.

The pressure due to the cork is transmitted undiminished to the bottom. The pressure exerted by the cork = 40 gm. wt. per cm.^{2} Hence thrust due to this at the bottom = pressure × area = $40 \times 30 = 1200$ gm. wt.

If h be the vertical height of the bottle, pressure at the bottom due to water = h gm. wt. per cm.^{2}. Hence thrust due to water = $30\ h$ gm. wt.

$\therefore$ Total thrust = $1200 + 30\ h$ gm. wt.

106. Hydrostatic Bellows. - The principle of transmission of fluid pressure is also illustrated by the *Hydrostatic bellows* designed by Pascal. It consists of a long glass tube (Fig. 147) attached to stout India rubber bladder or leather bellows B. The bladder is placed on a table while the tube is held vertically. The bladder and a part of the tube are filled with water. A piece of light board may be placed on the bladder. A heavy weight placed on the board can be supported simply by the weight of a column of water in the tube. The pressure exerted by the column of water in the tube is transmitted undiminished into the bladder and produces on the entire upper surface a vertical upward thrust proportional to its area, which supports the weight placed on the board. The board is taken for the uniform distribution of pressure exerted by the weight placed on the bladder.

Fig. 147

107. Hydraulic Press.—The principle of multiplication of force by transmission of fluid pressure is utilised in the working of a hydraulic press. This is used for exerting enormous force, as in compressing bales of paper, jute, cotton or cloth, in extracting oil from seeds, in testing the strength of iron beams, in punching holes through metal plates, etc.

It consists of two iron cylinders C and T communicating with each other through a stout metal pipe (Fig. 148), the cross-section of C being many times larger than that of T. There is in each cylinder a solid rod acting as a water-tight piston. The piston in T is worked by a lever L, having the fulcrum at the end. The piston in C is a large iron 'ram' R which carries a platform at its top. Above the platform there is a fixed plate supported on iron pillars S. The object to be pressed is placed on the platform on R. A reservoir filled with water is in communication with the cylinder T through a pipe fitted with a valve V which allows water to flow only in one direction from the reservoir to the cylinder. There is another valve V fitted to a side tube to pass water only in one direction from smaller to the bigger cylinder when the piston P is worked. The above improvement was devised by an engineer Bramah and the press is therefore sometimes called Bramah's Press.

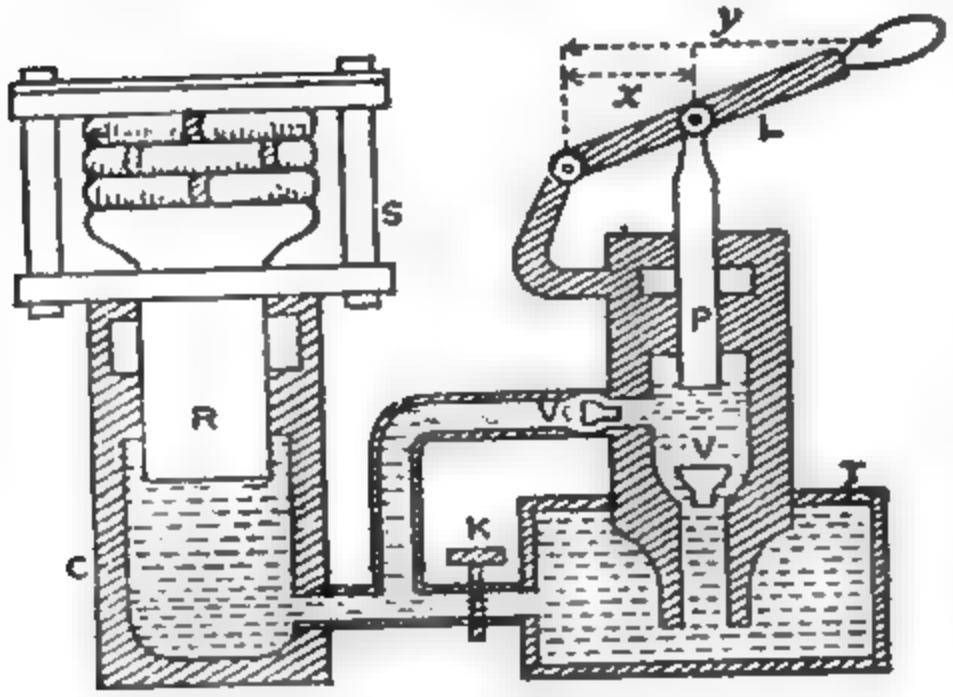


Fig. 148—Hydraulic Press

As the piston P is raised by the lever L, the valve within the side tube is closed and the inlet valve becomes open, whereby water from the reservoir enters into the cylinder T below the piston. If now the piston is lowered, the pressure inside increases, due to which the valve within the side tube is forced open and inlet valve is closed; consequently the water passes into the cylinder C. Thus, during each upward stroke of the piston a quantity of water is drawn into T, and during the next downward stroke this water is pushed into the cylinder C. The pressure applied on the piston is thus transmitted to the water in C and produces on the ram R an upward force which is many times greater than the force applied at the piston.

When the ram R is to be lowered, the tap K attached to the connecting pipe is opened and water, due to the weight of the ram is forced back into the reservoir. This tap must be closed before the press is operated. The ram may be kept under a pressure for a considerable period by closing the tap K.

Let the areas of cross-section of C and of T be α and β respectively and let a force F_1 be applied to the end of the lever L. If the thrust generated on the piston P be F_2 then from the theory of the lever.

$$\text{Mechanical advantage, } m = \frac{F_2}{F_1} = \frac{\text{Power arm}}{\text{Resistance arm}}$$

$$= \frac{\text{longer arm}}{\text{shorter arm}} = \frac{y}{x}$$

$$\therefore \qquad F_2 = mF_1 = \frac{y}{x}F_1 \qquad \ldots \quad (107.1)$$

Let the total force generated on the ram be F_s.

Then $$F_s = \beta \frac{F_2}{\alpha} = \frac{\beta}{\alpha} m F_1 \quad \ldots \quad (107,2)$$

$\therefore$ *Mechanical advantage* of the machine

$$= \frac{F_s}{F_1} = m\frac{\beta}{\alpha} = \frac{y\beta}{x\alpha} \quad \ldots \quad (107,3)$$

Here of course we neglect the effect of friction at different parts of of the machine. So by applying a small force at the lever, we get a much larger force exerted on the ram. Thus if the longer arm of the lever be 10 times that of the shorter one and the area of cross-section of the larger piston be 100 times that of the smaller one, the force on the ram is multiplied (10×100) or 1000 times the force applied at the lever, so that the mechanical advantage of the machine comes out to be 1000.

The principle of Conservation of Energy may be applied in this case in the following way. The decrease in the volume of water in T is evidently equal to the increase in the volume of water in C. If the ram is raised through a distance x_2, and the piston in T is pushed down through x_1 then $x_1\alpha = x_2\beta$.

$$\therefore \frac{x_1}{x_2} = \frac{\beta}{\alpha} = \frac{F_2}{F_1} \text{ whence } F_2x_1 = F_1x_2 \quad \ldots \quad (107,4)$$

Now work done on the smaller piston $= F_2x_1$

and work done on the larger piston $= F_1x_2$

From (107,4) work done by the larger piston

= work done on the smaller piston.

Examples :—

1. The diameters of two pistons of a hydraulic press are 1 in. and 6 in. respectively. What multiplication of force will be produced? (Del. H. S.—1940)

A. The pressures acting on the two pistons must be equal by Pascal's Law.

Now force = pressure $\times$ area of cross-section. Let F_1 and F_2 be the forces acting on smaller and longer pistons of radii 0·5 in and 3 in. respectively.

Then pressure $= \frac{F_1}{\pi \times \cdot 5^2} = \frac{F_2}{\pi \times 3^2}$

whence $\frac{F_2}{F_1} = \frac{\pi \times 3^2}{\pi \times \cdot 5^2} = 36$

2. The diameters of the pistons of a hydraulic press are 4 in. and 40 in. respectively. The shorter arms of the lever working the smaller piston is 8 in. and the longer arm is 4 ft. in length. Find the total force produced on the larger piston when a force of 75 lbs. is applied at the end of the longer arms of the lever.

A. Let the effective pressure at the smaller piston $= p$ lbs.

Then from principle of lever, $p = 75 \times 4 \times 12 \div 8 \quad \therefore \quad p = 450$ lbs.

Cross-sectional area of the smaller and the larger pistons $= 4\pi$ sq. in, and 400π sq. in. respectively.

Let the total force produced on the larger piston $= P$ lbs.

$$\therefore \quad \frac{P}{400\pi} = \frac{p}{4\pi} = \frac{450}{4\pi} \text{ when } P = 45000 \text{ lbs. wt.}$$

108. Conditions of Equilibrium of a Liquid.—When a quantity of a liquid is *at rest*, the following conditions hold good.—

(*1*) *Considering any elementary volume within the liquid, it is acted on by a vertical upward thrust which is equal to the weight of that elementry volume.*

The explanation is obvious, since any elementary volume of the liquid has a weight and this weight is always acting downwards. Since this element is at rest, there must be an upward thrust which balances the weight. If the forces in two opposite directions were not equal, this element would tend to move in the direction of the greater force and the equilibrium would be lost.

(*2*) *The free surface of a liquid at rest is horizontal.*

If possible, let the liquid surface AC be not horizontal, as shown in fig. 149. Let B and D be two points in the same horizontal line within the mass of the liquid. Let the vertical depths of this two points below the free surface of the liquid be h' and h.

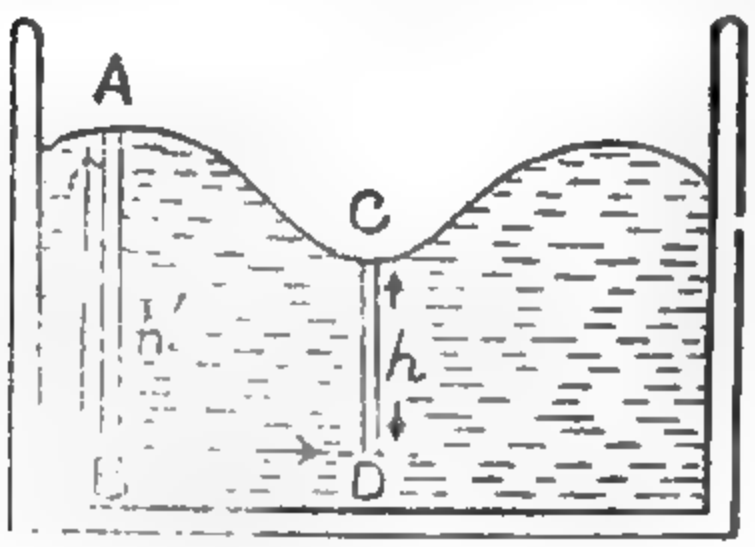


Fig. 149

The pressure at $D = P + h\rho g$, pressure at $B = P + h'\rho g$, where P is the atmospheric pressure. Hence the pressure at B would be greater than that at D and so an element of liquid would move away from B to D and equilibrium would be lost. This flow will persist in the direction BD so long as the pressures at B and D are not equalised. So for equilibrium, the pressures at B and D must be equal, *i.e.*, the depths of the two points from the free surface must be equal. As B and D are on a horizontal plane, points A and C must then be upon another horizontal plane, at an upper level. Since A and C are any two points on the surface, it proves that, *the liquid surface at rest is horizontal.*

109. Equilibrium of a Liquid in Communicating Vessels.—If any liquid is poured into a number of communicating vessels of whatever shape and size, as in fig. 150, then at rest, the liquid stands to the same height in all the vessels, so that the free surface of the liquid in all vessels are on the same horizontal plane.

If a horizontal plane be drawn within the liquid, cutting all the vessels, the pressure at all points on this plane must be the same. As the pressure at any point depends on the depth of the point from the upper surface of the liquid it shows that the pressure on the same horizontal plane must be the same and so *the free surface of the liquid in different vessels are on the same horizontal plane.* This fact is usually expressed by saying that a *liquid finds its own level.*

Equilibrium of different liquids in the same Vessel.—If several liquids of different densities, which do not mix with one another, be at rest in the same vessel, they would arrange themselves in the order of their densities, the heaviest liquid occupying the lowest position. Further, the free surface as also the surface of separation in each case are found to be horizontal. If small quantities of mercury, water (coloured, if necessary) and paraffin oil be poured into a narrow bottle or a test tube, the mercury occupies the lowest position and the oil the topmost. If the liquids be well shaken, they appear to mix, but on standing they separate and arrange themselves as before.

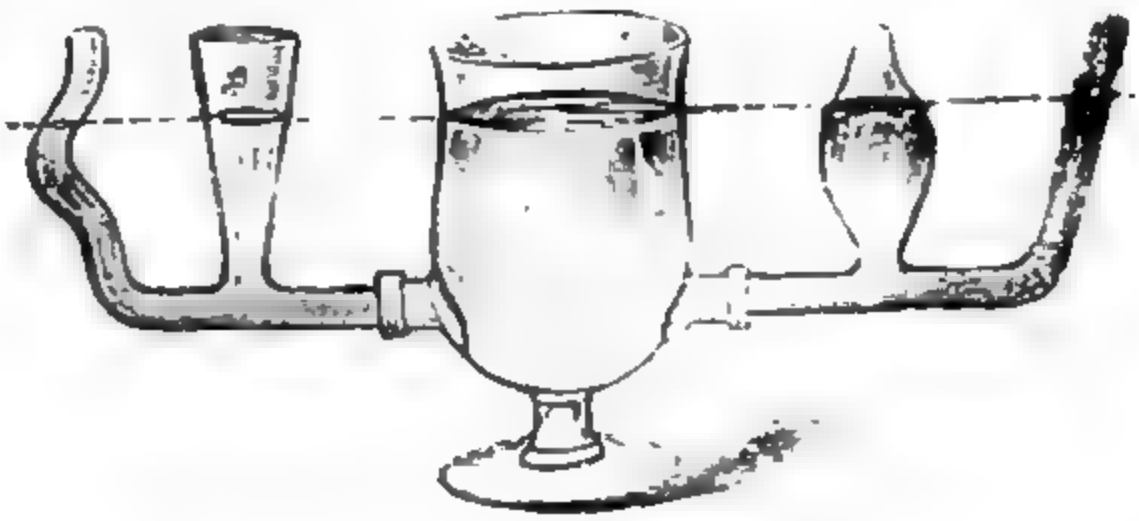


Fig. 150

Equilibrium of two different Liquids in a U-tube.—If two liquids of different densities, which do not mix with each other, be poured into the two limbs of a U-tube or any other kind of communicating vessel, then for equilibrium, the free surfaces of the two liquids do not generally stand at the same height.

ABC is a glass U-tube (Fig. 151); let mercury be poured into the limb BC, which is found to attain the same level in both the limbs. Now a quantity of a liquid which does not mix with mercury is poured into the limb AB. Let the common surface of contact with the two liquids stand at D, while the free surfaces of the liquid and of mercury stand at A and C respectively. Let the horizontal plane through D meet the other limb at E.

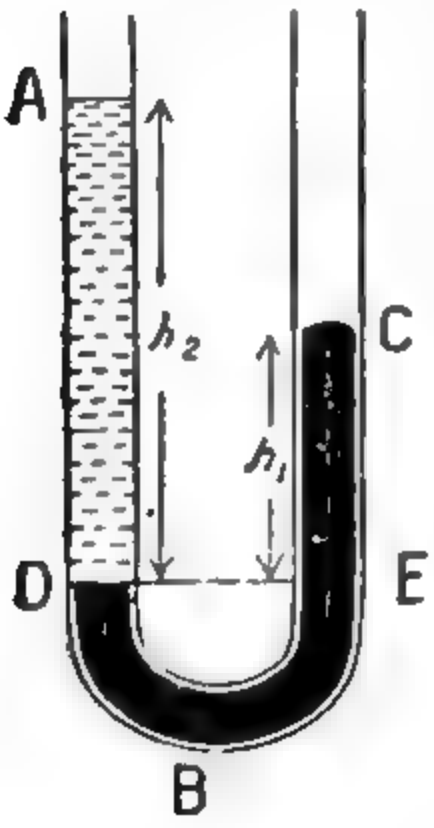


Fig. 151

Then the pressure at D = pressure at E.

Now pressure at $D = P + h_2\rho_2 g$ and pressure at $E = P + h_1\rho_1 g$, where P is the atmospheric pressure, $h_2 = AD$, $h_1 = CE$ and ρ_2 and ρ_1 are the densities of the liquid and of mercury respectively.

$$\therefore \quad P + h_1\rho_1 g = P + h_2\rho_2 g \text{ or } h_1\rho_1 = h_2\rho_2$$

$$\text{whence } \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \qquad \ldots \quad (109,1)$$

Hence for equilibrium, the heights of the two liquid columns above the common surface of contact are inversely proportional to their densities.

Taking the liquids to be water (density 1) and mercury (density 13·6) the height of the water column above the common surface is 13·6 times that of the mercury column, as measured from the same level.

110. Some Applications of Equilibrium of Liquid.—The following cases work on the principle of equilibrium of liquids.

Spirit Level.—This instrument is used to test the level, *i.e.*, to examine whether the plane surface on which it is placed, is horizontal or not. It consists of a glass tube slightly curved and closed at both ends. It is nearly filled with spirit so as to leave room for a small bubble of air, which naturally occupies the highest part of the tube (Fig. 152, *a*). The tube is mounted in a metal case, with convex side uppermost, the base of the case being perfectly plane. When the instrument is placed on a perfectly horizontal surface, the air bubble occupies exactly the middle position between two marks on the convex side of the tube equidistant from the middle. If the surface is not horizontal, the bubble moves to a different position. Fig. 152 (*b*) shows a spirit level of circular type.

(*a*) Fig. 152—Spirit levels (*b*)

Water-supply in Towns.—Water duly filtered is pumped up into a suitable reservoir and placed at an elevation higher than any place at which water is to be supplied. Main pipes run from the reservoir, while smaller service pipes branch off from the main to different houses. Water runs down from the reservoir and in finding its own level, is likely to be available in all places at lower levels than the reservoir. The 'head' of water which means the vertical height of the water surface in the reservoir thus furnishes the necessary pressure for the water supply. But owing to friction of water against the sides of the pipes and the viscous resistance between different layers, the water falls short of the expected height.

It is to be mentioned here that for supply of water in Calcutta, water is filtered at Palta, 29 miles north of Calcutta and is stored in tanks there at a height of about 100 ft. above the level of the city. But as the water in reaching Calcutta loses much of the pressure, a huge water reservoir has been constructed at Tala just to the north of Calcutta at the height about 100 ft. into which water is forced up by pumps and stored before distribution. But even with this arrangement the required pressure is not available at all places.

BLAISE PASCAL (1623—1662)

Pascal was born at Clermont-Ferraud in France in 1623. He was a genius in mathematics and he wrote a Treatise on Conic Section at the age of sixteen. Just at this period his health broke down seriously and he had to take rest for two or three years. He was appointed a teacher in mathematics at the Polytechnic School. While in this service he carried on researches in the properties of fluids. His most famous experiment was the demonstration of atmospheric pressure. He also devised various experiments for the measurement of pressure communicated by water and in 1646 enunciated the famous law of

pressure which goes by his name. At the age of twenty-four he gave up his scientific work and devoted the rest of his life to theology and religious activity at the Jansenist community at Port Royal. He died in Paris in 1662.

Summary

Pressure at a point in a liquid is the normal force exerted by the liquid on unit area containing that point. The pressure at a point or at different points on the same horizontal plane is the same in all directions.

The **magnitude** of the pressure at a point within a liquid is equal $h\rho g$

Pascal's Law.—*Pressure exerted anywhere upon a mass of a confined liquid is transmitted by the liquid in all directions, so as to act with undiminished force per unit area and at right angles to the surface exposed to the liquid.*

The principle of **multiplication of force** follows from the above fact. The principle is illustrated in the construction of a **hydraulic press** where a large mechanical advantage is secured.

The free surface of a liquid at rest must be horizontal. The principle is illustrated in (*i*) *Spirit-level and* (*ii*) *Water-supply in towns.*

Exercises on Chapter VIII

	Reference
1. Describe an experimental arrangement to show that water exerts pressure in all directions. (C. U.—1927)	
2. Obtain an expression for the total liquid thrust in absolute units on the plane horizontal bottom of a vessel, containing a liquid and describe briefly an experiment to show that this thrust does not depend on the shape of the vessel. (P. U.—1922; C. U.—1920)	Arts. 101 & 103
3. Distinguish between Thrust and Pressure. Give the units in which they are expressed. (Del. H. S.—1952)	Arts. 101 & 102
4. Define pressure at a point within a liquid at rest and obtain an expression for it. (Mad. U.—1950; Anna. U.—1951)	Art. 101
5. The density of sea water is 1·025. Find the pressure at the depth of 10 feet below the surface in pounds per square foot, given that one cubic foot of water weighs 62·5 lbs. (C. U.—1927)	Art. 101
A. 640·625 lbs. per sq. ft.	
6. If we have a vertical pipe half inch in diameter and 100 ft. high, filled with water, what will be the hydrostatic pressure at the lower end? What is the hydrostatic pressure at a point 100 ft. below the surface of water in a lake? In both cases, assume that density of water is 64 lbs. per cu. ft. and express the pressure in lb. per square inch. (P. U.—1949)	Art. 101
A. $44\frac{4}{9}$ lbs. per sq. in. in both cases.	
7. A hole, 6 inches square, is made in a lock-gate at a depth of 20 ft. below the surface of water. If the density of the water contained be 1·02, find the force that must be exerted on a plate to pre-	Art. 101

Reference

vent the water from going out by holding the plate against the hole (given 1 cu. ft. of pure water = 62·5 lbs.).

A. 322·575 lbs. wt.

Art. 101 8. A vertical tube 3 ft. long and having an internal diameter of 1 inch, is filled with equal volumes of water and mercury. Assuming that weight of mercury is 13·56 times the weight of water, calculate the pressure in lbs. per sq. in. at the bottom of the tube due to the head of the liquid. Also, what is the weight of water in the tube?

A. 9·45 lbs./sq. in.; 0·51 lb.

Art. 102 9. A lockgate is 35 ft. wide, and the heights of water above the bottom of the gate on the two sides are 26 and 13 ft. respectively. Find the resultant pressure and the height, measured from the bottom of the gate, at which it acts, the mass of water per cu. ft. being 64 lbs.

A. 253·5 tons; 10·11 ft.

Art. 102 10. A vertical wall, 2 ft. thick and 18 ft. high, weighing 124 lbs. per cu. ft. supports the pressure of water on one side. How high may the water rise without causing the resultant force on the base of the wall to pass more than 8 in. from the middle of the wall's width?

A. 6·59 ft.

Art. 103 11. Explain why the thrusts on the bases of Pascal's vases are not equal to the weight of the contained liquid. (Utkal. U.—1951)

Art. 104 12. A hollow right circular cone of base diameter 20 cm. and height 15 cm. is completely filled with a liquid of density 1·2 gm./c.c. Find the mass of the liquid and the thrust at the base of the cone.

A. Mass = 1884 gm.; thrust = 5652 gm. wt.

Art. 105 13. State Pascal's Law regarding the transmission of pressure in a liquid and describe suitable experiment to verify it in the laboratory. (C. U.—1931)

Art. 105 14 State Pascal's law as to the transmission of pressure in a liquid.

Art. 107 Describe the principle and action of a Hydraulic Press, giving a sectional diagram. (C. U.—1940, '44, '50; Del. U.—1951; Del. H. S.—1949, Pat. U.—1952; P. U.—1935, Dac. U.—1934)

Art. 107 15. Make a diagram showing the construction of a hydraulic press.

A force of 50 kgm., is applied to the smaller piston of such a machine. Neglecting friction, find the force exerted on the larger piston, the diameters of the pistons being 2 and 10 cm. respectively. (P. U.—1925)

A. 1250 kg. wt.

CHAPTER IX

ARCHIMEDES' PRINCIPLE

111. Resultant Thrust on a Body immersed in a Liquid.— When a body is immersed in a liquid, every point of its surface in contact with the liquid is subjected to a hydrostatic pressure. The pressure at any point is normal to the element of surface at that point and is proportional to the depth of the point below the free surface of the liquid. These pressures combine into a *resultant* thrust, which tends to move the body upwards.

Imagine a rectangular block ABDC of a solid to remain immersed vertically in a liquid (Fig. 153). The whole liquid pressure on the block consists of (i) thrusts on the vertical sides AC and BD and (ii) thrusts on the horizontal surfaces AB and CD. The pressure at any point on a side is horizontal and is exactly counter-balanced by an equal and opposite pressure at the point on the opposite side on the same horizontal line. The horizontal force due to pressures at all points on the vertical sides are thus mutually neutralised and we are left only with the vertical thrusts on the faces AB and CD. Let the depths of AB and CD from the free surface of the liquid be h and h' respectively and cross-sectional areas of the faces AB and CD be a.

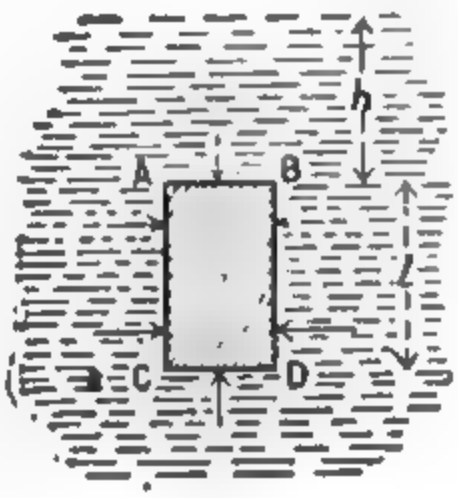


Fig. 153

Let height of the block $= l$

density of the liquid $= \rho$

$\therefore$ Thrust on AB acting vertically *downwards* $= ah\rho g$

Also thrust on CD acting vertically *upwards* $= ah'\rho g$

Since $h' > h$, the upward thrust is larger than the downward thrust.

$\therefore$ Resultant thrust on the block, acting *vertically upwards*

$= a(h' - h)\,\rho g = al\rho g$

= weight of volume ABCD of the liquid displaced by the block.

Thus *the resultant thrust on a body immersed in a liquid is equal to the weight of volume of the liquid displaced by the body and acts vertically upwards through the centre of gravity of this portion of the liquid.*

If the solid is not completely immersed in the liquid, the upward thrust acts on the immersed portion of the solid. In this case the resultant thrust is equal to the weight of the liquid displaced by the immersed portion of the solid. The resultant upward thrust on a solid immersed wholly or partly in a liquid, is called the *buoyancy* of the liquid. Its magnitude is therefore equal to the weight of the volume of

the liquid displaced. The buoyancy acts vertically through the centre of gravity of the volume of the displaced liquid, which is therefore called the **centre of buoyancy** or the *centre of floatation*. Evidently the buoyancy does not depend on the depth to which the body is immersed, nor on the shape, mass or material of the body itself. It simply depends on the immersed volume of the solid and the density of the liquid displaced.

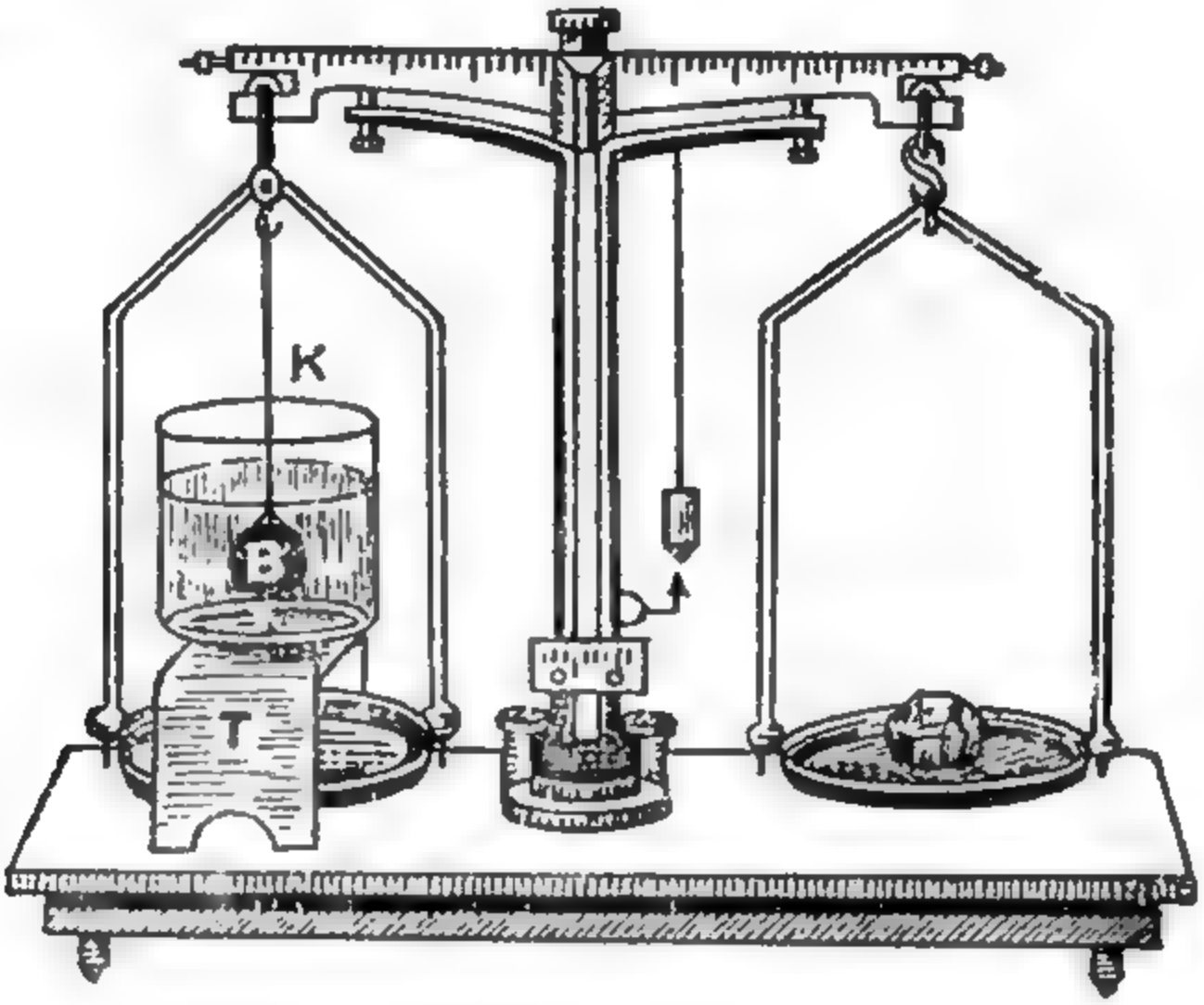


Fig. 154—Hydrostatic Balance

Apparent Weight.—As the weight of a body acts vertically downwards, the weight of any body immersed in a liquid must appear *less* than its true weight. If the true weight of the body be W and the buoyancy be W′, the *apparent weight* of the immersed body is equal to (W − W′). A body therefore appears lighter when immersed in a liquid. Thus it is easier to carry a pitcher filled with water or to move a stone when inside water than when outside.

112. Principle of Archimedes.—*When a body is immersed either wholly or partly in a fluid at rest, it appears to lose a part of the weight, which is equal to the weight of the fluid displaced.*

Experimental **Verification of the Principle.**—To verify the principle, *a hydrostatic balance* is usually taken. It is simply an ordinary balance with arrangement to get conveniently the weight of a body in air and also when immersed in liquid. In some special forms, one of the pans is suspended by a shorter frame and has a hook attached below, from which the body to be weighed can be suspended by a string. In the ordinary form a wooden bridge T (Fig. 154) is placed on the floor of the balance across one of the pans, so that a beaker containing liquid may be placed on the bridge. (For experimental details vide J. Chatterjee's Intermediate Practical Physics).

Take a solid cylinder A, which just fits into the hollow cylinder B, so that the volume of A (Fig. 155) is exactly equal to the internal volume of B. Suspend the cylinder B from a hook attached to one arm of the balance and suspend A from a hook fixed to the bottom of B. Counterpoise the whole by placing weights on the other scale pan. Now place an empty glass beaker D on the bridge C, so that A hangs within the beaker. Pour any liquid-say water, into the beaker till A hangs completely immersed in water ; take care that it does not touch any side of the beaker. The equilibrium is found to be lost and the other pan goes down, showing as if A has lost some weight. Now pour water into B till it is completely filled ; the equilibrium now is found to be restored, when the beam becomes horizontal again.

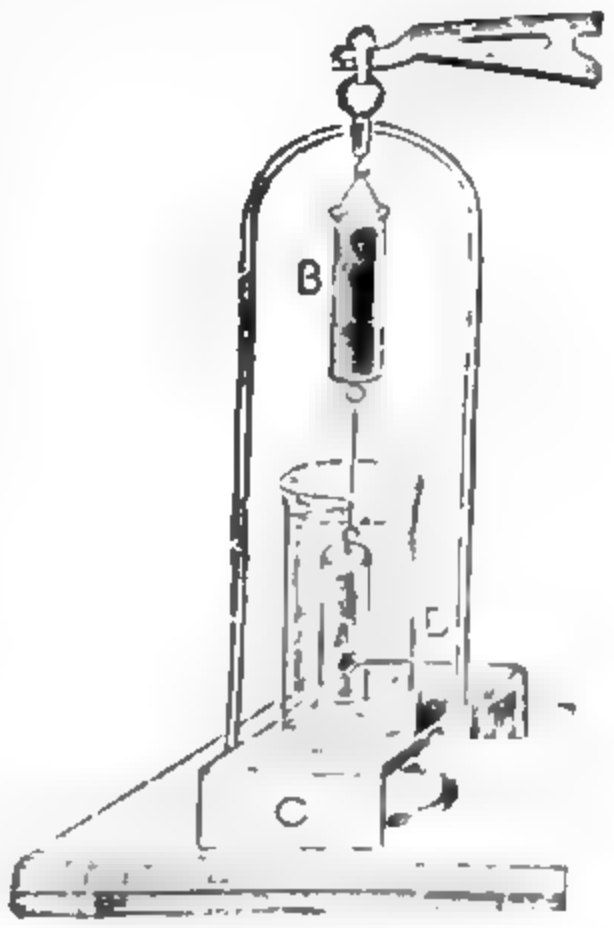


Fig. 155

Hence it is found that the apparent loss in the weight of A, due to the upward thrust of water is exactly equal to the weight of water completely filling B. As the inner volume of B is exactly equal to volume of A, *the apparent loss in the weight of A is equal to the weight of water displaced by it.* The experiment may be performed with any other liquid. The fact can also be very quickly verified by a spring balance.

Apparent Loss.—It is to be noted that the loss observed in the weight of the immersed solid is only *apparent*. Thus, if the beaker containing the liquid is placed on the scale pan, there will be no change whether the cylinder A is inside the liquid or outside. When the cylinder is immersed in the liquid, it experiences an upward thrust which tends to raise the arm of the balance ; but the cylinder at the same time exerts as *reaction* a downward force on water in the beaker. As by Newton's Third Law of motion, the reaction is exactly equal and opposite to the upward thrust, the balance remains always in the same condition.

Now if a finger is dipped into the water in the beaker, the pan carrying the beaker at once goes down. Here the upward thrust on the finger is taken by the hand but the reaction acts on the water and so on the beaker, causing the pan to go down. Similar effect is observed by suspending a solid into a liquid from a support placed outside.

113. Some Practical Applications of the Principle.—The principle is utilised in the following cases.—

(1) Determination of Volume of a Solid.—The volume of a solid, heavier than and insoluble in water, can be easily determined in the following way :—Weigh the body in air ; let it be W gm. Then weigh the body, when suspended in water with a hydrostatic balance ;

let it be W′ gm. Let the density of the water taken be ρ gm. per c.c. Then the mass of water displaced by the solid $W - W'$ gm.

Hence volume of the displaced water $= \frac{W - W'}{\rho}$ c.c.

= volume of the body.

If weights are given *in pounds*, the density of water is usually 62·5 lbs. per cu. ft. and so the volume of the body is $\frac{W - W'}{62{\cdot}5}$ cu.ft. The volume of a solid of *irregular shape* is usually determined by the above way. For example, suppose that the mass of a piece of stone is 52·12 lb. in air and its apparent mass in water is 31·54 lb. Then its volume comes out to be $(52{\cdot}12 - 31{\cdot}54) \div 62{\cdot}5 = 0{\cdot}33$ cu. ft.

(2) Determination of Density of a Solid.—As density is mass per unit volume, the density ρ' of the solid is given by

$$\rho' = \frac{\text{mass}}{\text{volume}} = W \div \frac{W - W'}{\rho} = \frac{W\rho}{W - W'}$$

If expressed in C. G. S. units, the density ρ' of the solid

$= \frac{W}{W - W'}$ gm. per c.c. (taking $\rho = 1$)

If expressed in F. P. S. units, the density ρ' of the solid

$= \frac{W}{W - W'} 62{\cdot}5$ lb. per cu. ft.

For the same specimen of stone its density in the F. P. S. unit from the data supplied comes out to be $2{\cdot}53 \times 62{\cdot}5$ lb. per cu. ft. and in the C. G. S. system it is simply 2·53 gm./c.c.

114. Equilibrium of Immersed and Floating Bodies.—When a body is immersed in a liquid, it is simultaneously subjected to two forces—(*i*) the weight W of the body, and (*ii*) the buoyancy W′ of the displaced liquid acting vertically upwards through the C. G. of the displaced liquid. The equilibrium of the body depends upon the relative magnitudes of the two forces and consequently *three* cases may arise, according as the weight W is greater than, equal to or less than the buoyancy W′.

(1) If $W > W'$, that is, if the weight of the body is *greater than* the vertical upward thrust, the body *sinks down*. The weight of the body in this case is greater than the weight of an equal volume of the liquid ; which means that the body is heavier than the liquid. Thus a piece of stone or iron sinks in water.

(2) If $W = W'$, that is, if the weight of the body is *equal* to the vertical upward thrust, the body *floats completely immersed* and may remain in equilibrium at any position within the liquid. Evidently the density of the body in this case is equal to that of the liquid. A drop

of olive oil may stand anywhere within a mixture of equal quantities of water and alcohol.

(3) If $W < W'$, that is, if the weight of the body is *less than* the vertical upward thrust, the body *floats partly immersed* in the liquid. A piece of cork or wood floats on water and iron floats on mercury. Evidently the weight of the body in this case is less than that of an equal volume of the liquid.

The third case gives the condition under which a body may float on a liquid. So the **conditions of equilibrium of a floating body** are

(1) The floating body must displace a volume of the liquid, whose weight is equal to the weight of the body itself.

(2) The C. G. of the body and the C. G. of the displaced liquid (centre of buoyancy) must lie on the same vertical line called the *central line* of the body and in general the former is *above* the latter. In case of a body floating just completely immersed, the C. G. of the body should be vertically *below* the centre of buoyancy.

Metacentre.—Let us take a body floating in equilibrium, say, a ship floating on an even keel. As already stated, the C. G. of the ship and the centre of buoyancy lie on the same vertical line, the former being above the latter. If the body is now displaced *e.g.*, when the ship heels over its C. G. and the centre of buoyancy are both displaced in the direction in which the body leans. The point where the vertical line through the new position of the centre of buoyancy intersects the central line of the body is called the **metacentre** of the body.

The equilibrium of a floating body is stable or unstable, according as the metacentre is above or below the C. G. of the body. When the two coincide as in the case of a floating spherical body, the equilibrium is neutral. By putting ballast in the ship, her C. G. is kept below the metacentre so as to have an increased stability.

A body may sink in one liquid but float on another. Thus a lump of iron sinks in water but floats on mercury. A fresh egg sinks in ordinary water but floats on strong brine. A fully loaded boat on sea is to be partly unloaded for safety before entering a river of fresh water.

115. Application and Illustration of the principle of Floatation.—The following cases furnish some illustrations of the principle of floating.

Floating of Ice on Water.—When water freezes into ice, it increases in volume, such that 11 volumes of water at 0°C, become 12 volumes of ice at the same temperature. Hence the density of ice is less than that of water and so ice floats on water.

∴ From the principle of floatation,

$$\frac{\text{Volume of ice under water}}{\text{Total volume of ice}} = \frac{11}{12}.$$

Hence a mass of ice will float on water with $\frac{11}{12}$th of its volume under water and only $\frac{1}{12}$ of the volume exposed. The density of sea water is about 1·08 and so an iceberg floats on sea-water with only one-ninth of its volume exposed. Again glacier ice is somewhat porous

and so an iceberg from a glacier floats on sea-water with about one-seventh of its volume exposed.

Floating of a Ship.—A solid heavier than a liquid may be given a suitable shape as to float on a liquid. The shape of the body would have to be such that the weight of the liquid displaced by the immersed portion of the body is equal to the weight of the body. Thus a porcelain saucer or iron cauldron floats on water partly immersed, for owing to its concave shape it displaces sufficient water for the purpose.

The body of a ship is constructed with iron plates, but owing to its concave shape the interior is hollow. It can therefore displace, even when partly immersed, a volume of water whose weight is equal to the weight of the ship with her contents; hence the ship floats. If the ship is loaded more, she sinks further to make the upthrust due to the newly displaced volume of water equal to the additional weight.

The carrying capacity of a ship is represented by her **tonnage.** A ship whose tonnage is 50,000 tons can carry an equal mass of material. It is to be noted that for the sake of safety there should be a suitable margin in the loading of a ship and sufficient *free-board* must be left above the water line. It has now been legalised that every ship should bear a mark called the **Plimson line,** which indicates the limit of permissible immersion.

Bodies heavier than water can also be made to float by attaching lighter bodies of suitable size. This explains the working of life-belts, life-buoys etc. A **life-belt** consists of a hollow air-tight tube, floating on water. When a person tries to swim with its help, it sinks a little further to counterbalance the force necessary for the man to float.

Swimming.—The human body is lighter than water, but his head is heavier. So the dead body of a human being floats with head leaning downwards. In swimming therefore we are to keep the head up by movements of the limbs, which produce sufficient reaction upwards to keep the body afloat. For a beginner the body may be attached to a life-belt. It is easier to float on water after taking a deep breath, as the lungs being inflated by air like a life-belt help the man to float. As sea water is heavier than river water the immersed portion of the body would experience greater upward thrust in sea water and so it is easier to swim in sea water than in river water. In case of quadrupeds, the front part is lighter than the hinder parts and so they swim naturally with head on water. Birds such as duck swim easily owing to the thick layer of impervious downs on the lower parts of their bodies.

116. Cartesian Diver.—The different kinds of equilibrium of a body floating on a liquid as well as the principle of transmission of fluid pressure can be strikingly demonstrated by a well-known hydrostatic toy known as the *Cartesian diver.* It is so called after its inventor Descartes, who devised it to illustrate the principle of Archimedes. The diver consists of a small hollow porcelain doll with a small tubular tail in communication with the inside and open at the end (Fig. 156). It is placed in water within a glass jar, top of the jar being closed airtight by a sheet of rubber. The doll is hollow within

and is partly filled with air and partly with water. Its mass is so adjusted as to be slightly less than that of water displaced so that it floats with a very small part exposed. In some cases the doll is solid and is attached to a hollow ball with a small aperture at the bottom (shown separately), so that the two together float in equilibrium.

Now if the rubber sheet is pressed, the diver sinks down, but on releasing the pressure the diver rises again to the surface. By regulating the pressure, the toy may be made to remain stationary at any level.

Explanation.—When the rubber sheet is pressed, the air enclosed within the jar above water is compressed. The pressure is transmitted through water to the air inside the diver. The air is thus compressed to a smaller volume, due to which an additional quantity of water enters into the body of the toy through the opening in the tail making it heavier than the water displaced. Hence the toy begins to sink. When the pressure on the rubber sheet is released, the air inside the toy expands again and drives out the additional water. The toy thus becomes lighter than the water displaced and it floats up.

Fig. 165—Cartesian Diver

It should be noted that as the diver goes down, more and more water is forced into the inside owing to increased liquid pressure. If the toy could be forced to such a depth that owing to the larger liquid pressure at that region, the inside air is reduced to a very small volume, the toy will not rise up even when the pressure on the rubber sheet is removed. Most fish have an air-bladder below the spine and by dilating or compressing it, they can rise up or dive in water at pleasure.

Submarine.—The modern submarine works on the principle of a *Cartesian diver* and may therefore be regarded as a ship with variable and controlled mass. It is provided with several large ballast tanks T (Fig. 157) in the bow, middle and stern of the ship. These are fitted with valves of trap doors, through which water can be admitted into

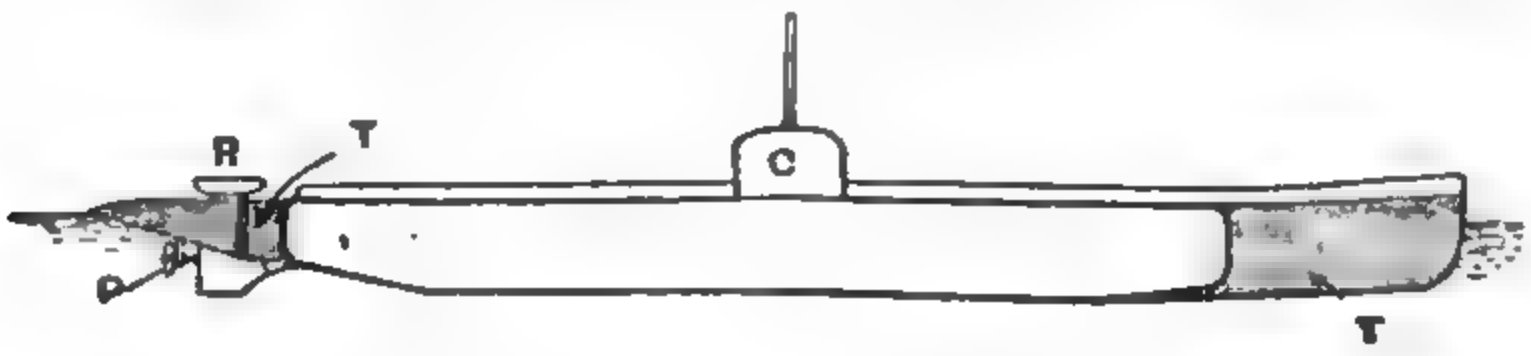


Fig. 157—Principle of a Submarine

the tanks. When to *sink*, requisite amount of water is taken into the tanks, so that the ship becomes slightly heavier than the displaced water and goes down. To *rise* to the surface, the water is forced out of the

tanks by powerful pumps worked by compressed air, whereupon the ship becomes lighter than the water displaced and she rises to the surface.

The operation of filling or emptying the tanks may be done swiftly. Besides the vertical steering rudder, there are horizontal rudders and by suitably tilting these, the ship may be kept at any desired depth. The *periscope* is fitted to the *conning tower* C and remains above the surface of water for getting view of objects on or above the surface of water.

Examples :—

1. Two bodies are equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 gm. and its density is 5·6; if the mass of the other is 36 gm. what is the density? (Pat. U.—1926)

A. Let the required density $= \rho$ gm. per c. c.

Here the apparent weights of the two bodies are equal.

Apparent weight of the first body $= 28 - (28 \div 5{\cdot}6) = 23$ gm.

Apparent weight of the second body $= 36 - 36/\rho$

$$\therefore \quad 36 - \frac{36}{\rho} = 23 \text{ whence } \rho = 2{\cdot}77 \text{ gm./c.c.}$$

2. Show that a hollow sphere of radius R made of metal of specific gravity of S will float on water if the thickness of its wall is less than R/3S. (Nag. U.—1952)

A. Let x be the thickness of the wall for which the hollow sphere can *just float*. Evidently if the thickness be less than x, the mass of the sphere would be less, and it would float out with a portion of it above water.

Now volume of the *metal* part of the sphere = volume of the outer sphere — volume of the hollow part of the sphere

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R-x)^3 = \frac{4}{3}\pi\left[R^3 - (R-x)^3\right]$$

and its mass is $\frac{4}{3}\pi\left[R^3 - (R-x)^3\right]S$.

For the sphere *just to float*, the mass of water displaced is $\frac{4}{3}\pi R^3$

$$\therefore \quad \tfrac{4}{3}\pi\left[R^3 - (R-x)^3\right]S = \tfrac{4}{3}\pi R^3$$

$$\text{or } \left[R^3 - (R-x)^3\right]S = R^3 \qquad \text{or, } 1 - \left[\frac{R-x}{R}\right]^3 = \frac{1}{S}$$

$$\text{or } \frac{R-x}{R} = \left[1 - \frac{1}{S}\right]^{\frac{1}{3}} = 1 - \frac{1}{3S} \text{ very approximately}$$

$$\text{or, } \quad x = \frac{R}{3S}.$$

3. A body of density δ is dropped gently on the surface of a liquid of density δ' (δ' being less than δ). Show that it will reach the bottom of the liquid after a time $\sqrt{\frac{2d\delta}{g(\delta-\delta')}}$, where g is the acceleration due to gravity and d is the depth of the liquid. (Pat. U.—1931)

A. Let m be the mass of a body; then its volume $= m/\delta$

$\therefore$ The upward thrust due to the displaced liquid $= \frac{m}{\delta} \times \delta' g$

So the resultant downward force on the body within liquid $= mg - \frac{m}{\delta} \times \delta' g$

Hence the acceleration f of the body, while going down $= g\left(1 - \frac{\delta'}{\delta}\right)$

Since the body starts from rest from the surface of the liquid,

$d = \frac{1}{2} f t^2 = \frac{1}{2} g \left(1 - \frac{\delta'}{\delta}\right) t^2$ whence $t = \sqrt{\frac{2d\delta}{g(\delta - \delta')}}$

ARCHIMEDES (287--212 B.C.)

Archimedes was born in 287 B. C. at Syracuse in Sicily, then under Greek empire. His father used to take interest in Mathematics and

Fig. 168—Archimedes

Geometry, which in those days were in a most elementary and crude form. In his boyhood, Archimedes was educated in a school at Alexandria. Inheriting his father's inclinations, he was also an ardent student of Mathematics and the rest of his life was dedicated to the advancement of Mathematics and Mechanics.

He is regarded as the most perfect type of scientific intellect that has ever appeared on earth. His researches were mostly theoretical and almost all his mechanical invention came out as things of urgent necessity to his country and friends. As Plutarch tells us, Archimedes regarded his inventions only as among the amusements of Geometry.

As occasions demanded, his inventions among many other things are pulleys, windlasses, water screws, hydraulic and compressed air machines. When Romans invaded Syracuse, Archimedes devised large concave mirrors out of small plane mirrors and focussed by these mirrors sun's rays upon the Roman fleet to destroy them completely. The story regarding the discovery of the famous Archimedes' principle runs thus. Hiero, the King of Sicily, had given a known weight of gold to an artificer with which to make a crown for him. When the crown was made, he suspected a cheat and asked Archimedes to identify, of course without breaking the crown, whether any base metal had been substituted for any of the gold. The scientist was at first puzzled and kept on thinking for some days regarding the behaviour of different metals. One day while taking a bath in a tub, he observed some displacement of water and at once it flashed to his mind that different substances of equal mass immersed in water might displace different volumes of water and this method might serve a clue to test the purity of a metal. He became so sanguine about the project that he jumped up from the bath and ran through the streets shouting "Eureka, Eureka" (I have found it) in complete forgetfulness of his covering body with clothes.

During an invasion of a Syracuse by Romans in 212 B. C. the soldiers broke open the city gate and a few of them entered Archimedes' house to find him sitting motionless and meditating over an abstruse geometrical figure drawn upon sand. They challenged Archimedes who did not make a reply. He was then slain. It is said that just before he was killed he cried out—"Kill me but do not kill my figure."

Summary

The **resultant thrust** *on a body immersed in a liquid is equal to the weight of the liquid displaced by the body and acts vertically upwards through the centre of gravity of this portion of the liquid.* This is called the **buoyancy of the liquid.**

Principle of Archimedes.—*When a body is immersed in a fluid at rest, it appears to lose a part of its weight which is equal to the weight of the fluid displaced.*

The conditions of floatation of a body are,—

(*i*) The floating body must displace a volume of a liquid, whose weight is equal to the weight of the body itself.

(*ii*) The C. G. of the body and that of the displaced liquid, called the centre of buoyancy must lie on the same vertical line and in general the former is above the latter. In case of a body floating completely immersed, the C. G. of the body is vertically below the centre of buoyancy.

EXERCISES ON CHAPTER IX

Reference

1. State Archimedes' principle, and show how it can be verified experimentally (Nag. U.—1951 ; *cf.* Raj. U.—1953 ; Utkal. U.—1951 ; Gau. U.—1952 ; C. U.—1943, '47, '49 ; Dac. U.—1941). — Arts. 111

2. State the principle of Archimedes. Show how it may be proved from the laws of liquid pressure. How does it lead to the law of floating bodies. (P. U.—1950) — Arts. 112 & 114

3. Describe hydrostatic balance and explain clearly the principle on which the working of the instrument is based. (C. U.—1932) — Art. 112

4. Show how Archimedes' principle can be applied to find the volume of the solid. (C. U.—1949) — Art. 113

5. The crown of Hiero weighed 20 lbs. Archimedes found that it lost 1·25 lbs., when immersed in water. The crown was made of gold and silver. Find the weights of these metals. Sp. gr. of gold = 19·3 ; sp. gr. of silver = 10·5. (Dac. U.—1941) — Art. 113

A. gold 15·078 lb. ; silver 4·922 lb.

6. State the conditions of equilibrium of a floating body. (*Cf.* Anna. U.—1950 ; Gau. U.—1952 ; Del. H. S.—1951 ; C. U.—1927 ; P. U.—1927 ; U. P. B.—1941) — Art. 114

7. When does a solid body, immersed in a liquid, sink or remain suspended or buoy up ? (Raj. U.—1953)

8. A piece of iron weighing 272 gm. floats on mercury of density 13·6 with ⅛ of the voume immersed. Determine the volume and density of iron. (Dac. U.—1937, '39 ; C. U.—1930) — Art. 114

A. 32 c. c. ; 8·5 gm. per c. c.

9. 52 gm. of an alloy of two metals of specific gravities 8 and 12 respectively, is found to weigh 46 gm. in water. Find the masses of the metals in the alloy. (Utkal. U.—1951) — Art. 114

A. 42 gm. ; 10 gm.

10. State the law of floatation and describe an experiment to verify the law. (Mys. U.—1952 ; Nag. U.—1952 ; U. P. B.—1941) — Art. 114

11. A hollow sphere made of glass ball just and only just sinks in methylated spirit of specific gravity 0·8. It floats in sulphuric acid (specific grovity 1·8). Explain why this is so and calculate the portion of the volume of the sphere immersed in the acid. (C. U.—1937) — Art. 115

A. 0·44 of volume nearly.

12. An elephant weighing two tons is taken abroad a barge of length 20 ft. breadth 6 ft. standing in a river. How many inches will the barge sink ? (1 cu. ft. of water weighs 62·5 lbs). (Mys. U.—1952) — Art. 115

A. 7·17 in.

13. Two pieces of metals are suspended from the arms of a balance and are found to be in equilibrium when kept immersed in — Art. 115

Reference

water. The mass of one piece is 32 gm. and its density is 8 gm./c.c. The density of the other is 5 gm./c.c. What is its mass?
(C. U.—1949)

A. 35 gm.

Art. 115 14. A raft is constructed of four empty barrels lashed together and planks weighing 40 lbs. are placed on them. If the barrels be regarded as approximately cylindrical and if the height, radius and mass of each be respectively 4 ft., 1·5 ft. and 50 lbs., find the load that the raft can carry. (1 cu. ft. of water weights 62·5 lbs.)

A. 6825 lbs. nearly.

Art. 115 15. A glass tube of mass 150 gm., length 30 cm. and of internal cross-section 1 sq. cm. closed at one end by a plate of negligible weight, is filled with mercury and suspended vertically, its open end being under mercury contained in a trough. Find the force required to hold it in this position when 20 cm. of the tube are out of the mercury in the trough. (Density of glass = 2·7 gm. per c. c.)

A. 170 gm. wt.

Art. 115 16. A wooden sphere of radius 1·0 cm. sinks in water. On coating it with a layer of uniform thickness 3 mm. of wax of density 0·8 gm. per c.c., it just floats. What is the density of wood?

A. 1·24 gm. c.c.

Art. 115 17. A block of ice (sp. gr. = 0·91) floats in a vessel of water filled upto the brim. What fraction of the volume of the block will be above the surface of water? Describe and explain the changes taking place when ice melts. (U. P. B.—1941)

A. ·09 of the volume.

Art. 116 18. A pontoon with perpendicular sides has a rectangular base 30 yds. by 10 yds. and is loaded so that the base is submerged to a depth of 5 ft.; 4 ft. of the pontoon remaining above water. It commences to leak, taking in 10 gallons of salt water per minute. How long will it be before it sinks; (A gallon of salt water weighs 10·3 lbs.)

A. 112·5 hours.

Art. 115 19. (*a*) Explain why an iron ship floats on water.
(C. U.—1920, '37; Dac. U.—1933)

(*b*) An iron nail does not float on water, but a much heavier steel ship does. (Mys. U.—1952)

Art. 115 20. Explain why it is easier to swim in sea water than in river water. (C. U.—1948)

Art. 115 21. Describe the Cartesian diver and explain how it acts. Do you know of any modern appliance which is based on this principle?

Art. 116 22. A solid body floating on water has one-sixth of its volume above the surface. What fraction of its volume will project if it floats in a liquid of specific gravity 1·2? (P. U.—1927; C. U.—1938, '43)

A. $\frac{1}{36}$

CHAPTER X

SPECIFIC GRAVITY

117. Specific Gravity.—If equal volumes of different substances are weighed, they are found to have *different* masses. Thus, if two cubes of the same size, one of marble and the other of iron, be taken, the mass of the latter is found to be greater than that of the former which means that iron is *heavier* than marble. The *specific gravity* of a substance indicates how much a given volume of the substance is relatively heavier than an equal volume of a standard substance. Hence,

The ***specific gravity*** *of a substance is the ratio of the weight of any volume of the substance to the weight of an equal volume of some standard substance.* The standard substance taken in case of *solids* and *liquids* is water at 4°C and in case of *gases* it is hydrogen at normal temperature and pressure.

∴ Specific Gravity (Sp. Gr.) of a substance,

$$= \frac{\text{weight of any volume of the substance}}{\text{weight of equal volume of water at 4°C}}$$

$$= \frac{\text{mass of that volume of the substance}}{\text{mass of equal volume of water at 4°C}}$$

$$= \frac{\text{mass of unit volume of the substance}}{\text{mass of unit volume of water at 4°C}}$$

$$= \frac{\text{density of the substance}}{\text{density of water at 4°C}}$$

Hence the specific gravity of a substance is sometimes called its **relative density.** The specific gravity is a pure ratio ; therefore it has no dimension.

Density and Specific Gravity.—As already stated, *density* of a substance is defined as its mass per unit volume and is therefore expressed in some unit, while the specific gravity being a ratio is a *pure number.*

If S be the sp. gr. of substance, ρ' its density and ρ that of water at 4°C, then from the above relation $\rho' = S\rho$. In C.G.S. unit, $\rho = 1$ gm. per c. c. ; so $\rho' = S$. Hence the *specific gravity of a substance in C.G.S. units is numerically equal to its density.* Again in F.P.S. units, $\rho = 62{\cdot}43$ lb. per cu. ft. at 4°C. so $\rho = 62{\cdot}43 \times S$ lb. per cu. ft, *i.e.*, the density of a substance in the F.P.S. unit is numerically equal to $62{\cdot}43 \times$ specific gravity of the substance. The mass of 1 cu. ft. of water is usually taken to be 62·43 lb. at ordinary temperature.

Thus in C.G.S. units, density of mercury = 13·6 gm. per c.c. and the sp. gr. of mercury = (13·6 ÷ 1) = 13·6.

Again in the F.P.S. unit the *density* of mercury = (13·6 × 62·43)' lb. per cu. ft.

$$\text{And the sp. gr. of mercury} = \frac{13{\cdot}6 \times 62{\cdot}43}{62{\cdot}43} = 13{\cdot}6$$

Thus the statement the specific gravity of mercury is 13·6, merely means that a certain volume of mercury is 13·6 times heavier than an equal volume of water at 4°C. in whatever unit the volume is measured.

Temperature Correction.—In determining the specific gravity of a solid or a liquid, water at 4°C should be taken for comparison. But water, usually available in the laboratory, has a temperature higher than 4°C. As the mass of unit volume of water at any other temperature except 4°C is less than that at 4°C, a correction for this is necessary. Let the temperature of water available be t°C.

$$\text{Then observed Sp. Gr.} = \frac{\text{weight of 1 c. c. of substance}}{\text{weight of 1 c. c. of water at } t°\text{C}}$$

$$\text{But, true Sp. Gr.} = \frac{\text{weight of 1 c. c. of the substance}}{\text{weight of 1 c c. of water at 4°C}}$$

$$= \frac{\text{weight of 1 c.c of the substance}}{\text{weight of 1 c.c. of water at } t°\text{C}} \times \frac{\text{weight of 1 c.c. of water at } t°\text{C}}{\text{weight of 1 c.c. of water at 4°C}}$$

= Observed specific gravity × *density* of water at t°C.

Fig. 159

The variation of density of water for a moderate change of temperature is very small. So unless a great accuracy is required, temperature correction need not be done.

118. Determination of the Specific Gravity of Solids.—It is now evident that in measuring the specific gravity of a substance, solid or liquid, by any method, we are to determine two weights—(1) the weight of a certain volume of the substance and (2) the weight of an equal volume of water. The *specific gravity of a solid* may be determined by the following methods.—

(1) By Hydrostatic Balance.—The hydrostatic balance has already been described in Art. 112 (For further details vide J. Chatterjee's Intermediate Practical Physics).

(*a*) *Solid heavier than and insoluble in water.*—Take a suitable piece of the solid. Suspend the solid by fine thread (Fig. 159), from the hook of the pan and weigh it. Then place a bridge on the table above the pan without touching it. Slide a beaker over the bridge, so that the body hangs freely within it without touching the walls. Now slowly pour water within the beaker so that it hangs completely immersed in water, and then weigh it in this condition.

Let weight of the solid in air $= W$
weight of the solid in water $= W'$
Then the upward thrust on the solid $= W - W'$
= wt. of water of the same volume as the solid.

Hence specific gravity S of the solid $= \frac{W}{W - W'}$ ...(118,1)

If the temperature of the water be t°C, the corrected specific gravity = S × *density* of water at t°C.

(*b*) *Solid lighter than but insoluble in water*—Here the given body is tied with a small heavy body unaffected by water so that the two together might sink in water. The heavy body so used is called a *sinker*. Take a suitable piece of given solid and weigh it in air. Then with the solid still on the pan suspend the sinker in water from that arm of the balance and counterpoise it with necessary weights. Next fasten the solid and the sinker together, suspend them in water and counterpoise.

Let weight of the solid in air $= W$
weight of the solid in air with the sinker in water $= W_1$
weight of the solid and the sinker, both in water $= W_2$
∴ wt. of the solid in air − wt. of the solid in water $= W_1 - W_2$

Hence sp. gr. S of the solid $= \frac{W}{W_1 - W_2}$ ...(118,2)

If water be at t°C, the corrected specific gravity
= S × density of water at t°C.

(*c*) *Solid soluble in water*—In this case the relative density S_1 of the solid is determined with respect to a liquid in which it is not soluble and then the specific gravity S_2 of the liquid is determined by any suitable method (see later) It can be shown that true specific gravity S of the solid is equal to the product of S_1 and S_2. Thus,

$$\text{Sp. gr. S of the solid} = \frac{\text{weight of V vol. of the solid}}{\text{weight of V vol. of the water}}$$

$$= \frac{\text{weight of V vol. of the solid}}{\text{weight of V vol. of liquid}} \times \frac{\text{weight of V vol. of the liquid}}{\text{weight of V vol. of water}}$$

∴ S = relative density of the solid with respect to the liquid × sp. gr of the liquid $= S_1 \times S_2$ ...(118,3)

For example, take a piece of alum. Find its density S_1 relative to the kerosene and then find the specific gravity S_2 of kerosene. Then the sp. gr. of alum $S = S_1 \times S_2$. (for Experiments with a hydrostatic balance vide J. Chatterjee's Intermediate Practical Physics).

(2) **By Hydrometers.**—The *hydrometer* works on the principle of floatation and is so constructed that it can float vertically in a liquid. According to the method of working, various forms of the instruments may be put under two different classes, *viz.*

(1) *Constant Immersion Hydrometer.*

(2) *Variable Immersion Hydrometer.*

Nicholson's Hydrometer.—It consists of a hollow metal cylinder A (Fig. 160) ending in two cones. One of these is connected by a short thin stem C to a pan B above, while from the other a conical hollow closed pan D hangs. The pan D is so weighed inside with lead shots or mercury that the instrument may float vertically in a liquid. There is a mark on the upper steam, which remains a little above when the instrument floats on a liquid. During an experiment, the hydrometer is *always* made to sink up to this mark. It is therefore a type of a constant immersion hydrometer.

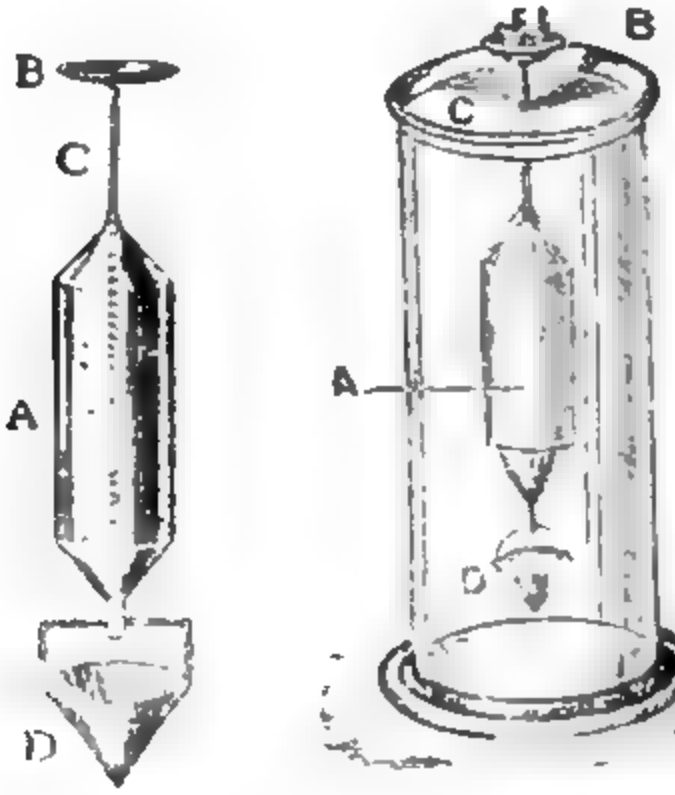


Fig. 160

Fig. 161

Float the hydrometer in water contained in a tall and wide glass jar, so that it does not touch the side of the jar (Fig. 161). A bent piece of wire is sometimes placed round the stem C across the mouth of the jar to prevent the instrument from sinking too much. Place suitable weights on the upper pan B to sink the hydrometer upto the mark and note the weights. Remove the weights. Now place on B a suitable piece of the given solid and add weights on B to sink the instrument again upto the mark. Since in both the cases, the hydrometer sinks to the same extent, the downward force on the upper pan must be equal. In the former case, it is the weight of the 'weights' placed and in the latter case it is the weight of the body and some weight. Hence the difference between the two weights gives the weight of the solid in air.

Next place the solid on the lower pan in water. If the solid is lighter than water, tie the solid with the lower pan. Weights are now to be added on the pan B to sink the instrument up to the mark; note the total weight now on B.

Let weight to sink the hydrometer upto the mark = W

wt. to sink, when the body is on the upper pan = W_1

wt. to sink, when the body is on the lower pan = W_2

Then weight of the solid in air = $W - W_1$

And weight of the solid in water = $W - W_2$

Hence weight of water displaced by the solid

$$= (W - W_1) - (W - W_2) = W_2 - W_1$$

$$\therefore \text{Sp. gr. S of the solid} = \frac{W - W_1}{W_2 - W_1} \qquad (118,4)$$

If required, the temperature correction may be made in the usual way.

(3) By Specific Gravity Bottle.—This method is specially suitable for finding the specific gravity of a solid given in the form of powder or small fragments.

The *specific gravity bottle* consists of a small flat bottomed glass bottle, having a narrow neck (Fig. 162). It is carefully fitted with a ground glass stopper having a capillary bore running through its length. If the bottle is filled to the brim with any liquid and the stopper is then carefully inserted, the excess of liquid escapes through the hole in the stopper, thus leaving the bottle completely filled with the liquid.

Fig. 162

Take the bottle, clean it and dry it carefully. Weigh the empty bottle in a balance. Then put the solid inside the bottle and weigh again. If the solid is a lump too big to go into the bottle, crush it and introduce one or two chips of the specimen into the bottle. The difference between two weights gives the weight of the solid put into the bottle. Now fill the bottle completly with water, replace the stopper carefully, so that no air bubble stricks inside. Wipe off any liquid that oozes out and weigh again. Next pour out the contents of the bottle, wash it thoroughly and fill it completely with water ; wipe the outside dry and weigh again.

Let weight of the empty bottle $= W_1$

weight of bottle with solid inside it $= W_2$

wt. of bottle with solid inside and water to fill it $= W_3$

weight of bottle with water to fill completely $= W_4$

$\therefore$ Weight of the solid in air $= W_2 - W_1$

and weight of bottle completely filled with water + solid

$= W_4 + W_2 - W_1$

Hence weight of water of the same volume as the solid

$= (W_4 + W_2 - W_1) - W_3 = W_4 - W_3 + W_2 - W_1$

$\therefore$ Specific gravity of solid, $S = \dfrac{W_2 - W_1}{W_4 - W_3 + W_2 - W_1}$ ...(118,5)

For a powder *soluble* in water, use a liquid in which the solid is not soluble and proceed as usual. The sp. gr. S_1 of the solid with respect to this liquid is thus obtained, Next determine the sp. gr. of the liquid with respect to water. Then the sp. gr. of the solid, $S = S_1 \times S_2$. The temperature correction may be made as usual.

(4) By Floatation.—If the solid is lighter than and insoluble in water and if it is obtained in a regular from, *e.g.*, a cube or a cylinder, the specific gravity can be readily determined by the method of *floatation.*

Measuring the height and the cross-section of the solid. Let these be l cm. and α sq. cm. respectively. Float the solid on water and mark the height exposed (Fig. 163). If the density of water be ρ, then the density of the solid is $S\rho$ and its mass is $l\alpha S\rho$. If the height immersed be h, then the height exposed is $(l-h)$.

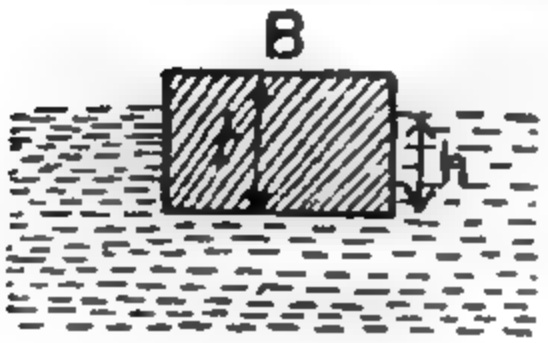


Fig. 163

Hence volume of water displaced $= h\alpha$

So mass of water displaced $= h\alpha\rho$

$\therefore$ For floatation, $l\alpha S\rho = h\alpha\rho$ whence $S = \frac{h}{l}$

119. Determination of Specific Gravity of Liquids.—To determine the specific gravity of a liquid methods are the same as those of a solid. The usual temperature correction may be applied if necessary. The specific gravity of a liquid may be determined in the following ways.—

(1) By Hydrostatic Balance.—Take a piece of glass or similar solid which is heavier than and chemically unaffected by the liquid as also by water. Weigh the solid in air. Suspend it from the left arm of the balance and weigh it first in water and then in the given liquid.

Let weight of the sinker in air $= W$
weight of the sinker in water $= W_1$
weight of the sinker in the given liquid $= W_2$
Then weight of liquid displaced by the sinker $= W - W_2$
Weight of water displaced by the sinker $= W - W_1$

$\therefore$ Specific gravity S of the given liquid $= \frac{W - W_2}{W - W_1}$ ...(119,1)

(2) By Nicholson's Hydrometer.—Find the weight of the hydrometer in an ordinary balance. Float the hydrometer in water and add weights on the upper pan to sink it upto the mark ; note down the weights. Remove the hydrometer and float it in the given liquid. Place weights on the upper pan to sink the hydrometer again upto the mark and note down the weights. Then from the principle of floatation, the sum of the weights placed on the pan together with that of the hydrometer is in each case equal to the weight of the corresponding liquid displaced by the hydrometer upto the mark. As the hydrometer sinks to the same point in both cases, the volumes of water and of the liquid displaced are the same.

Let the weight of the hydrometer $= W$
weight to sink it upto the mark in water $= W_1$
weight to sink it upto the mark in liquid $= W_2$
Then weight of water displaced $= W + W_1$
weight of the liquid displaced $= W + W_2$

$$\text{Hence the specific gravity S of the liquid} = \frac{W + W_2}{W + W_1} \qquad (119,2)$$

Examples :—

1. A Nicholson hydrometer sinks to a given mark in water when 3·32 gm. is placed on the upper pan and to the same mark in a liquid of specific gravity 1·02 when 9·41 gm. is placed on the upper pan. Find the weight of Nicholson's hydrometer. (Pat. U.—1953)

A. The specific gravity S of the liquid = 1·02; weight W_1 to sink upto the mark in water = 3·32 gm.; weight W_2 to sink upto the same mark in the liquid = 9·41 gm. If W be the weight of the hydrometer, then

$$S = \frac{W + W_2}{W + W_1} \quad \text{or} \quad 1{\cdot}02 = \frac{W + 9{\cdot}41}{W + 3{\cdot}32}$$

$$\text{or,} \quad 1{\cdot}02\ W + 1{\cdot}02 \times 3{\cdot}32 = W + 9{\cdot}41$$

whence W = 301·2 gm.

2. A Nicholson's hydrometer floats in a liquid with a part of its stem immersed. On putting a weight of 40 milligrams on the upper pan, one centimetre more of the stem goes under the liquid. If the diameter of the stem is 2 mm. find the density of the liquid. (Nag. U.—1953)

A. When the hydrometer floats in a liquid, then it displaces a volume, say V c. c. of the liquid whose weight is equal to the weight of the hydrometer. When a load of 40 mgm. is put on the upper pan, then the weight of the hydrometer virtually increases by this amount and in this case the volume of the liquid displaced is V c. c. plus the additional volume displaced. Thus weight of this additional volume must be equal to 40 mgm.

Now additional volume of liquid displaced = height × cross section of the stem = $1 \times \pi \times {\cdot}1^2 = {\cdot}0314$ c.c.

If the density of liquid be ρ then mass of this liquid = ·0314 ρ which is equal to 40 mgm. = ·04 gm.

Thus ·0314 ρ = ·04 whence ρ = 1·274 gm./c.c.

(3) By Variable Immersion or Constant Weight Hydrometer.— In this, the weight of the instrument as whole remains constant, while the portion of the hydrometer immersed varies with the nature of the liquid. Instrument is made of glass and in its usual form consists of a narrow hollow stem attached to a hollow body ending in a bulb at the bottom (Fig. 164). This bulb is weighed with mercury or lead shots to make the instrument float vertically. The stem contains a paper scale inside; the mark at which the liquid surface meets the stem directly gives the specific gravity of the particular liquid.

Fig. 164

The instrument is graduated by floating it in the liquid of known specific gravities The higher is the sp. gr of the liquid, the more the hydrometer floats out while the lower is the sp. gr. the farther does it sink down. The graduation are therefore made from above downwards. There is usually a set of hydrometers enclosed in a box for different ranges of specific gravities of values from 0·7 to 2. When made to float in water at 4°C. the reading is 1000, *i.e.*, sp. gr. = 1·000; a reading of 1750 indicates that sp. gr. is 1·750.

These instruments are used for industrial purposes where the specific gravity of a liquid is to be readily determined. For special purposes instruments of different types are constructed and are differently named. The sp. gr. of milk is tested by an instrument called a **lactometer** which may be graduated by immersing it in mixture of milk and water in various different proportions. Thus, if it sinks upto the mark ½, it indicates a mixture of milk and water in equal proportions. The sp. gr. of ordinary milk is lighter than skimmed milk. The sp. gr. of urine is tested by *urinometer* and that of alcohol by *alcoholometer*.

Examples :—

1. A common hydrometer has a uniform stem on which the readings 1·00 and 1·10 are 8 cm. apart. Find the distance of the reading 1·06 from 1·10.

(Mad. U.—1950)

A. Let M be the mass of the hydrometer and V its volume from the bottom upto the graduation marked 1·10. When made to float on a liquid of sp. gr. 1·10, then according to the law of floatation, $V \times 1{\cdot}1 = M$. If α sq. cm. be the area of cross-section of the stem, then when made to float on a liquid of sp. gr. 1, $M = V + 8\alpha$.

Thus $M = V \times 1{\cdot}1 = V + 8\alpha$, whence $\alpha = \frac{V \times {\cdot}1}{8}$ sq. cm.

Let l cm. be the distance of graduation 1·06 from 1·10. Then from the law of floatation,

$M = (V + l\alpha) \times 1{\cdot}06 = (V + lV \times 0{\cdot}0125) \times 1{\cdot}06$

and since $M = V \times 1{\cdot}1$, $(V + lV \times {\cdot}0125) \times 1{\cdot}06 = V \times 1{\cdot}1$

or, $l = 3$ cm. very approximately.

2. The stem of a common hydrometer is cylindrical and the highest graduation corresponds to a specific gravity of 1·0 and the lowest to 1·4. What specific gravity corresponds to a point exactly midway between the divisions? (Utk. U.—1953)

A. Let M be the mass of the hydrometer and V its volume from its bottom upto the graduation marked 1·4. When made to float on a liquid of sp. gr. 1·4, then according to the condition of floatation,

$M = V \times 1{\cdot}4$. Call it eqn. 1.

Suppose l be the length of the stem from graduation 1·4 to 1·0 and α the area of cross-section of the stem assumed uniform. When made to float on a liquid of sp. gr. 1·0, then from law of floatation,

$M = (V + l\alpha) \times 1 = V + l\alpha$. Call it eqn. 2.

If now S be the sp. gr. of the liquid on which it floats exactly $\frac{1}{2}\,l$ exposed, then $M = (V + \frac{1}{2}l\alpha)S$. Call it eqn. 3.

Then from 1, $V = 0{\cdot}714\ M$

From eqn. 2, $M = V + l\alpha = 0{\cdot}714M + l\alpha$ or, $0{\cdot}286\ M = l\alpha$

From eqn. 3, $M = (V + \frac{1}{2}l\alpha)S = (0{\cdot}714\ M + 0{\cdot}143M)S$

or, $S = \frac{1}{0{\cdot}857} = 1{\cdot}167$.

Hence the graduation corresponding exactly to the mid-point is 1·167

(4) By Specific Gravity Bottle.—Weigh a clean, dry and empty bottle with the stopper. Then fill it completely with water and weigh again. Pour out the water and dry the bottle carefully. Now fill it completely with the given liquid and weigh again.

Let weight of the empty bottle = W
weight of bottle filled with water = W_1
weight of bottle filled with the given liquid = W_2
Then weight of water filling the bottle = $W_1 - W$
weight of the liquid filling the bottle = $W_2 - W$

Hence the specific gravity S of the liquid = $\frac{W_2 - W}{W_1 - W}$ ...(119,3)

(5) By Hare's Apparatus or Watt's Hydrometer.—The specific gravities of two liquids which mix with each other can be readily compared by this apparatus It is merely an inverted U-tube, having a branch tube at the top end, to which a piece of rubber tubing is attached (Fig. 165) this is again fitted with a clip and has a short piece of glass tube attached at the other end. The U-tube is held vertically in a suitable frame and the lower ends of the two limbs dip into the separate beakers, each containing a liquid. When air is partially withdrawn from inside by sucking at the open tube, the pressure of air inside is decreased and the external atmospheric pressure forces the liquid to convenient heights, when the rubber tube is closed by the clip. The height of each liquid column above the surface of the corresponding liquid in the beaker is either read off from the scale attached or is measured by a scale.

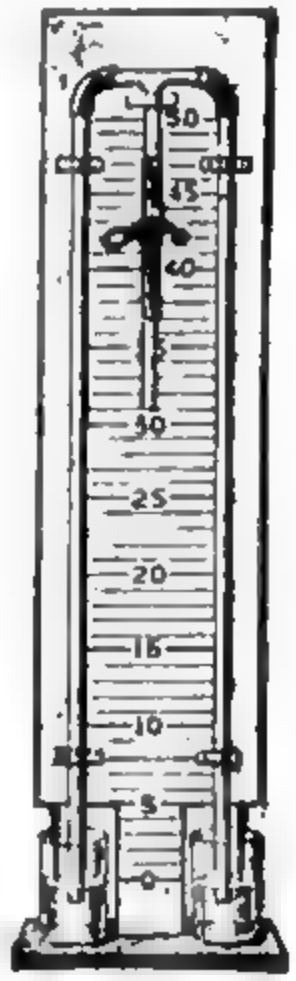


Fig. 165

Let densities of the two liquids respectively = ρ, ρ'
corresponding heights of the liquid columns = h, h'
atmospheric pressure at the time = P

Now the pressure of air inside the tube is the same throughout ; let it be p. Also atmospheric pressure at the surface of the liquid in each vessel is P.

$$P = p + h\rho g = p + h'\rho' g \text{ or } h\rho = h'\rho'$$

whence $\frac{\rho'}{\rho} = \frac{h}{h'}$ or $\rho' = \frac{h}{h'}\rho$. ... (119,4)

Thus the densities of the two liquids are inversely as the heights of liquid columns, measured from the respective surfaces. Hence the densities of the liquids can be readily compared and if one of them be known, the other is obtained. For example if one of the liquids be water, the density and so the sp. gr. of the other is directly found. It is to be noted that the cross-sections of the tubes do not come into consideration and the method can be applied for all liquids.

(6) By Balancing Columns (U-tube).—This is a convenient and ready method of comparing the specific gravities of two liquids, which do not mix nor have any chemical action with each other *e.g.*, mercury and water.

Take a glass U-tube ABC, as in fig. 151 and pour the heavier liquid into it. Then pour the ligter liquid carefully into the limb AD. The two liquids have a common surface of contact at D and let the hori-

zontal line through D meet the other limb at E. If the heights of the liquid columns from the common surface be h_1 and h_2 respectively and if ρ_1 and ρ_2 be their respective densities, then proceeding as in Art 109, it can be shown that.

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1} \text{ whence } \rho_2 = \rho_1 \frac{h_1}{h_2} \quad \ldots (119,5)$$

whereby the specific gravities of two liquids can be compared or the specific gravity of one liquid can be found if that of the other is known.

The following Table supplies the Specific Gravities of commonly observed substances :—

Specific Gravities of Common Substances

Substances	Sp. Gr.	Substances	Sp. Gr.	Substances	Sp. Gr.
Steel	7·7 – 7·9	Duralumin	2·8	Celluloid	1·4
Iron (pure)	7·87	Gunmetal	8·0 – 8·4	Cork	·22 – ·26
Brass	8·4 – 8·7	Manganin	8·5	Glycerine	1·26
Gold	19·32	Marble	2·0 – 2·8	Turpentine	·87
Copper	8·93	Quartz	2·66	Kerosene	·80
Silver	10·5	Sand	2·8 – 2·6	Castor oil	·97
Platinum	21·5	Glass, crown	2·4 – 2·8	Olive oil	·92
Lead	11·37	Sugar	1·52	Paraffin oil	·96
Nickel	8·9	Alum	1·70	Mustard oil	·12
Tin	7·3	Paraffin Wax	·87 – ·96	Petrol	·71
Copper-Sulphate	2·1	Sealing Wax	1·8	Milk	1·08
		Common Salt	1·6		

Examples :—

1. The specific gravity of ice is 0·918 and that of sea-water is 1·03; what is the total volume of an iceberg which floats with 700 cu. yds. exposed?

(C. U.—1932)

A. Let the total volume of the iceberg = V cu. yds.

Then volume under water = (V – 700) cu. yds.

∴ Mass of the iceberg = V × 27 × 0·918 × 62·5 lbs.

Also mass of sea water displaced = (V – 700) × 27 × 1·03 × 62·5 lbs.

∴ V × 27 × ·918 × 62·5 = (V – 700) × 27 × 1·03 × 62·5.

whence V = 6437·5 cu. yds.

2. Find a mathematical expression for the density of a mixture when the densities and the masses of the components are known.

Calculate the quantity of pure gold in 100 gm. of an alloy of gold and copper of density 16. Density of gold = 19 and that of copper = 9.

(Dac. U.—1930, '32)

A. Let the masses of the components be m_1 and m_2 and their densities be ρ_1 and ρ_2 respectively. Let the required density of the mixture be ρ

Now volume of the mixture = Total volume of the components.

$$\therefore \quad \frac{m_1 + m_2}{\rho} = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} \text{ whence } \rho \text{ can be calculated.}$$

Problem.—Let the quantity of pure gold be x gm.; then quantity of copper = (100 – x) gm.

Now volume of the alloy $= \frac{100}{16}$ and volume of gold $= \frac{x}{19}$

and volume of copper $= \frac{100 - x}{6}$

$\therefore \quad \frac{100}{16} = \frac{x}{19} + \frac{100 - x}{6}$ whence $x = 83{\cdot}125$ gm.

3. The densities of three liquids are in the ratio 1 : 2 : 3. What will be the relative densities of the mixture by combining (*a*) equal volumes, (*b*) equal weights? (Gau. U.—1953; C. U.—1954)

A. Let D be the density of the first liquid. Then the densities of the second and the third liquid are 2D and 3D.

(*a*) If V be the volume of each liquid mixed together, provided there is neither contraction or expansion of the resulting volume, the volume of the mixture is 3V. If ρ be the resulting density, then the mass of the mixture is $3V\rho$.

Then masses of the liquids in the mixture are VD, 2VD and 3VD. Hence from the principle of conservation of mass,

$$3V\rho = VD + 2VD + 3VD \quad \text{whence} \quad \rho = \frac{6D}{3} = 2D$$

(*b*) If m be the mass of each liquid mixed together, then the total mass of the liquid is $3m$ and the total volume is $3m/\rho$.

The volume of individual liquids are $\frac{m}{D}$, $\frac{m}{2D}$ and $\frac{m}{3D}$

$\therefore \quad \frac{3m}{\rho} = \frac{m}{D} + \frac{m}{2D} + \frac{m}{3D}$ whence $\rho = \frac{18}{11}D$.

4. A cylinder of wood whose specific gravity is 0·25 has another cylinder of metal (specific gravity 8·0) attached to one end. The cylinders are 2 inches in diameter, they have the same axis, and are respectively 20 in. and 1 in. long. If the whole is placed in water, find how much of it will be above the surface. (Utk. U.—1952)

A. Let the height below water be h in.

Volume of the metal cylinder $= \pi \times 1 \times 1$ cu. in. $= \frac{\pi}{1728}$ cu. ft.

$\therefore$ Mass of the metal cylinder $= \frac{\pi}{1728} \times 62{\cdot}5 \times 8$ lbs.

Volume of wooden cylinder $= \pi \times 1 \times 20$ cu. in. $= \frac{\pi \times 20}{1728}$ cu. ft.

$\therefore$ Mass of wooden cylinder $= \frac{\pi \times 20}{1728} \times 62{\cdot}5 \times 0{\cdot}25$ lbs.

Vol. of water displaced $= \pi \times 1 \times h$ cu. in $= \frac{\pi \times h}{1728}$ cu. ft.

$\therefore$ Mass of water displaced $\frac{\pi \times h}{1728} \times 62{\cdot}5$ lbs.

Since the total mass of the cylinder is equal to the mass of water displaced, we get the equation whence $h = 13$ in.

So height above the surface $= (21 - 13)$ in. $= 8$ in.

5. A silver ornament is suspected to be hollow. Its weight is 288·75 cm. and it can displace 30 c.c. of water. If the specific gravity of silver is 10·5, find the volume of the cavity. (P. U.—1926)

A. True volume of the ornament $= 288 \cdot 75 \div 10 \cdot 5 = 27 \cdot 5$ c.c.

But outer volume of the ornament $= 30$ c.c.

$\therefore$ Volume of the cavity $= (30 - 27 \cdot 5)$ c.c. $= 2 \cdot 5$ c.c.

120. Surface Tension.—It has already been stated that there is a cohesive force between the same kind of molecules of a solid or a liquid. The action of such cohesive forces on the *free surfaces of a solid or a liquid* of limited mass is to contract the bounding surface, and this contractile force is different for different substances. Since a liquid can very easily take up any shape, this force of contraction acting along the surface may be demonstrated very conveniently with a liquid as shown in the following experiment.

ABCD represents a metal wire bent twice at right angle at points B and C, so that the arms AB and CD are parallel to each other (Fig. 166). Another straight piece of *thin* wire EF which is tied with a thin piece of thread at the middle point can slide over the parallel arms. A soap solution is made in a vessel and this wire combination is dipped into the solution. On taking it out, a soap film is formed into the space BCFE and the wire EF would be found to be moving

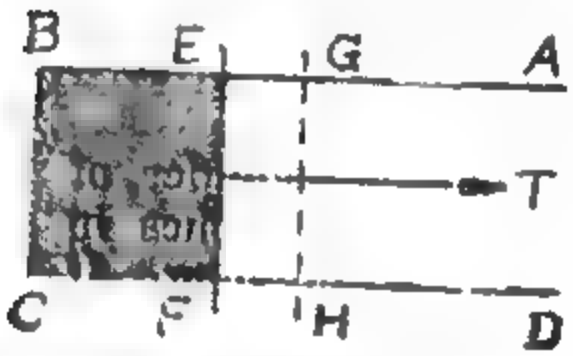


Fig. 166

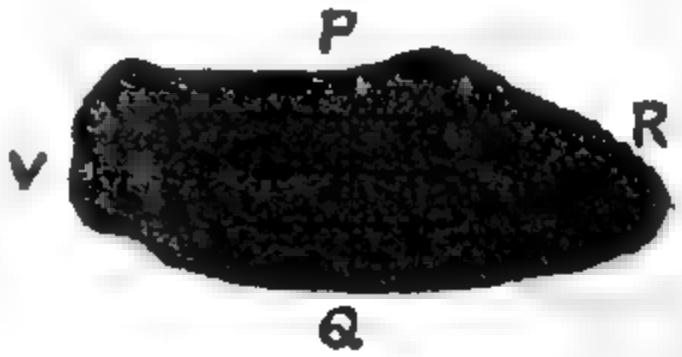


Fig. 167

towards BC. The force with which the wire moves is due to the contractile force acting on the surface of soap solution. To maintain the film, a certain amount of force, although very small, would have to be exerted on the thread outwards. Unless this force is applied the wire EF would gradually approach BC and would ultimately coincide with it and consequently the film would collapse. The surface of the liquid therefore behaves as if it were in a state of tension like a thin stretched elastic membrane.

The phenomenon is explained by supposing that there is a system of forces acting along the surface of a liquid. Let PVQR be an element of the surface of a liquid and let PQ be an imaginary line drawn upon it (Fig. 167). At any point of this line, there are two forces equal and opposite and acting at right angles to this line. If we imagine a unit length along PQ, the force acting over this unit length is called the *force of surface tension*. It is to be borne in mind that such forces of surface tension *act over free surface of a liquid only and not within the interior of the liquid.*

Let the force applied along the outward direction just to balance the force of surface tension be T and if the length of the wire EF touching the film be l, then remembering that the soap film has got two free surfaces, upper and lower, on each of which the surface tension is acting,

$$T = 2Sl \quad \therefore \quad S = \frac{T}{2l} \qquad \ldots(120,1)$$

The unit of surface tension is of the dimension of a force per unit length and may be measured in *dynes per centimetre* or poundals per inch. Phenomena due to surface tension may alternatively be explained in terms of potential energy developed when a liquid surface is increased against the forces of surface tension. Suppose for instance that the wire EF is pulled against the force of surface tension through a distance EG. This pulling should be so slow as not to *disturb the temperature* of the liquid. The work done to stretch the film surface is T × EG.

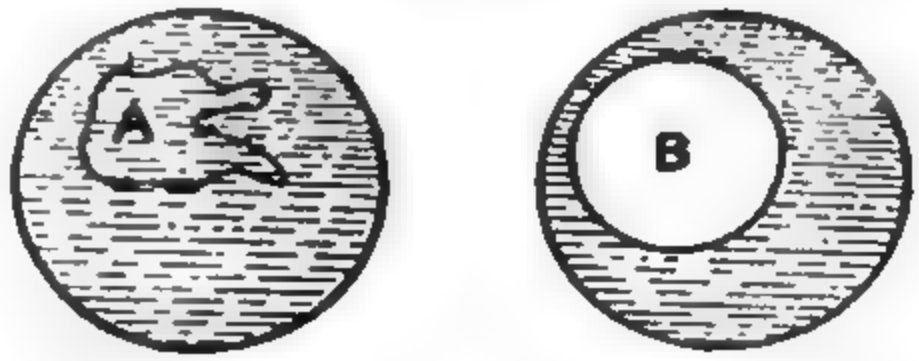
$$\therefore \quad T \times EG = 2Sl \times EG = 2S \times \text{area } FEGH$$

The film surface, because of its two sides, is actually increased by twice the area FEGH. Hence work done per unit area is,

$$\frac{T \times EG}{2 \times \text{area } EFGH} = S.$$

So surface tension is numerically equal to the work done to increase the surface area of a liquid by unity under isothermal condition. If temperature changes, the force of surface tension would also change. The dimension of surface tension has a force divided by a length and may be expressed as $MLT^{-2} \div L = MT^{-2}$.

121. Illustrations of Surface Tension.—The following illustrations prove the existence of surface tension of liquid. A round wire frame is dipped into a soap solution so as to form a film and a short silk thread A with a knot at the ends is placed on the film. The closed loop of thread may lie on the film in any manner as shown in fig. 168(*a*) On now touching any portion of the film inside the loop with a needle point so as to pierce the surface, this point of the film disappears and immediately the loop of thread assumes a round figure B (Fig 168*b*). This is due to the uniform surface tension of the film acting along the external boundary line of the loop of thread and for such a system of forces, the equilibrium condition is a circle.



(*a*) (*b*)

Fig. 168

It is always found that small drops of rain or due or small floating particles of liquid in air are round in shape. Such roundness of shape can be explained in terms of surface tension in the following way. Work must be done to form a liquid surface and consequently

some potential energy is associated with the surface of the liquid in question. It is a general law that for a stable equilibrium of a system, its potential energy would be minimum. Hence a liquid drop would assume such a surface as to have minimum potential energy. When forces of surface tension acting on the surface of the liquid drop are very great as compared to the weight of the liquid, the form of the liquid is round ; since for a given volume, the minimum surface area is that of a sphere.

A slightly greased sewing-needle, if carefully placed on the surface of water in a vessel, is found to float although it is much heavier than water. It can be explained by supposing that due to the surface tension the free surface of water behaves as an elastic membrane. Whenever the needle is placed on this membranous surface, the part of it below the needle is depressed and consequently the surface as a whole is enlarged. The curved portion of the liquid surface below the needle in its attempt to straighten out offers a vertical force on the needle to support its weight. Spiders and insects are found to move and run on the surface of water without sinking. This is also due to the surface of water behaving as an elastic membrane strong enough to support the insects.

If a soap bubble, blown at the end of a pipe, be allowed to stand, it gradually shrinks in volume. This shows that the surface tension tends to reduce the liquid surface to a minimum. If a camel-hair brush is dipped in water, the individual hairs project in different directions ; but when taken out of water, the hairs are held together as is coated by a membrane. When the cohesive forces between the molecules of the liquid are less than the adhesive forces between the molecules of the liquid and the solid, the liquid wets the solid *e.g.*, water in contact with glass. But if the case is reverse, the liquid does not wet the solid ; *e.g.*, mercury in contact with glass. Mercury sprinkled on a glass plate collects into small spherical drops, while water would spread out over the glass surface. If two glass rods are introduced in water and mercury, then after withdrawal, the former is found to be moist while the latter is dry.

***122. Various Surfaces in Contact.**—When there are various surfaces in contact, for example when a liquid is in contact with a solid in air, there are three surface tensions to be considered. Let a quantity of liquid rest on a solid surface in contact with air (Fig. 169). Let

Fig. 169

Fig. 170

the point O be the junction of three surfaces. Let S_1 be the surface

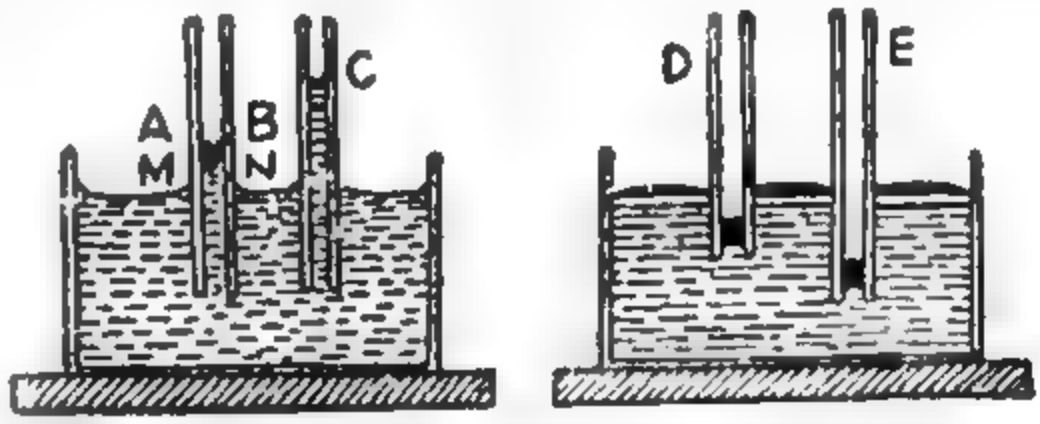
tension between the liquid and air surfaces acting tangentially at the point of contact. If θ be the angle between tangent plane and the solid surface, it is called the *angle of contact* of the liquid and the solid surfaces. Let S_2 and S_3 represent surface tensions between the liquid and the solid and between the solid and air respectively. The three surface tensions tend to make the air-liquid, liquid-solid and air-solid surfaces as small as possible and the form assumed by the liquid under these circumstances depends on the values of these three surface tensions. The condition of stability of the shape of the liquid mass is that the potential energy due to these three factors and gravitational forces on the mass shall be a minimum.

The effect of surface tensions between liquid and air and that between liquid and solid tend to cause the liquid to form a drop; while the surface tension between solid and air together with the gravitational forces tend to cause the liquid to form a film on the solid surface. For a very small drop on a solid surface, the effect of gravitational forces may be neglected in comparison to the forces due to surface tension. In such a case when the drop is in equilibrium state, the horizontal component of S_1 together with S_2 and S_3 must vanish.

Hence $S_3 - S_2 - S_1 \cos\theta = 0$ or, $S_3 - S_2 = S_1 \cos\theta$ ... (122,1)

If S_3 is larger than S_2 as is the case with a liquid adhering to a solid, θ would be such that $S_1 \cos\theta$ would contribute in the direction of S_3, in proportion to the amount of adhering or wetting property. This is shown in fig. 161, which represents a water drop on a glass surface. If S_3 is less than S_2 as is the case when liquid does not adhere to the solid, S_1 and θ would contribute in the direction of S_3 for equilibrium. This is shown in fig. 170, which represents a drop of mercury on glass surface.

123. Capillarity.—If glass tube of a fine bore, called a *capillary* tube (from Lat. *capillus*, a hair) is dipped vertically in any liquid, then the surface of the liquid inside and outside the tube are found not to be in the same horizontal plane. In the case where the liquid wets the walls of the tube, the liquid surface in contact with the outside is slightly raised, while the surface of the liquid inside the tube is raised a little *above* the free surface, being concave outwards, as at AB in fig. 171(i). Another tube of a narrower bore is dipped in to the liquid, the level inside is still higher as shown at C. In the case where the liquid does not wet the tube *e.g.*, mercury, the liquid surface in contact with the outside of the tube is somewhat depressed, while that within the tube is a little below the free surface, being convex upwards, as in (Fig. 171 ii). These phonomena are due to what is known as **capillarity** or *capillary action* for they are well marked in tubes of narrow bore.



(i) Fig. 171—Capillary Action (ii)

Ascension and Depression of Liquids.—When a glass tube is dipped in water, various surface tensions act at any point along the line of contact of water, air and glass. The resultant effect of these surface forces at any point acts parallel to the surface of the glass tube and at right angles to the line of contact. The net effect of these forces is a pull in the upward direction over the line of contact of glass and water. As a result the edges of the liquid are raised and the liquid surface within the tube is enlarged. Due to surface tension, free liquid surface tends to reduce its area as much as possible. Hence the concave surface of a liquid within the tube tends to straighten. But as soon as it does so, it rises higher in the tube. Thus the liquid continues to rise up in the tube until this tendency of the surface to contract is balanced by the weight of the liquid column. The narrower is the tube, the less is the weight of a column of the liquid and so the greater must be the corresponding height. In the case of a liquid which does not wet the tube, the effect of various surface tensions acting over the line of contact is a pull in a downward direction. Hence the liquid column must continue to fall, till the tendency to go down is balanced by upward liquid thrust over the section of the tube.

Laws of Capillarity.—The laws of capillarity, as experimentally established, may be summarised as below.—

(*i*) When a capillary tube is dipped into a liquid, the liquid rises or is depressed in the tube, according as it does or does not wet the material of the tube.

(*ii*) The elevation or the depression of the liquid in the tube is inversely proportional to the radius of the tube at the place of contact. This is known as **Jurin's law** of capillarity. If r is the radius of the internal bore at the level of the liquid and h is the height *i.e.*, the difference of the level from the free surface $h \propto 1\ r$ or rh is a constant.

(*iii*) The height is also dependent on the nature of the liquid, the medium above the free surface of the liquid and decreases as the temperature of the liquid rises.

Illustrations of Capillarity.—The rise of sap in plants, of oil in wicks of oil lamps or molten wax in those of candles, the retention of water by a piece of sponge, the rise or flowing down of ink in the narrow bore of a pen, soaking up of ink by blotting paper, the wetting of a towel with one end dipping in water, etc. are due to capillary action.

124. **Determination of Surface Tension.**—From an observation of the rise of liquid in a capillary tube, the value of the surface tension of the liquid with respect to air and glass may be determined. Let h be the height of the liquid column within a vertical capillary tube, as shown in fig. 172. If r be the radius of the capillary tube, then weight of the liquid column is $\pi r^2 h\rho g$ where ρ is the density of the liquid and g is the acceleration due to gravity.

Since the angle of contact between water and glass surface is 0°, the force of surface tension of water incontact with glass and air acts parallel to the glass walls. The surface tension S acts over a circular contour line of length $2\pi r$. Hence the total force of surface tension is $2\pi r$S. Thus for equilibrium.

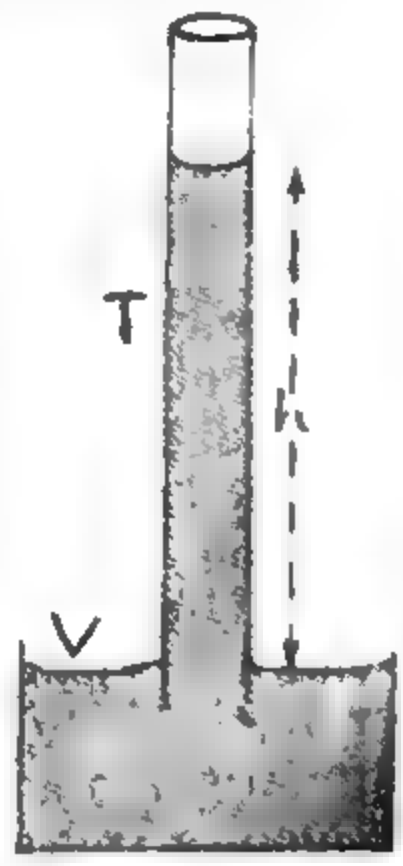


Fig. 172

$$2\pi r\mathrm{S} = \pi r^2 h\rho g, \qquad \text{whence } \mathrm{S} = \frac{rh\rho g}{2}$$

A small correction of the weight of the column may be made by considering the hemispherical liquid surface with the capillary tube. With a given liquid, place of experiment and temperature, S, ρ a nd$g$ are constants. Thus $r \times h$ is constant, which proves the truth of Jurin's law.

*125. **Molecular Theory of Surface Tension.**— The phenomena of surface tension are due to the force of molecular attraction. A molecule can exert its attractive influence upto some distance from it in any direction. If there be one or more molecules within this range from this particular molecule, there would be attraction between them. If this range of influence be r, and if a sphere be drawn with centre as the molecule and r as radius, then the sphere drawn is called the sphere of molecular influence. In fig. 173, A, B and C represent three molecules of a liquid and circles round them are the spheres of influence ; MN is the free surface of the liquid. Now if any molecule lies within this imaginary sphere, it would attract the central molecule.

When a molecule lies within the body of a substance, such that the sphere of influence is fully covered by the molecules, as at A the pull on the central molecule is equal in all directions producing an equilibrium. Hence there is no resultant force of molecular attraction on this molecule (Fig. 173) and the molecule within a liquid is free to move in any direction without doing any work, so long as its sphere of influence is filled up completely with molecules,

Now suppose that a molecule is approaching the surface MN of the liquid from the interior. At a certain position of a molecule as at B a portion of top part of the sphere of influence is projected above the liquid surface, and this portion contains no liquid molecules to attract the molecule at the centre. The lower half of the imaginary sphere now contains more liquid molecules than the upper half. Hence there would be a resultant pull on the central molecule in the vertically downwards direction. When a molecule is on the surface as at C, all the attracting molecules lie within the lower half of the

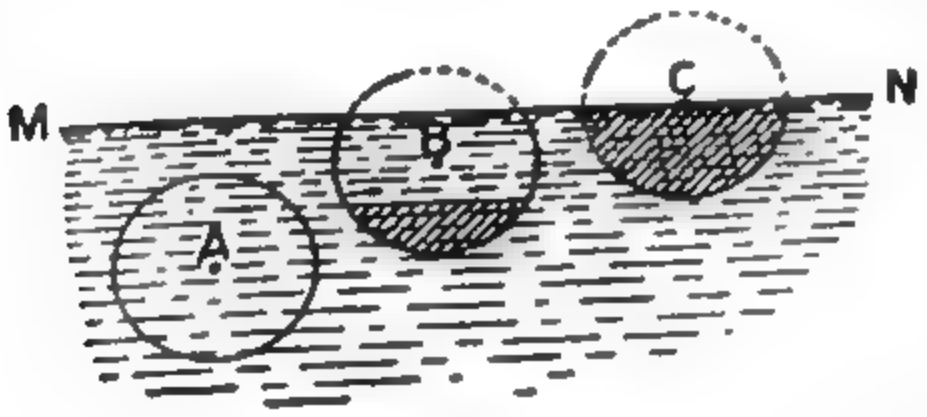


Fig. 173

sphere, and consequently there is the maximum pull on the molecule towards the interior of the liquid. Hence all molecules *on or very near the surface* tend to go to the interior of the liquid by the attractive forces and thus the surface area tends to diminish. An increase in surface area of a liquid is equivalent to forcing fresh molecules to rise up to the surface from the interior. The larger is the fresh surface made, the more would be the work done. Let S ergs be the amount of work done to increase the surface by 1 sq. cm. Then S is the potential energy per unit area of the surface. The same result may be arrived at if we assume a force of S dynes to act per centimetre length of the liquid surface. Then on dragging this line parallel to itself through 1 cm., S ergs of work are done and this is stored up as potential energy of the surface. This contractile force on the liquid surface produces the force of surface tension. The surface tension depends upon the medium which lies in contact with the free surface of a liquid. Such a contractile force is also present along the free surface of a solid body, although there is no flow of molecules within it.

EVANGELISTA TORRICELLI (1608—1647)

Torricelli was born at Faenza, Italy, on October 15, 1608. From his childhood he showed remarkable talents in scientific matters. At the age of nineteen he was sent to Rome as a student of Castelli, a favourite pupil of Galileo. Castelli was then engaged with some work in hydraulics and Torricelli took up the matter and wrote some papers on "Fluids in Motion". These papers were the forerunners of what is nowadays known as the branch of Hydro-dynamics. He then went to Galileo and worked under him for some time during which period the great scientist died.

Galileo observed that water rises about 32 feet in an evacuated tube but it was Torricelli who explained successfully that it is the pressure of the atmosphere which forces the water into the tube to that extent. He then employed mercury instead of water and showed that the given height of mercury in a tube devoid of air exerts the same pressure at the base as an equivalent column of water under the same condition. Thus with a student named Viviani he constructed the first mercurial barometer in 1643. He died a premature death on October, 25, 1647.

Summary

The **Specific Gravity** of a substance is the **ratio of the weight of any volume** of the substance to the weight of an equal volume of some standard **substance. It** is a mere *number.*

The Sp. Gr. of *solids* can be determined by (*i*) ***Hydrostatic balance.*** (*ii*) ***Nicholson's hydrometer,*** (*iii*) ***Specific gravity bottle.***

The Sp. Gr. of *liquids* can be determined by (i) *Hydrostatic balance.* (ii) *Nicholson's hydrometer,* (iii) *Specific gravity bottle.* (iv) *Hare's apparatus* and (v) *U-tube.*

Surface Tension is defined to be the force acting on the surface of a liquid per unit length in any direction parallel to the surface.

EXERCISES ON CHAPTER X

Reference

1. Distinguish between density and specific gravity. Art. 117
(E. P. U.—1951; Utk. U.—1949; C. U.—1936; Dac. 1940, '42)

2. A mixture is made of 7 c.c. of a liquid, specific gravity 1·85 and 5 c.c. of water. The specific gravity of mixture is found to be 1·615. Determine the amount of contraction. (C. U.—1927) Art. 117

A. 0·89 c.c.

3. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 gm. and its density is 5·6 gm./c.c. If the mass of the other body is 30 gm. what is its density? (Dac. U.—1942) Art. 117

A. 2·77 gm./cc.

4. A piece of alloy consisting of gold and silver weighs 20 gm. in air and 18·7 gm. in water. How much gold is there in the mixture? (Sp. gr. of gold = 19·3; of silver = 10·5). (C. U.—1936) Art. 117

A. 13·93 gm. approx.

5. A body weighs 121·5 gm. in air, 100·11 gm. in one liquid and 102·35 gm. in another. Find the apparent weight of the body in a mixture formed by equal volume of the two liquids. Art. 117

A. 107·73 gm.

6. 1 c.c. of lead (sp. gr. 11·4) and 21 cc. of wood (sp. gr. 0·5) are fixed together. Show whether the combination will float or sink in water. (C. U.—1933) Art. 117

A. The combination floats with ·1 c.c. above water.

7. A piece of lead of density 11·2 when attached to a piece of wax of volume 90 c.c. and density 0·88 appears to have no weight when the two together are immersed in a liquid of density 1·2. Find the volume of the piece of lead. Art. 117

A. 2·88 c.c.

8. A Nicholson's hydrometer requires 16·84 gm. on the upper pan to sink it in water upto the mark. It requires 4·06 gm. and 9·71 gm. on upper pan when a piece of glass is placed respectively on the upper and the lower pan. Find the specific gravity of glass. Art. 171

A. 2·5.

9. A piece of wax of volume 22 c.c. floats in water with 2 c.c. above the surface, find the weight and the specific gravity of the wax. Art. 117

A. 20 gm.; 0·9.

Reference

Art. 117 10. Describe in detail how the specific gravity of a solid soluble in water can be actually determined.
(Pat. U.—1952; C. U.—1926; Cf. And. U.—1952)

Art. 117 11. Describe fully an experimental method of measuring density of powdered sugar. Show the calculations clearly. (R. P. B.—1943)

Art. 117 12. A specific gravity bottle weighs 12·64 gm. when empty and 61·54 gm. when some pieces of iron are put into it. On being filled up with water, the bottle weighs 80·02 gm. When the bottle is filled with water only, it is found to weigh 37·6 gm. Find the specific gravity of iron.

A. 7·54.

Art. 118 13. A specific gravity bottle weighs 16·59 gm. when empty, 26·69 gm. when some quantity of salt is put into it. The bottle is then filled with kerosene oil and weighs 42·77 gm. The salt is now thrown out and the bottle is completely filled with kerosene oil, when it weighs 36·59 gm. If the specific gravity of kerosene oil be 0·8, find the sp. gr. of salt.

A. 2·08.

Art. 118 14. Describe any method of measuring specific gravity of a substance lighter than and insoluble in water. (Utk. U.—1949)

Art. 118 15. How would you proceed to find out the sp. gr. of (*i*) a solution of copper sulphate, (*ii*) a solid crystal of copper sulphate? Find the sp. gr. of a solid from the following data;—

Weight of solid in air = 0·4 gm.

Weight of sinker in air = 4·0 gm.

Weight of solid and sinker in water = 3·37 gm.

Sp. gr. sinker = 8·00. (Dac. U.—1934)

A. 0·75.

Art. 119 16. Describe a Nicholson's hydrometer and explain how you would determine the specific gravity of liquid with its help.
(Pat. U.—1953; C. U.—1943; '48; R. P. B.—1951)

Art. 119 17. A Nicholson's hydrometer sinks to a certain mark in liquid of sp. gr. 0·6; but it takes 120 grammes to sink it to the same mark in water. What is the weight of the hydrometer? (C. U.—1948)

A. 180

Art. 116 18. Explain the principle of a Nicholson's hydrometer. How would you use it for finding out relative density of cork.
(R. P. B.—1946, '48 *Cf.* Anna. U.—1951; *Cf.* E. P. U—1953)

Art. 119 19. Describe a constant weight hydrometer and explain how would you construct and use it to measure the specific gravity of a liquid.
(Gau. U.—1953; C. U.—1954, *Cf.* Utk. U.—1953)

Art. 119 20. A glass tube 80 cm. long and ½ sq. cm. in cross-section is closed at one end; its weight is 4 gm. and 10 gm. of mercury are poured into it. What will be the sp. gr. of a liquid in which it floats vertically with 2 cm. length of its stem above the surface? (P. U.—1926)

A. 1.

Reference

21. Explain how a test tube float may be used as a variable immersion hydrometer to determine the relative density of a solid. (Anna. U.—1950; And. U.—1952) — Art. 119

22. The stem of a common hydrometer is cylindrical and the range of the instrument is from 1·0 to 1·25. Find the specific gravity of a liquid in which the instrument floats with one-fourth of its stem exposed. (Anna. U.—1950) — Art. 119

A. 1·05.

23. The densities corresponding to the lowest and highest divisions on the stem of a hydrometer are 2 units and 1 unit respectively. What is the density corresponding to the middle of the stem? (Anna. U.—1951) — Art. 119

A. $1\frac{1}{3}$ units.

24. A specific gravity bottle weighs 14·72 grams when empty, 39·74 grams when filled with water, and 44·15 grams when filled with a solution of common salt. What is the specific gravity of the solution? (C. U.—1934) — Art 119

A. 1·17.

25. The lower ends of two vertical tubes, whose cross-section are 1 sq. in and 0·1 sq. in respectively are connected by a tube. The tubes contain mercury. How much water in ounce must be poured in one to raise the level of mercury in the narrower tube by 1 inch? 1 cu. ft. of water weighs 1000 oz. and density of mercury = 13·6 gm./c.c. (Utk. U.—1952) — Art. 119

A. 113·3 oz.

26. Write a short note on lactometer (Utk. U.—1952)

27. A U-tube contains some mercury at the bottom. Water is poured into one limb and oil into the other. When the heights of the columns are respectively 7·6 cm. and 9·5 cm. the mercury stands at the same level in both the limbs. Find the density of the oil — Art. 119

A. 0·8 gm./c.c.

28. A hollow stopper made of glass of density 2·6 gm./c.c. is found to weigh 23·4 gm. in and ond only 3·9 gm. in water. What is the volume of the air in the internal cavity of the stopper? (Utk. U.—1952) — Art. 118

A. 10·5 c.c.

29. Describe and indicate the use of a Hare's apparatus. (C. U.—1943, '49, *Cf.* Anna. U.—1951, Mad. U.—1950) — Art. 119

30. Explain with illustration the phenomena of surface tension Describe an experiment to find the surface tension of a liquid. (And. U.—1951; *Cf.* E. P. U.—1952) — Art. 120

31. What do you understand by the term 'surface tension' of a liquid? How do you account for it? Describe some simple experiments to demonstrate its effects. (P. U.—1952; *Cf.* E. P. U.—1953) — Art. 120

CHAPTER XI

PROPERTIES OF GASES

126. Gases and Liquids.—As already mentioned, gases have several properties in common with liquids and both are commonly termed *fluids*. Gases like liquids exert pressure and possess elasticity of volume but not of shape. A gas does not possess a free surface as a liquid. On the other hand, gases being much lighter than liquids, are very compressible and have got the property of indefinite expansion. Any volume of a gas possesses weight. The fact that air has weight can be shown by the following experiment, first undertaken by Otto Von Guericke in 1650.

A fairly large glass globe, about 3 to 4 inches in diameter and provided with a stop-cock, is taken. By means of air pump the globe is evacuated as much as possible and the stop-cock is closed. The globe is now carefully weighed in a balance. The stop-cock is then opened and air re-admitted into the globe. On weighing the globe again, it is found to be heavier. The difference between the two weights evidently gives the weight of the air admitted into the globe. The above experiment can be performed with any other gas, which shows that all gases possess weight.

127. Pressure of air : Atmospheric Pressure.—The air is a mixture of a few gases, mainly of oxygen and nitrogen. It envelopes the earth and extends up to a considerable height from the surface of the earth. This envelope is called the atmosphere. The atmosphere may be supposed to be divided into a number of horizontal layers parallel to the surface of the earth ; each layer bears the weight of all other layers above it and is thus subjected to a pressure due their weight. This pressure is called the **atmospheric pressure.** Evidently this pressure is greatest at the surface of the earth and decreases as we move higher. Further in case of air at rest the pressure at all points on the same horizontal plane is the same. As different layers are compressed to different extents, they possess different densities, the lowest one being densest.

The *existence of the atmospheric pressure* can be demonstrated by the following experiments. Take a glass tumbler filled with water to the brim and slide carefully a sheet of cardboard on its rim. If the tumbler is now carefully inverted, the water does not fall, showing that the upward pressure of the atmosphere on the lower surface of the cardboard supports the water and the cardboard.

Depress an empty glass tumbler into water with mouth downwards. The level of water inside the tumbler is found to be lower than that outside. This is due to the pressure of enclosed air which depresses the level of water inside.

Tie a piece of thin rubber sheet R air tight over a stout glass cylinder G open at both ends (Fig. 174). Place the other end on the plate B of an air pump, after having it well greased. Now evacuate the air gradually from inside the cylinder. The rubber sheet is depressed more and more and finally it bursts with a loud report. This is due to the fact that when air is taken out, the inside pressure falls and the pressure of the outside air depresses the rubber sheet which ultimately bursts when this pressure becomes too much. If the upper end of the cylinder is pressed by the palm of the hand in place of the rubber sheet, then on working the pump, the pressure of the air can be felt. When the exhaustion has proceeded far, the hand is pressed so much that it is matter of effort to lift it up at this stage.

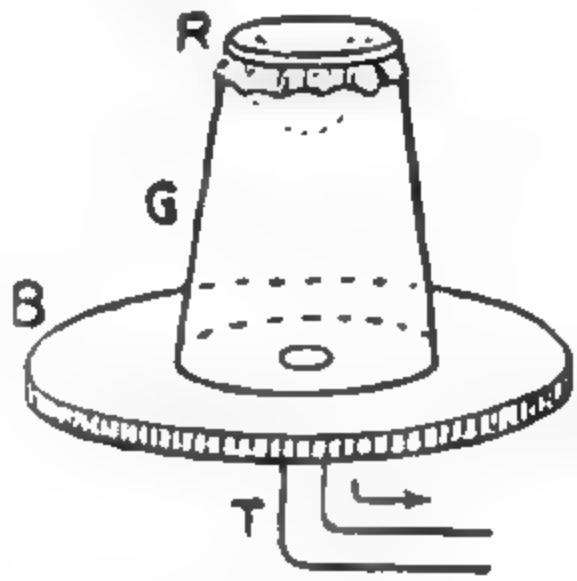
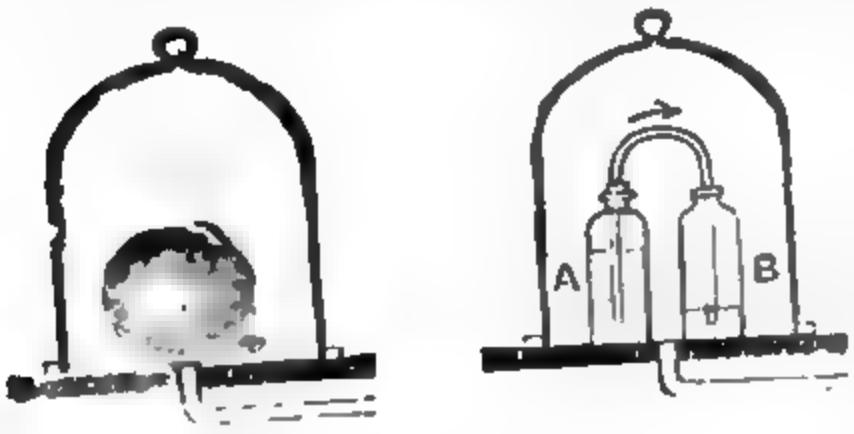


Fig. 174

Take a toy rubber balloon partially filled with air ; tie its mouth with a string and put it under the receiver of an air pump. On working the pump, the balloon gradually swells up and may burst, if the receiver is sufficiently evacuated (Fig. 175 *a*). Now re-admit air in the receiver, when the balloon again collapses to its original form. When the receiver is full of air the air inside the balloon exerts pressure on the wall, which is balanced by the pressure of air within the receiver ; but when the receiver is gradualy evacuated, the outside pressure decreases, the pressure inside the balloon becomes greater and so it swells up.



(*a*) Fig. 175 (*b*)

Take two bottles A and B connected together by a bent tube as shown in (175 *b*). One contains some liquid and is provided with an *air-tight* stopper ; through which end of the glass tube in this bottle dips inside the liquid. The tube within the other bottle passes very loosely, so that the inside of this bottle is an communication with the outside air. Put the two bottles under the receiver of an air pump and gradually evacuate. It will be found that the liquid flows from the closed bottle to the open one through the bent tube. This happens because when the receiver is evacuated, the pressure of air within the open bottle falls and hence the enclosed air within the closed bottle which has now a higher pressure, forces the liquid through the tube.

Magdeburg Hemispheres Experiment.—The existence of atmospheric pressure was demonstrated with brass hemispheres by Otto Von Guericke in 1654. The apparatus consists of two hollow brass hemispheres (Fig. 176), whose edges are so ground that they fit each other exactly. One of these is provided with a handle and the other with a stop-cock, with a handle screwed to the latter. The edges of

the hemispheres are well-greased and put together so that they fit air tight. The air from inside is then exhausted. A very great force is now required to separate the hemispheres.

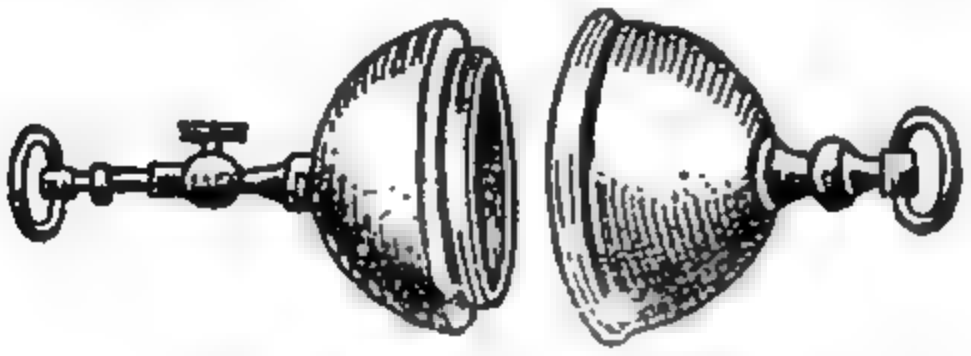


Fig. 176—Magdeburg Hemispheres

When there is air inside, the internal pressure balances the external pressure ; but when the air from inside is removed the atmospheric air presses the hemispheres from all sides and keeps them very tightly pressed against each other. On re-admitting air the hemispheres can be easily separated.

128. Rise of Liquid in an Exhausted Tube.—When the lower end of an open tube is dipped into a liquid and the air is sucked out at the upper end, the liquid rises up the tube. Similarly water rises up in a syringe or in a pump as the piston is pushed up. All these facts were formerly explained by saying that *nature abhors vacuum*. It was found afterwards that water rises only to a height of about 30 ft. and not to *any* height, as conceived before. The explanation was given by Torricelli that the rise of the liquid is due to the pressure of the atmosphere. He then argued that a heavier liquid should consequently rise to a smaller height and that the height for a particular liquid would change with a change of atmospheric pressure. To verify these facts, Torricelli in 1643, devised an experiment which goes by his name.

Torricelli's Experiment.—Take a thick-walled glass tube, about a metre long and closed at one end. Fill it completely with pure dry mercury, taking care that no air bubble sticks to the inside of the tube. Now close the open end tightly with the thumb and invert the tube vertically over a basin D containing pure dry mercury so that the open end is under the mercury surface (Fig. 177). On removing the thumb, the mercury column sinks down to some extent but remains stationary at a certain height. The height of this level above the mercury surface in the basin is found to be nearly 76 cm.

Explanation—The mercury surface in the basin is subjected to the pressure of the atmosphere which is transmitted undiminished in all directions. Again the pressure due to the mercury column in the tube at a point in level with the outside surface, is due to the weight of the mercury column of height equal to the vertical distance between the two mercury surfaces. As the mercury column is in equilibrium, these two pressures balance each other and must therefore be equal in magnitude. Hence the atmospheric pressure maintains the weight of the mercury column. If an opening be made at the top of the tube, the atmospheric pressure would also act upon the mercury inside the tube and so the column would come down the level of mercury in the basin. Evidently then *the atmospheric pressure is equal to the pressure due to a column of mercury 76 cm. in height.*

The space within the tube above the mercury surface contains no air and is usually referred to as **Torricellian vacuum.** Really however, the space contains some mercury vapour and is not a perfect vacuum. But as the pressure due to mercury vapour at the ordinary temperature is extremely small, its pressure may practically be neglected.

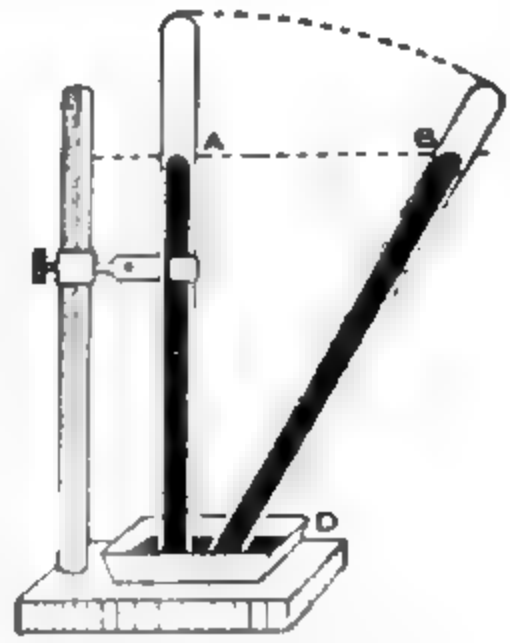


Fig. 177

The atmospheric pressure is given by the *vertical height* of the mercury column. If the tube is slowly inclined (Fig. 177). it is found that the space above mercury column gradually decreases but the vertical height measured from the mercury surface in the basin remains the same. When the tube is sufficiently inclined, the mercury strikes the top of the tube with a smart metallic sound, showing that there is no air inside. The height of the mercury column changes from time to time and also at different places, which shows that the *atmospheric pressure is not constant.*

If there is a change of pressure above the free surface of mercury, the height of the mercury column is found also to change. Place the basin under the receiver of an air pump while the tube passes through an air-tight rubber cork fitted to the open top of the receiver. As the pump is worked, the level of the mercury column gradually sinks down. On re-admitting air into the receiver, the mercury column gradually rises to its original level in the tube.

Take the tube as in fig. 177 and *raise* it gradually with the open end always under mercury. The space above mercury increases, but the height of the mercury column measured from the surface of mercury in the basin remains the same. If a very deep cistern is taken and the tube is gradually *pushed* in it, the space above mercury decreases but the height of the mercury column above the outside surface remains the same. The mercury ultimately touches the top of the tube and if the latter is pushed further down, the tube remains always filled with mercury. The height of the mercury column will not remain the same, if the space above mercury contains air or any other gas.

Pascal's Experiment.—Pascal suggested that since the atmospheric pressure decreases as we go up, the height of the mercury column would also decrease accordingly. In 1648, Perrier went up to the top of Puy-de-Dome, a mountain in France about 1000 metres in height and found that the mercury column actually came down through 8 cm. It was also found that with different liquids, columns stood at different heights.

129. Magnitude of the Atmospheric Pressure.—Since the pressure of the atmosphere supports a column of mercury, 76 cm. in height, the atmospheric pressure per sq. cm. is equal to the weight of a

column of mercury of 1 sq. cm. in cross-section and of 76 cm. in height. Taking the mean value of g to be 981 and density of mercury as 13·6, the atmospheric pressure per sq. cm.

= wt. of 76 c.c. of mercury = 76 × 13·6 gm. wt.

= 76 × 13·6 × 981 dynes = 1,013,961 dynes.

So the atmospheric pressure is approximately equal to one million dynes or 10^6 dynes per sq. cm.

Expressed in F. P. S. units, atmospheric pressure per sq. inch = wt. of a column of mercury 30 inches in height and 1 sq. inch in cross-section = wt. of 30 cu. inches of mercury.

$$= 30 \times 13{\cdot}6 \times \frac{62{\cdot}5}{1728} = 14{\cdot}8 \text{ lbs. wt.} = 14{\cdot}8 \times 32 = 473{\cdot}6 \text{ poundals.}$$

Hence the average atmospheric pressure is approximately equal to *15 lbs. wt. per sq. inch* or nearly one ton per sq. ft. When any gas or liquid exerts an equivalent pressure, the pressure is often mentioned as that of *one atmosphere*.

The height of *water column* supported by atmospheric pressure

= 30 × 13·6 in. = 34 ft. nearly.

Similarly with glycerine of density 1·29, the height of the liquid column = (34 ÷ 1·29) = 27 ft. nearly.

Normal or Standard Atmospheric Pressure.—The *normal* or *standard* atmospheric pressure is defined as the pressure exerted by column of mercury, 76 cm. in height, and of density 13·5951 gm./c.c. subject to the gravitational acceleration of 980·665 cm./sec^2. This is taken to be the standard with which other pressure may be compared.

Hence **standard atmospheric pressure**

= 76 × 13·5951 × 980·665 = 1,013,250 dynes per sq. cm.

In *meteorology*, a pressure of one million dynes per sq. cm. is called a **bar.** This is also equivalent to a pressure of 29·52 inches of mercury. One millionth part of a *bar* is called a *millibar*. The standard atmospheric pressure is thus equal to 1013·25 millibars.

Pressure on the human body.—Taking the total area of the body of a man of middle size to be about 16 sq. ft. the pressure on the body amounts to (15 × 12 × 12 × 16) lbs. or over 15 tons wt. It is very surprising to think how a human being can withstand such an enormous thrust without least discomfort while it is absurd for him even to carry a load of one ton ! The explanation is not difficult to find, for there are openings and innumerable pores on the skin of the body, through which air goes within the system and there is a free communication between inside and outside. The pressure of air inside counterbalances the pressure outside and consequently just as a lump of cotton in air can fly about safely without being squeezed by atmospheric

pressure, any fine part of a human system is equally safe. The pressure of blood within veins is matched against the outside atmospheric pressure. But if the outside pressure *considerably* alters, for example in climbing up high altitudes or in diving under sea, precautions should be taken to compensate for the difference of pressures inside and outside.

130. Barometers.—A barometer is an instrument for measuring the pressure of the atmosphere at any time in terms of the height of the liquid column (Gk. *Baros*-weight). Mercurial barometers are of two types—(*i*) *Cistern barometer*, and (*ii*) *Siphon barometer*. There is another type of barometer, called *Aneroid barometer*, in which no liquid is used.

Cistern Barometer.—An evacuated tube standing vertically on a cistern of mercury constitutes a very simple form of cistern barometer. The height of the mercury column measured from the mercury surface in the cistern measures the atmospheric pressure at the time. When the atmospheric pressure changes, the height of the mercury column in the tube and consequently the level of mercury in the cistern also changes. Hence a scale cannot be permanently fixed with its zero mark in level with the mercury surface in the cistern. To get an accurate reading of the pressure, therefore, either the scale should be variable in position or the level of mercury in the cistern must necessarily be kept the same. The latter condition is attained in a type of barometer, as given below.

Fortin's Barometer.—This is most convenient and accurate form of mercurial barometer of the cistern type and is therefore widely used. It essentially consists of a barometer tube which is completely filled with pure dry mercury with no air bubble sticking to the inside, (Fig. 178) and is inverted over a cistern of mercury called the *reservoir*. The mercury stands at a certain height, depending on the atmospheric pressure. The upper part of the reservoir consists of a glass cylinder A (Fig. 179) which allows the mercury surface to be seen ; the cylinder is fitted at its lower end into a box-wood cylinder B, which is surrounded by a brass case. The lower end of B, is closed by a piece of flexible leather which thus forms the true base of the cistern. The leather base is provided with a wooden bottom C at the lower surface. A screw S passes through the bottom of the cistern and presses against C. By turning the screw, the bottom can be raised or lowered, so that the level of mercury of the reservoir can be altered at will. The upper end of the cistern is closed by a strip of leather through the pores of which communication is maintained with the external air and thus the atmospheric pressure is transmitted to the inside. A small ivory *pointer* P (Fig. 179) is fixed vertically to the lid of the cistern, such that its lower tip is in the same horizontal plane as the zero of the scale.

The *barometer tube* is of larger bore at the upper side to avoid the

effect of the surface tension and is drawn to a small opening at the lower end to reduce any oscillation which may be set up in the mercury column during adjustment; the tube may be constricted some way above the lower end. The barometer tube is enclosed within a long brass tube. In the upper part of this tube, two wide vertical slits are cut, diametrically opposite to each other, so that the upper level of mercury may be seen through them.

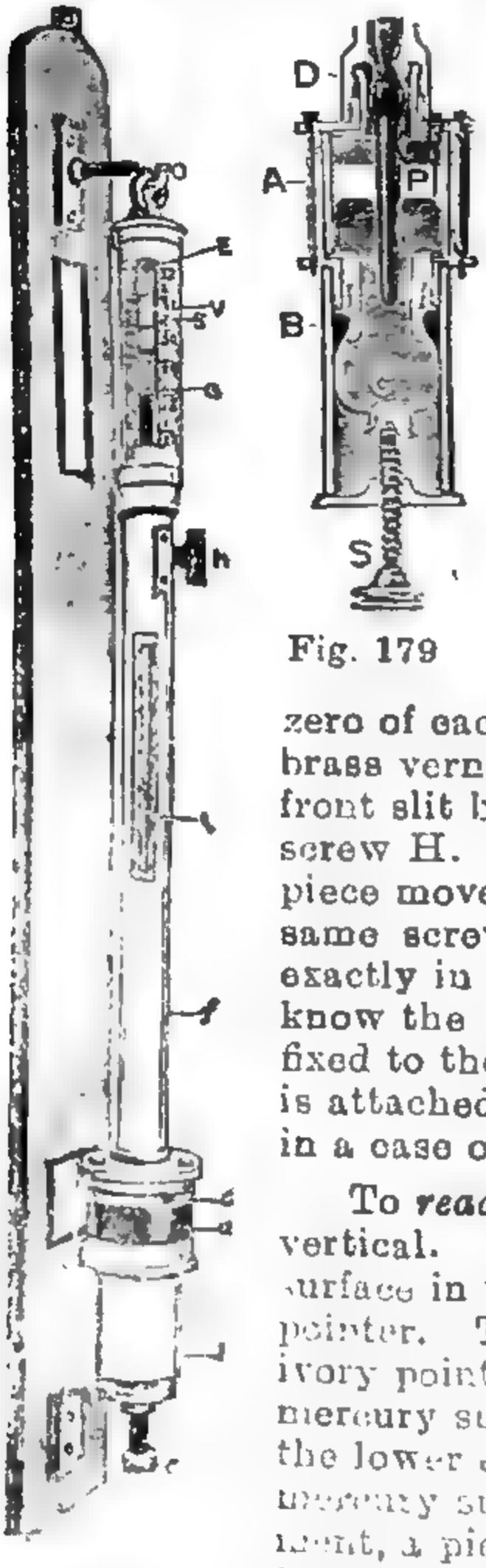


Fig. 179

The *scale*, S (Fig. 178) to be used is engraved on the brass tube along the edges of the front slit, one edge being graduated in centimetres and the other in inches. The scales are not continued throughout the length of the case but are usually graduated from about 27 to 32 inches and 68 to 80 cm., the zero of each corresponding to the tip of the pointer. A brass vernier scale V can be moved up and down in the front slit by rack and pinion arrangement, worked by the screw H. A plane piece of brass joined to the vernier piece moves inside the back slit, being worked by the same screw. The lower edges of the two pieces are exactly in the same horizontal plane in all positions. To know the atmospheric temperature, a *thermometer* T is fixed to the lower part of the brass case. The barometer is attached to a wooden board which may be hung within a case or against the wall of the room.

Fig. 178

To *read the barometer* the instrument is at first made vertical. The screw S is then worked until the mercury surface in the cistern just touches the tip of the ivory pointer. The adjustment is correct when the tip of the ivory pointer appears just to touch its image on the clean mercury surface. The vernier scale is then moved until the lower edge appears to be tangential to the curved mercury surface in the tube. To facilitate this adjustment, a piece of milk glass is usually fitted to the back board of the instrument. Keeping the eye in level with the mercury surface, the vernier scale is moved until the piece of white glass is just cut off from view. When the adjustment is complete, the reading is taken from the scale and the vernier. For accurate determination of the pressure, a *temperature correction* of the reading is necessary. (For details vide Heat. Art. 29 & J. Chatterjee's Intermediate Practical Physics).

*131. **Siphon Barometer.**—This is more convenient and portable than a simple cistern barometer. It has no cistern but consists simply of a U-shaped tube having unequal limbs. The longer limb AB (Fig. 180) is closed at the top and is about 90 cm. long, while the

shorter limb CD is open to the atmosphere. The space above mercury column within the limb AB is vacuous and the mercury column in CD

Fig. 180

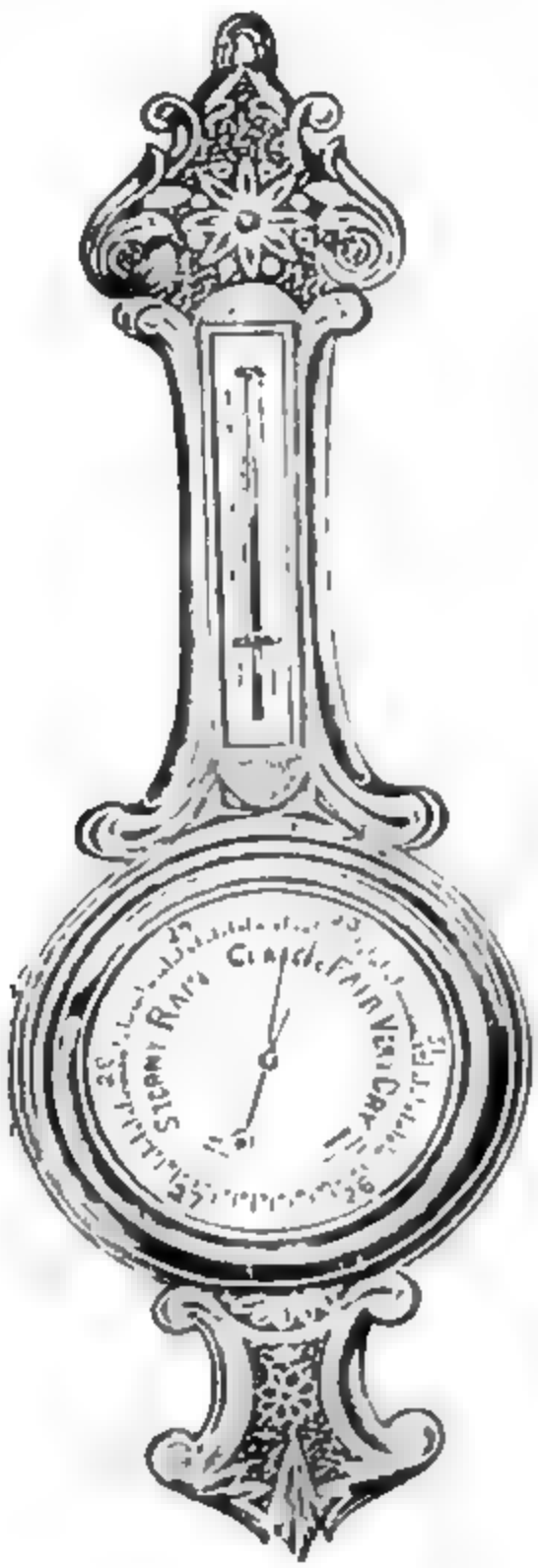


Fig. 181

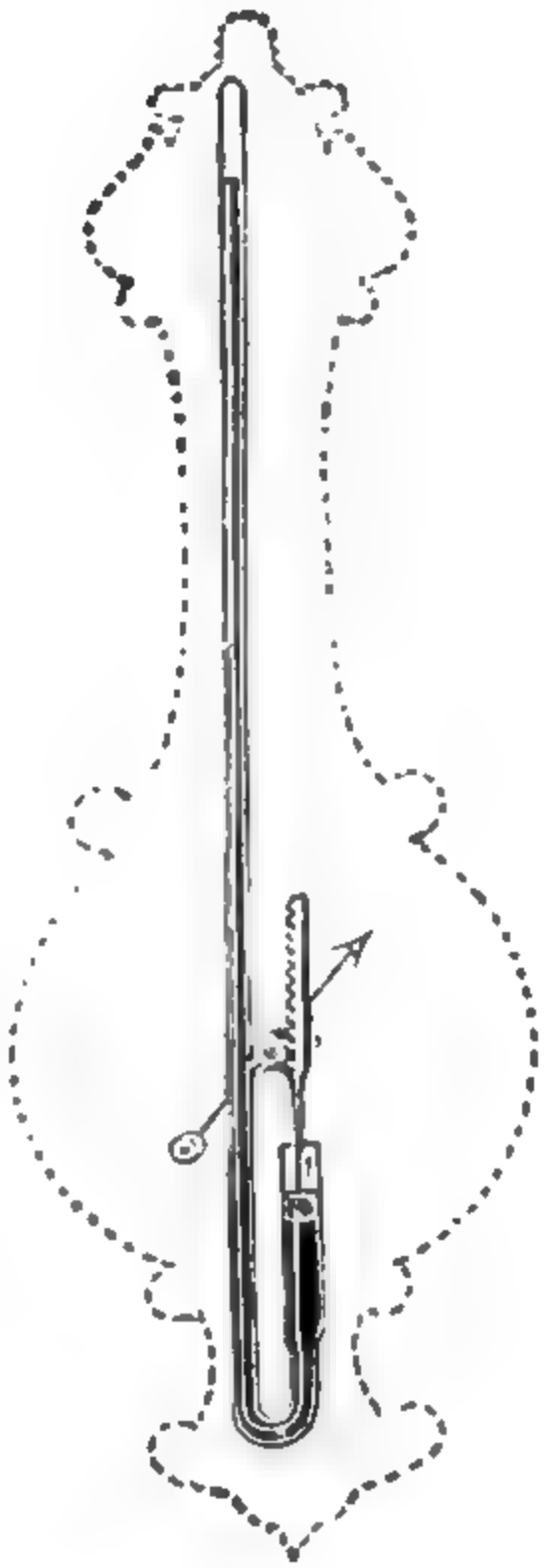


Fig. 182

is exposed to the atmospheric pressure. The difference of mercury levels in the two limbs gives the barometric height.

To protect the mercury surface at C, the limb CD is very nearly closed, leaving a very small opening at D through which communication with the extrenal atmosphere is maintained. To prevent the entrance of air into the limb AB when the instrument is inclined, the two limbs are joined by a U-tube of very narrow bore, as shown in the figure. The instrument is fixed on a wooden board and a scale is attached to the board against each of the two limbs to read the difference of levels of mercury.

The **weather glass** or a *household barometer* (Fig. 181) is a form of Siphon barometer. In the shorter limb, there is a float on the mercury surface, which moves up or down with the surface. This float in connected through a rack and pinion arrangement to a wheel, to the centre of which a pointer is attached (Fig. 182). The pointer moves over a dial mounted in front of the tube. The dial is graduated in

inches and is also marked with indications such as *fair*, *variable*, *rain*, *stormy* etc. The position of the pointer therefore, gives the atmospheric pressure in inches and the probable state of the weather in immediate future. Hence this instrument is termed a *Weather glass*.

Aneroid Barometer.—In this barometer no liquid is used (*a* without ; *neros*, liquid). In the present form, it is sufficiently accurate and at the same time very light and easily portable. It consists of a shallow cylindrical metal chamber C (Fig. 183), which is evacuated as completely as possible and is hermetically closed with a thin elastic metallic diaphragm. This is corrugated in circles to make it readily yeild to the changes of atmospheric pressure. Any variation of pressure causes a deformation, of the corrugated top, proportional to the change of pressure. The deformation, which is always small, is magnified by a delicate system of levers and is ultimately transmitted to a pointer at the top moving over a circular scale The scale is previously graduated in comparison with a standard mercury barometer, so that the pressure can be directly read on it in centimetres or inches. To prevent the instrument form being permanently deformed due to a sudden rise of pressure, the lid is supported on a strong still spring which presses against the lever L acting about a fulcrum. Words *rain*, *fair*, *storm*, *change*, *dry*, *etc.* are written on the dial and the pointer at any position would *correctly* indicate the corresponding state of the weather, provided the instrument is kept at approximately the same level for which it is meant to be used. A complete picture of the instrument with its outer casing and dial is shown in fig. 184.

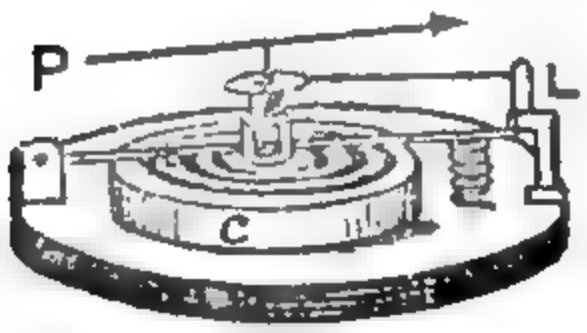


Fig. 183

The aneroid barometer is lighter, more easily portable and less liable to injury than a mercurial barometer. An aneroid barometer may also be provided with a scale to indicate directly height in feet or in metres. Such barometers, which may be as small as watches and are used by mountaineers and aviators to record the heights attained, are called **altimeters.**

Barograph.—A barograph is an instrument for automatically recording on a chart a continuous variation of the atmospheric pressure (Fig. 185). It consists of an aneroid barometer C which is kept well covered in a glass box. The top of the barometer is connected through a system of levers L and S. Any change in the atmospheric pressure is recorded by the end of the lever S rising up or down. A recorder is attached to the end of a long lever and its point presses against the drum over which a graph paper is wound. The drum is slowly rotated by a clock-work arrangement and revolves once a week. The pressure variation is recorded on the graph paper as a continuous wavy line.

Advantages of Mercury as a Barometric Substance.—The *ad-*

vantage of mercury as a barometric substance will be apparent from the following facts :—

(1) On account of the high specific gravity of mercury, a comparatively short column of it, about 30 inches, balances the atmospheric pressure.

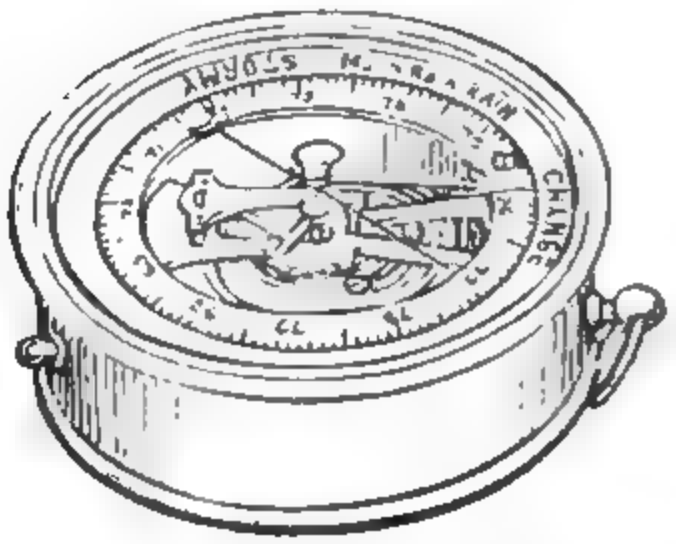


Fig. 184

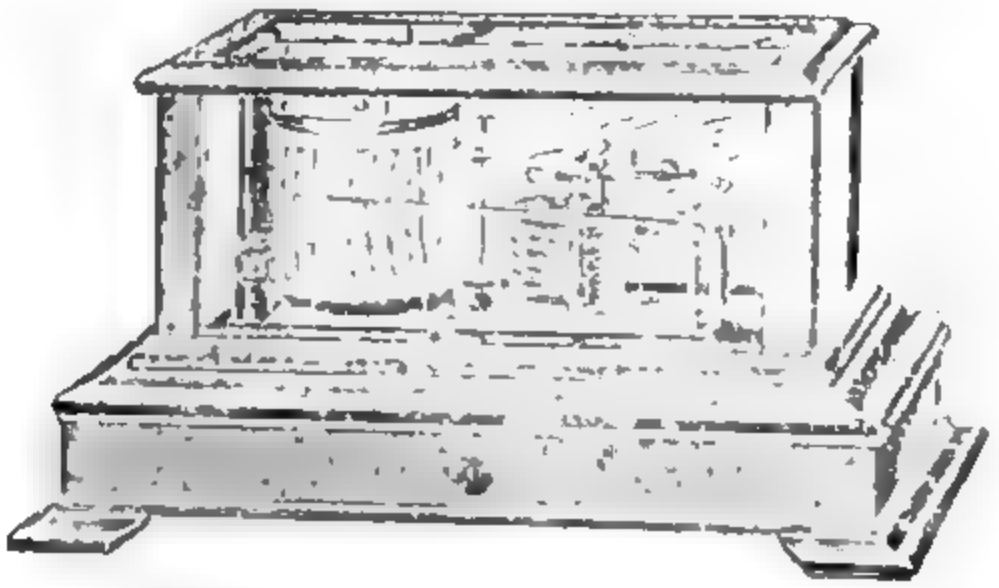


Fig. 185

(2) In the Torricellian vacuum, there is very little mercury vapour, so the pressure due to mercury vapour present is negligible. On the other hand water gives off vapour even at the ordinary temperature, and which will exert appreciable pressure on the liquid column and so the observed height would be less than the true height. Glycerine vapour has, of course, very low pressure at ordinary temperatures : but as glycerine readily absorbs moisture from the atmosphere, its density changes and also this moisture is given off by the liquid within the tube.

(3) Mercury when pure does not wet glass.

***132. Use of a Barometer.**—A barometer may be used either for the measurement of atmospheric pressure or for the determination of height.

Determination of Height.—At the sea level, mercury barometer stands at about 76 cm. On ascending through the atmosphere, a certain height of the air column is left out, so the atmospheric pressure and consequently the length of barometric column decrease. Let the barometer readings at lower and upper stations be h_1 and h_2, respectively.

Vertical height between the two stations. $= H$ cm.

Average density of air between the two stations $= \rho$ gm./c.c.

So $H \times \rho \times g = (h_1 - h_2) \times 13{\cdot}6 \times g$ whence $H = \frac{h_1 - h_2}{\rho} \times 13{\cdot}6$ cm.

Taking the average density of air near the surface of the earth, it is found that for an altitude difference of 107·7 metres, the change of barometric height is 1 cm. This corresponds to a change of pressure of 1 inch for an altitude difference of 900 ft. But as air is highly compressible and as the temperature decreases in the upper regions,

the density of air gets continuously less as we go up. So the atmospheric pressure and hence the barometric readings do not fall in proportion to the height. A rigorous calculation of the *determination of height* is given in the part on Heat (Art. 81).

The height of a place above the sea-level *in metres* can be directly determined from the formula,

$$H = K(\log_{10} h_1 - \log_{10} h_2) \quad \ldots(132,1)$$

where h_1 and h_2 are the atmospheric pressures *in centimetres* at the sea-level and at the required altitude and K is a numerical constant equal to 18900 nearly.

Forecasting of Weather.—It is found that the atmospheric pressure at a place varies at different hours of the day, as also from day to day. The variations may be due to changes in the density of air consequent upon the changes of temperature and on the amount of water vapour in the atmosphere. Hence the barometer readings would give us some idea as to the possible condition of the weather at any time and also in near future. Water vapour is lighter than air, its density being $\frac{5}{8}$ of that of dry air. So when the proportion of water vapour in the atmosphere becomes large, the density and hence the pressure of the atmosphere decrease and so the barometer reading falls. A *considerable fall* in the barometer, therefore, indicates that ***rain*** is imminent. If there is a sudden decrease of pressure at the place, air from surrounding regions would rush to the place. A *sudden fall* in the barometer therefore predicts an *immediate* storm. *A slow fall* for serveral days, implies *continued bad weather*. On the other hand, a ***sharp rise*** indicate a fine weather but not lasting. A *steady rise* indicates a *continued fine weather* for sometime. These forecasts are only approximate.

Weather Charts containing informations about pressure, temperature and humidity of atmosphere, strength and direction of the wind, the state of the sky, etc are daily published by Meteorological Offices. Air pressures in millibars are shown on the charts by continuous lines, called isobars, each of which passes through places of equal barometric pressure at a certain time. The relative position of these lines indicate the state of expected weather. When the isobars are near, the variation in the pressure is rapid. The regions of low pressure are called cyclones and those of high pressure are called **anticyclones.** The instrument by means of which the strength and the direction of wind can be measured and recorded is known as **anemometer.**

133. **Extent of the Atmosphere.**—The air at the surface of the earth would tend to expand indefinitely in space. But at a certain height, this tendency is expected to be balanced by the action of gravity and so the atmosphere may be taken to extend to a limited height. But as the density of air gradually decreases it cannot be accurately said how far air extends into space.

If the atmosphere be taken to be *homogeneous i.e.*, to have a constant density throughout, we can calculate the extent of the atmosphere which is called the **height of the homogeneous atmosphere.** If

this height be H cm., then the weight of air H cm. in height and of unit cross-section must be equal to the weight of a column of mercury 76 cm. in height and also of unit cross-section. The density of dry air at sea-level is 0·001293 gm. per c. c. ; taking the density of air throughout to be equal to this, $H \times 0{\cdot}001293 \times g = 76 \times 13{\cdot}6 \times g$.

whence $H = \frac{76 \times 13{\cdot}6}{0{\cdot}001293} = 801200$ cm. or 5 miles nearly.

But air in a rarefied condition is found to exist at an altitude much higher than this limit. In 1862, a height of about 7 miles was attained by Glaisher and Coxwell in a balloon. On may 4, 1937 ; Captain Gray took arrangement for oxygen and rose without any troubles to a height of 8 miles.

On the 27th May, 1931, Prof. Piccard of Belgium rose to a height of about 10 miles On September 30, 1930, the Soviet Balloon "U.S.S R," rose into the stratosphere to an altitude of 19000 meters or about $11\frac{7}{8}$ miles. On August 20, 1934, Professor Max Cosyns started from Brussels in a balloon on a stratosphere ascent and reached a height of about 9·9 miles. On the 11th November, 1937 a stratosphere expedition was undertaken above Dakota in America under the National Geography Society by Capt. A. W. Stevens and Capt. Orville Anderson, who rose in a balloon to a height of 72395 ft. or nearly 13·7 miles.

In May, 1937, an Italian pilot rose in an aeroplane to a height of 51,363 ft. or about 9·73 miles. On the 30th June, 1937, Flight-Lieut. M. J. Adams formerly in service on the North-Western Frontiers of India, attained by aeroplane a height of 53,937 ft. or about $10\frac{1}{8}$ miles. In July 1937, Eugenia Suit, an woman engineer of Moscow reached a height of 24,000 ft. into the substratosphere. In August, 1951, an American test pilot Bill Bridgeman rose in a jet plane to a height of 79,494 ft. An unmanned balloon carrying self recording instruments has gone up to 21·8 miles, and their is evidence of sufficient air at a hight of even 45 miles to reflect sun's rays at sunset. Reflection of radio waves has revealed that traces of atmosphere extend to even 200 miles and beyond.

Difficulties of High Ascent.—It is to be mentioned here that great difficulties are experienced in a very high ascent, as in the *Everest Expedition*, for the following among other reasons :—

(*i*) *The density of Air.*—The lower density causes difficulty of breathing as a person will have to make a greater number of inhalations per minute to take the requisite supply of oxygen. With artificial inhalation of oxygen, this difficulty may be minimised.

(*ii*) *The low atmospheric pressure.*—Owing to this the pressure within our body will be in excess ; this would cause a rush of blood toward the nose, ears and eyes, which would even sometimes bleed. Further, the muscles shall have to put forth greater force in supporting the limbs failing which the limbs will feel to be stiff and without any strength.

(*iii*) *The low temperature.*—The temperature of the atmosphere falls as we go up. At a height of 5 miles, the temperature of the air is found to fall to −20°C, and at a height of 7 miles, it falls to about −58°C. Unless special precautions be taken, the aeronaut may be frozen to death.

134. Isothermal Volume changes of Gases.—It has already been mentioned that gases are highly compressible so that the volume of an enclosed mass of a gas readily changes with changes of pressure. The relation between the pressure and the volume of a given mass of a gas at constant temperature was first discovered in 1662 from experimental results by Robert Boyle, and also independently in 1676 by Mariotte in France. The relation is enunciated in a law, called the Boyle's Law in England and the Mariotte's Law in the Continent.

Boyle's Law.—*The volume of a given mass of any gas at constant temperature varies inversely as the pressure to which it is subjected.* Thus if v is the volume of a mass of a gas at a pressure p, then at constant temperature—

$$v \propto \frac{1}{p}, \text{ or } v = k\frac{1}{p} \text{ where } k \text{ is a constant}$$

whence $pv = k$, a constant ...(134,1)

Hence if a given mass of a gas at a constant temperature has volumes v_1, v_2, v_3, etc., under pressures p_1, p_2, p_3, respectively then we have $p_1 v_1 = p_2 v_2 = p_3 v_3 =$ a constant. This constant depends on the nature, mass and temperature of the gas. The equation (134,1) is known as the **isothermal equation of state.** For example, if the volume occupied by a gas is 20 c.c. under a normal pressure of 76 cm., then its volume would be reduced to 10 c c. when the pressure over it is increased to 152 cm.

Pressure and Density.—The relation given in Boyle's law can also be expressed in terms of pressure and density of the gas. Thus let a certain mass m of the gas at a constant temperature possess a volume v_1 and density ρ_1 at a pressure p_1, and a volume v_2 and density ρ_2 when the pressure is p_2 ; then since the mass remains constant.

$$\therefore m = v_1\rho_1 = v_2\rho_2 \text{ whence } \frac{\rho_1}{\rho_2} = \frac{v_2}{v_1} \quad ...(1)$$

Again from Boyle's law, $p_1 v_1 = p_2 v_2$ or $\frac{p_1}{p_2} = \frac{v_2}{v_1}$ ...(2)

$\therefore$ From (1) & (2), $\frac{p_2}{\rho_2} = \frac{p_1}{\rho_1}$ whence $\frac{p}{\rho}$ is constant. (134,2)

135. Verification of Boyle's Law.—The apparatus generally used for this purpose consists of a long glass tube AE (Fig. 186) of uniform bore and closed at the top permanently by being sealed off or by means of two air tight glass stoppers, as shown in the figure. This tube is connected at its lower end by a piece of long flexible thick-walled rubber tubing to a wider glass tube DB open at both ends. Both

the tubes can slide along vertical uprights and can be fixed at any desired position by clamps X and Y. The lower parts of both the glass tubes and the whole of the rubber tubing T are filled with pure dry mercury, while the upper part of A contains some amount of *dry* air or any other gas. There is a vertical scale S attached to the stand, which is graduated on each side in centimetres. The positions of the levels of the mercury in the tubes can be read from the scale S. As the tube AE is of uniform bore, the volume of the enclosed air proportional to the length of the enclosed air column within the tube. For accurate observation, the tube is graduated in fractions of cubic centimetres.

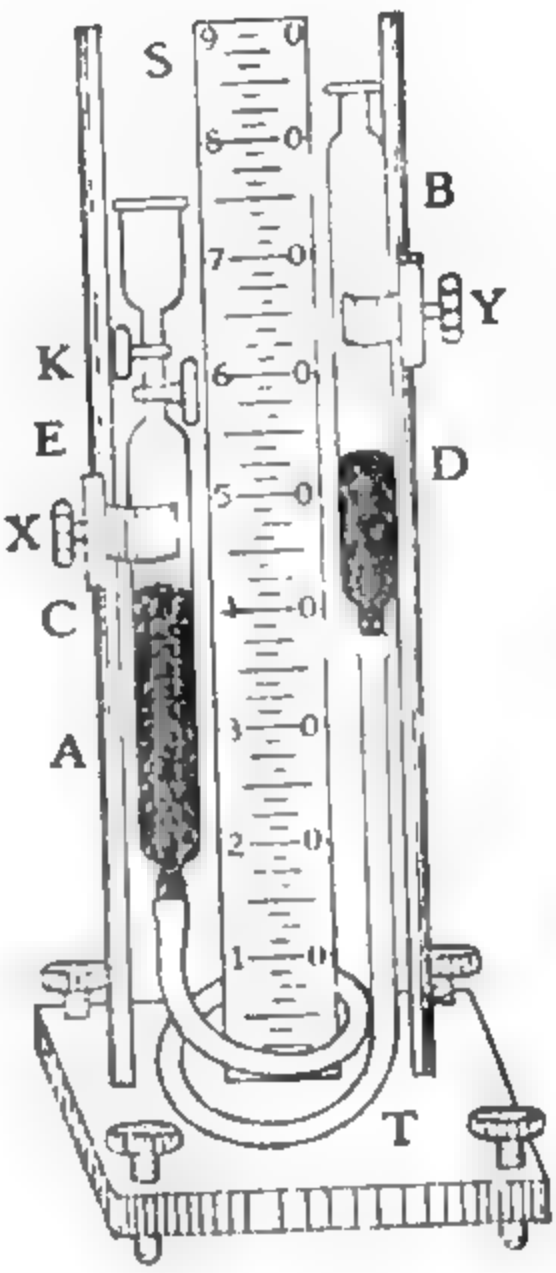


Fig. 186

For Higher Pressures.—Raise the open end, such that the mercury level in B is higher than A. Fix it up some where and wait for some time to allow the compressed gas to attain room temperature. The *volume* of the air in A is evidently proportional to the difference of the readings of the closed top E of the tube and mercury level at C. The *pressure* of the air in A is obtained by *adding* the difference of the mercury level D and C in the two tubes and the barometric height at the time. Take a few such readings of volumes and pressures with the mercury level in B higher in each case. Here *the pressure is always higher than the atmospheric pressure.*

For Lower Pressures.—Slide B down till the level of mercury in the two tubes is the same again. Now lower B step by step. The *volume* of the air in A gradually increases and the level of mercury in B is always lower than that in A The *pressure* of the air in A is in each case obtained by *subtracting* the difference of the mercury levels in the two tubes from the barometric height. Here *the pressure of the enclosed air is always less than the atmospheric pressure.* Note the volume of the air and the corresponding pressure at each step.

Find the product of the volume (v) and the corresponding pressure (p) in all cases. It is found that the *product* $p \times v$ *is approximately constant.*

Precautions.—The mercury as well as the air enclosed in A must be perfectly dry. To get dry air the stop-cocks at the mouth of the tube A may be opened and some fused calcium chloride is kept in the funnel tube. The tube B is then raised so that mercury level in A touches the lower stop-cock. Thus the air inside A is all forced out. Then B is slowly lowered, whereby mercury level in A coming down sucks atmospheric air through calcium chloride making it dry. Thus any required volume of the gas may be taken in at the start.

To account for any variation in the atmospheric pressure during the experiment

the mean of the barometer readings at the beginning and at the end of the experiment should be taken.

It is sometimes a difficult task to make glass stoppers absolutely air tight in a glass tube. If there is any leakage from the closed end tube, during experiment, then the experiment is spoilt. To ensure the safety, the upper end of the tube A is made permanently sealed.

The alteration in the volume of the enclosed gas must be made very slowly so as to avoid any change in temperature due to the sudden compression or expansion.

The relation between the pressure p and the volume v of a gas can be represented graphically. If pressure be represented as abscissa and the corresponding volume as ordinate a curve as shown in fig. 187 will be obtained. The curve so obtained is a branch of, a *rectangular hyperbola*. If the pressure p is plotted against reciprocal of corresponding volume $1/v$, the resulting graph is a *straight line*.

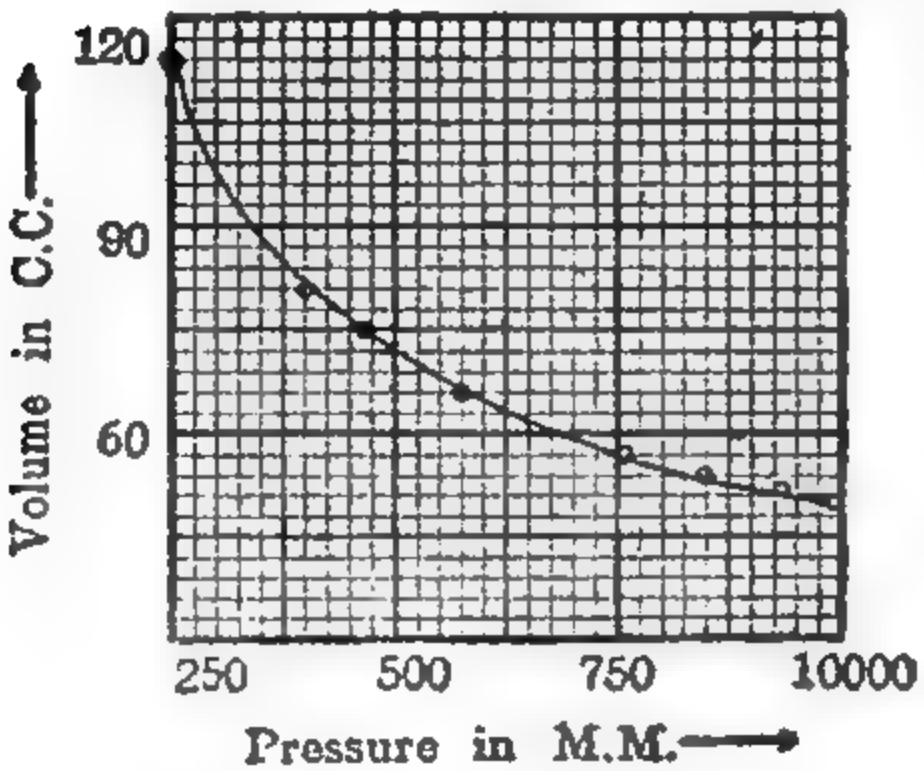


Fig. 187—Isothermal curve

The change of volume of a gas at *constant temperature* is known as *isothermal change* (Gk. *Isos.* same ; *thermos*, heat) and the curve representing the change in the volume of a gas with the change of pressure at any constant temperature is called an **isothermal curve** or simply an *isothermal* at that temperature.

Examples :—

1. A vertical cylinder of sectional area 10 sq. cm. is fitted with an airtight piston weighing 30 kgm. The cylinder contains some air and the height of the piston from the bottom of the cylinder is 28 cm. If the barometer reads 76 cm. at the time, find the total pressure inside. If an extra load of 30 kgm. be placed on the piston, find how far it will be depressed. ($g = 980$ cm. per sec.^{2}).

A. Pressure per unit area due to the weight of the piston.

$$= \frac{30 \times 1000 \times 980}{10} = 2{\cdot}94 \times 10^6 \text{ dynes}$$

Atmospheric pressure per unit area $= 76 \times 13{\cdot}6 \times 980 = 1{\cdot}01 \times 10^6$ dynes

$\therefore$ Total pressure inside $= (2{\cdot}94 \times 10^6 + 1{\cdot}01 \times 10^6) = 3{\cdot}95 \times 10^6$ dynes.

The additional load produces a pressure of $2{\cdot}94 \times 10^6$ dynes per sq. cm.

$\therefore$ Total pressure $= (3{\cdot}95 + 2{\cdot}94) \times 10^6$ dynes $= 6{\cdot}89 \times 10^6$ dynes.

Let the piston be depressed through x cm.

Then $(28 - x) \times 6{\cdot}89 \times 10^6 = 28 \times 3{\cdot}95 \times 10^6$ whence $x = 11{\cdot}9$ cm. nearly.

2. Some air is introduced into the space above mercury in a barometer tube. The height of mercury column is 20 in. and the space above it is 4 in. long. When the tube is pushed down into the cistern so that the space is reduced to 2 in. the height of mercury column becomes 28 in.. What will be the height of mercury column in air-free barometer ? **(All. U.—1931)**

A. Let the height of the mercury column in air-free barometer = P in. So intial pressure of enclosed air = (P − 29) in. and its vol. is proportional to 4 in. Final pressure, of the enclosed air = (P − 28) in. and its vol. is proportional to 2 in.

∴ From Boyle's law $(P - 29) \times 4 = (P - 28) \times 2$ whence $P = 30$ inches.

3. A tube 6 ft. in length closed at one end is half filled with mercury and it is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube? (C. U.—1931)

A. Let the height of mercury column in the tube when inverted = h in. Initially the air occupies (3×12) in. of the tube and its pressure = 30 in. Final volume of the air is $(72 - h)$ in. and the pressure = $(30 - h)$ in.

∴ $(72 - h)\ (30 - h) = 36 \times 30$ whence $h = 12$ in. or 90 in.

But as the length of the mercury column cannot be greater than the tube itself so the second value cannot be accepted. Hence the required height is 1 ft.

4. A faulty barometer containing air and of 1 sq. cm. cross-section reads 77 cm. when the true reading is 78 cm. and 70 cm. when the true reading is 70·8. Find the volume of air present at a pressure of 76 cm. (And. U.—1952)

A. Let l cm. be the length of the barometer tube measured from the mercury level of the cistern. In the first case air pressure within the borometer is $78 - 77 =$ 1 cm. of mercury and volume of air is $(l - 77)$ c.c. In the second case air pressure is $70{\cdot}8 - 70 = 0{\cdot}8$ cm. and the volume is $(l - 70)$ c.c. Applying Boyle's law,
$(l - 77) \times 1 = (l - 70) \times {\cdot}8$ whence $l = 105$ cm.

Thus the volume of air is $105 - 77 = 28$ c.c. under a pressure of 1 cm. If v be the volume of this air at a pressure of 76 cm., then from Boyle's law,

$28 \times 1 = 76 \times v$ whence $= 0{\cdot}37$ c.c.

136. Archimedes' Principle applied to Gases.—Just as a body immersed in a liquid experiences a vertical upward thrust equal to the weight of the liquid displaced by the body, so also a body is subjected to an upward thrust by the gas surrounding it, which is equal to the weight of the gas displaced. The upward force exerted on any body by the surrounding air is called the *buoyance of the air*. Hence the weight of a body in vacuum, which is its *true weight*, is greater than its apparent weight in air by the weight of the displaced air.

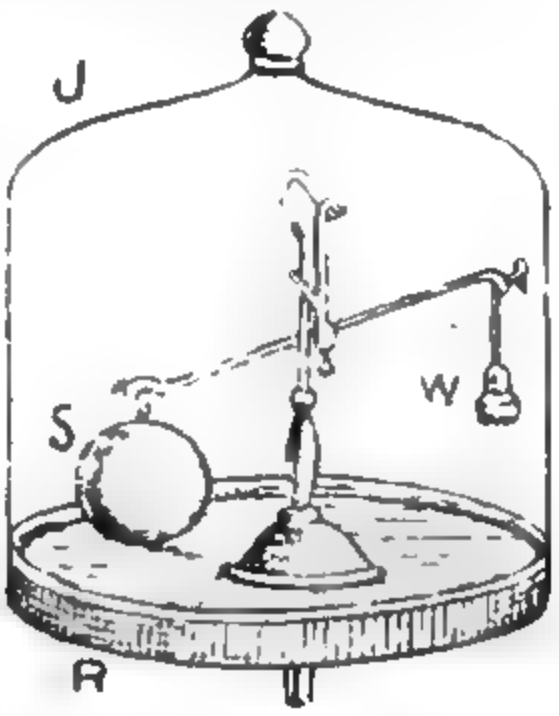


Fig. 188

This is clearly demonstrated by a baroscope, which consists of a small balance from the arms of which a small brass weight and a large cork sphere are suspended so as to counterpoise each other when in air. Therefore their apparent masses are equal. The whole is then placed under the receiver of an air pump. If now the air is pumped out, the cork sphere is seen to sink down becoming actually heavier (Fig. 188). When both are suspended in air the cork sphere owing to its larger volume displaces greater volume of air than the brass weight and so

experiences a larger upward thrust. But as the apparent masses of the two are the same, the true mass of the cork sphere must be greater than that of the brass weight. In the following calculation we use the word 'weight' to mean the mass of a body.

***Buoyancy Correction in Weighing.**—The fundamental assumption in a buoyancy correction in weighing is that the apparent mass of the body to be weighed is equal to the apparent mass of the 'weights' placed to counterpoise it.

Let the *true* weight of the body = W
true weight of the counterpoising weights = W′
density of the material of the body = ρ
density of the material of the weights = ρ'
density of air at the time = d

Then for equilibrium, $W - \frac{W}{\rho} d = W' - \frac{W'}{\rho'} d$

whence $$W = \frac{W'\left(1 - \frac{d}{\rho'}\right)}{\left(1 - \frac{d}{\rho}\right)} = W'\left(1 - \frac{d}{\rho'}\right)\left(1 - \frac{d}{\rho}\right)^{-1}$$

$$= W'\left(1 + \frac{d}{\rho} - \frac{d}{\rho'}\right) \quad \ldots \quad (136,1)$$

since d is small in comparison to ρ and ρ'

Example :—

The weight of a body in air of density 0·00125 gm. per c.c. is found to be 399·3 gm. If the density of the material of the body be 2·5 gm. per c.c. find the weight of the body in air of density ·00129 gm. per c.c. and also the true weight.

A. Let the true weight of the body = W gm.

Then the volume of the body = (W ÷ 2·5) = volume of the air displaced.

$\therefore$ $W - \frac{W \times 0{\cdot}00125}{2{\cdot}5} = 399{\cdot}8$ whence W = 400 gm.

Hence the volume of the body = (400 ÷ 2·5) = 160 c.c.

$\therefore$ Its wt. in air of density 0·00129 = (400 − 160 × 0·00129) = 399·793 gm.

137. Balloon.—The buoyancy of the air is utilised in the ascent of a balloon, an airship or any other *air floating* machine. If the weight of the machine with its accessories be less than that of the displaced air the machine will rise up in the air.

A balloon is essentially a large bag of silk or some other light material, the outside of which is thoroughly varnished to make it air-tight (Fig. 189). It is filled with some gas lighter than air *e.g.*, hydrogen, helium or coal gas. The balloon is provided with.—

(*i*) *A light car* to carry passengers which is suspended by cords attached to a large net-work of ropes covering the balloon.

(*ii*) *A safety valve* at the top ; by opening it with a cord, the contained gas may be allowed to escape and the balloon may be lowered at will.

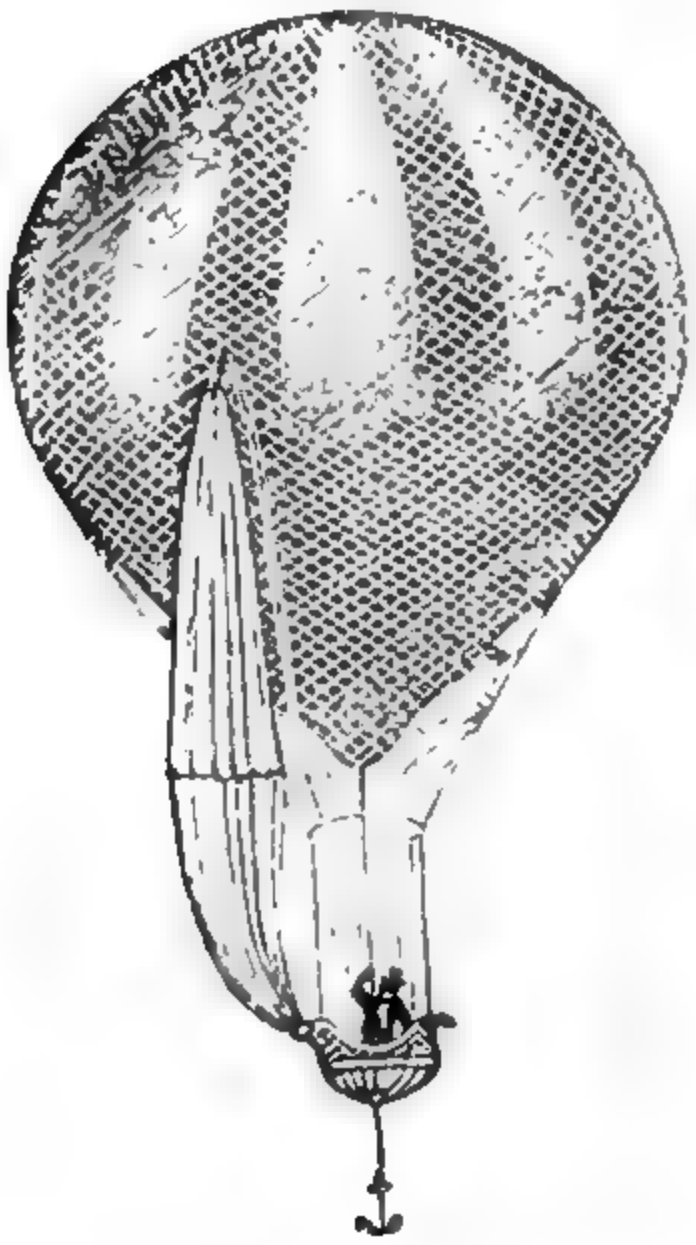


Fig. 189

(*iii*) Some bags of sand, which being emptied will make the balloon ascend further or descend less rapidly.

(*iv*) *A parachute* which is a large umbrella-like contrivance with cords hanging from its circumference and carrying a light frame at the other end (Fig. 190).

Fig. 190

The aeronaut may leave the balloon with the parachute, which then opens out like an umbrella due to the resistance of the air below and makes the descent slow. There is a small opening at its top through which the air slowly escapes whereby it descends upright.

Lifting Power of Balloon.—If the weight of the gas in the balloon be W_1 that of the balloon and other accompaniments be W_2 and the weight of the air displaced by the inflated balloon be W then the balloon will ascend if $W > (W_1 + W_2)$.

If d be the density of air outside the balloon ρ the density of gas enclosed in it having a volume V then taking the volume of the balloon to be practically equal to that of the gas the mass of the enclosed gas is equal to $V\rho$. The buoyancy due to the displaced air is Vd, which should be greater. Therefore the lifting power is $V\ (d - \rho)$. A part of this power is utilised in overcoming the weight of the balloon and the cargo it carries.

A balloon is not fully inflated at the start. As it rises up, the density and so the pressure of the surrounding air decreases. The volume of the balloon and so that of the displaced air increase almost in the same proportion upto some height. The weight of the displaced air, therefore practically remains the same and so also the lifting power of the balloon. But as the weight of the balloon and other parts practically remains the same, weight of the displaced air ultimately becomes equal to the weight of the balloon and its contents, when the balloon will not rise any more. If the sand bags are now emptied, the balloon will rise again. Coal gas being cheap and easily procurable, is generally used in a balloon.

Air Ship.—It is a modification of the balloon with arrangements to steer the machine in any direction with changable speed. After being lifted up in the air, it can be driven by *propellers* worked by motors and it can be steered in any direction by a suitable *rudder* ; zeppelin is an example. Hydrogen being extremely light is used in the air-ships : but now-a-days helium is preferred as it is non-inflam-

mable. It is due to the hydrogen taking fire that the German air liner the *Hindenburg*, the biggest of its type and used in trans-Atlantic flight, was destroyed on 6th May, 1937.

Examples :—

1. A litre of hydrogen and a litre of air weigh about 0·09 gramme and 1·3 grammes respectively at a certain temperature (t) and pressure (p). What would be the capacity of a balloon, weighing 10 kilogrammes, which just floats when filled with hydrogen having the same pressure (p) and the same temperature (t) as the air.

A. Let the capacity of the balloon $= V$ litres.

$\therefore$ Mass of hydrogen enclosed in the balloon $= V \times 0{\cdot}09$ gm.

And mass of air displaced by the balloon $= V \times 1{\cdot}3$ gm.

Now wt. of the balloon + wt. of hydrogen enclosed = wt. of air displaced.

$\therefore \quad 10000 + V \times 0{\cdot}09 = V \times 1{\cdot}3 \qquad$ whence $\qquad V = 8264{\cdot}46$ litres nearly.

2. A balloon, weighing 150 kgm., contains 1000 cu. metre of hydrogen and is surrounded by air. Calculate the additional weight it can lift. Also explain why the balloon will float in stable equilibrium at a constant altitude. Density of hydrogen 0·00009 gm./c.c., and density of air = 0·00129 gm./c.c. (Gau. U.—1952)

A. The mass of the balloon and the contained gas = 150 kgm. + mass of gas.

Now mass of 1000 cu. metre of hydrogen

$= 1000 \times 10^6 \times {\cdot}00009$ gm. $= 10^6 \times {\cdot}00009$ kgm. $= 90$ kgm.

$\therefore$ Total mass of balloon and hydrogen $= 150 + 90 = 240$ kgm.

Mass of air displaced $= 1000 \times 10^6 \times {\cdot}00129$ gm. $= 1290$ kgm.

Hence from Archimedes' principle force of buoyancy $= 1290 - 240 = 1050$ kgm. = its lifting power at the place where density of air is ·00129 gm./c.c. At a higher altitude the density of air is less, so that balloon can not ascend with the given load. Again at a lower altitude the density of air is higher than ·00129, so that the balloon with the given load could be forced upwards due to greater buoyancy. So it takes up a constant height determined by the required density of air (·00129 gm./c.c.).

3. A balloon is filled with a gas whose density is $\frac{1}{18}$ that of air at N. T. P. Compare the total lifting power of the balloon in air at 76 cm. to that in air at 74 cm, the temperature remaining always at 0°C and the volume of the balloon being supposed to be the same.

A. Let the density of air at N. T. P. $= d$; then that of enclosed gas $= \frac{d}{18}$

Again density of air at 0°C, and 74 cm. $= d \times \frac{74}{76}$

Let the volume of the balloon be V c.c. and the total lifting power in the two cases be W_1 and W_2 respectively.

Then $W_1 + \frac{Vd}{18} = Vd$ or $W_1 = Vd\ (1 - \frac{1}{18})$ ... 1)

Again $W_2 + \frac{Vd}{18} = Vd \times \frac{74}{76}$ or $W_2 = Vd\ (\frac{74}{76} - \frac{1}{18}) = \frac{443}{684}\ Vd$ ... (2)

From (1) & (2), $W_1 : W_2 = 456 : 443$.

4. A cylinder filled with air at atmospheric pressure is lowered in water with its mouth downwards till it is $\frac{1}{3}$ full of water. To what farther depth should it be lowered till it is $\frac{2}{3}$ full of water? Density of mercury is 13·6 and barometric height is 76 cm. (U. P. B.—1953)

A. Let l be the length of the cylinder in cm. and α its area of cross-section in sq. cm. Then the volume of air is $l\alpha$ c.c. under the atmospheric pressure of $76 \times 13{\cdot}6 \times g$ dynes/sq. cm.

Let the depth of water be h cm. measured from the top surface of the cylinder at which the cylinder is $\frac{1}{3}$ full of water. The volume of air is then $\frac{2}{3} l\alpha$ c.c. and the total pressure within the cylinder is $76 \times 13{\cdot}6 \times g + (h + \frac{2}{3} l) \times g$ dynes/sq.cm.

Now, if temperature does not change, then according to Boyles' law

$$l\alpha \times 76 \times 13{\cdot}6 \times g = \tfrac{2}{3} l\alpha \times [76 \times 13{\cdot}6 \times g + (h + \tfrac{2}{3} l) \times g]$$

$$\text{or, } 76 \times 13{\cdot}6 = 2h + \tfrac{1}{3} l \qquad \text{or, } h = \frac{76 \times 13{\cdot}6}{2} \text{ cm.}$$

neglecting the length of the cylinder as compared to h.

When the cylinder is farther lowered by h_1 cm. where it is $\frac{2}{3}$ full of water the volume of contained air is $\frac{1}{3} l\alpha$ c.c. and the total pressure is $76 \times 13{\cdot}6 \times g + (h + h_1 + \frac{1}{3} l) \times g$ dynes/sq.cm. Then again applying Boyle's law,

$$l\alpha \times 76 \times 13{\cdot}6 \times g = \tfrac{1}{3} l\alpha \times [76 \times 13{\cdot}6 \times g + (h + h_1 + \tfrac{1}{3} l) \times g]$$

$$\text{or, } 3 \times 76 \times 13{\cdot}6 = 76 \times 13{\cdot}6 + h + h_1 \text{ neglecting } l$$

$$\text{or, } 2 \times 76 \times 13{\cdot}6 = h + h_1 = \frac{76 \times 13{\cdot}6}{2} + h_1$$

$$\text{or, } \frac{3 \times 76 \times 13{\cdot}6}{2} = h_1 = 3830{\cdot}4 \text{ cm.}$$

ROBERT BOYLE (1627 –1691)

Robert Boyle, the fourteenth child of the Earl of Cock, was born at Lismore Castle, Ireland, on January, 25, 1627. Once in his boyhood, he was ill and a druggist served by mistake a poisonous medicine which was administered As an effect of the poison he very nearly lost his life. This accident led Boyle to read Chemistry.

After finishing his education in a London School, he spent six years of his life in France, Sweden and Germany for higher studies. He came back to London in 1644 and hired a house where a few people loving science used to meet and discuss scientific topics. This debating society was transformed by Charles II in 1662 into the Royal Society.

Having obtained an air pump in 1659, he began to study the properties of gases. His caution and thoroughness of investigation led him to discover the well-known law of gases. He also prepared alcohol from wood and acetone from lead and lime. He was the first scientist to make a systematic study of the elements by a method of chemical analysis. Although a scientist he was a devout follower of the Church

and he established in London the "Boyle Lectures" in propagating faith in Christianity. He died on December 30, 1691.

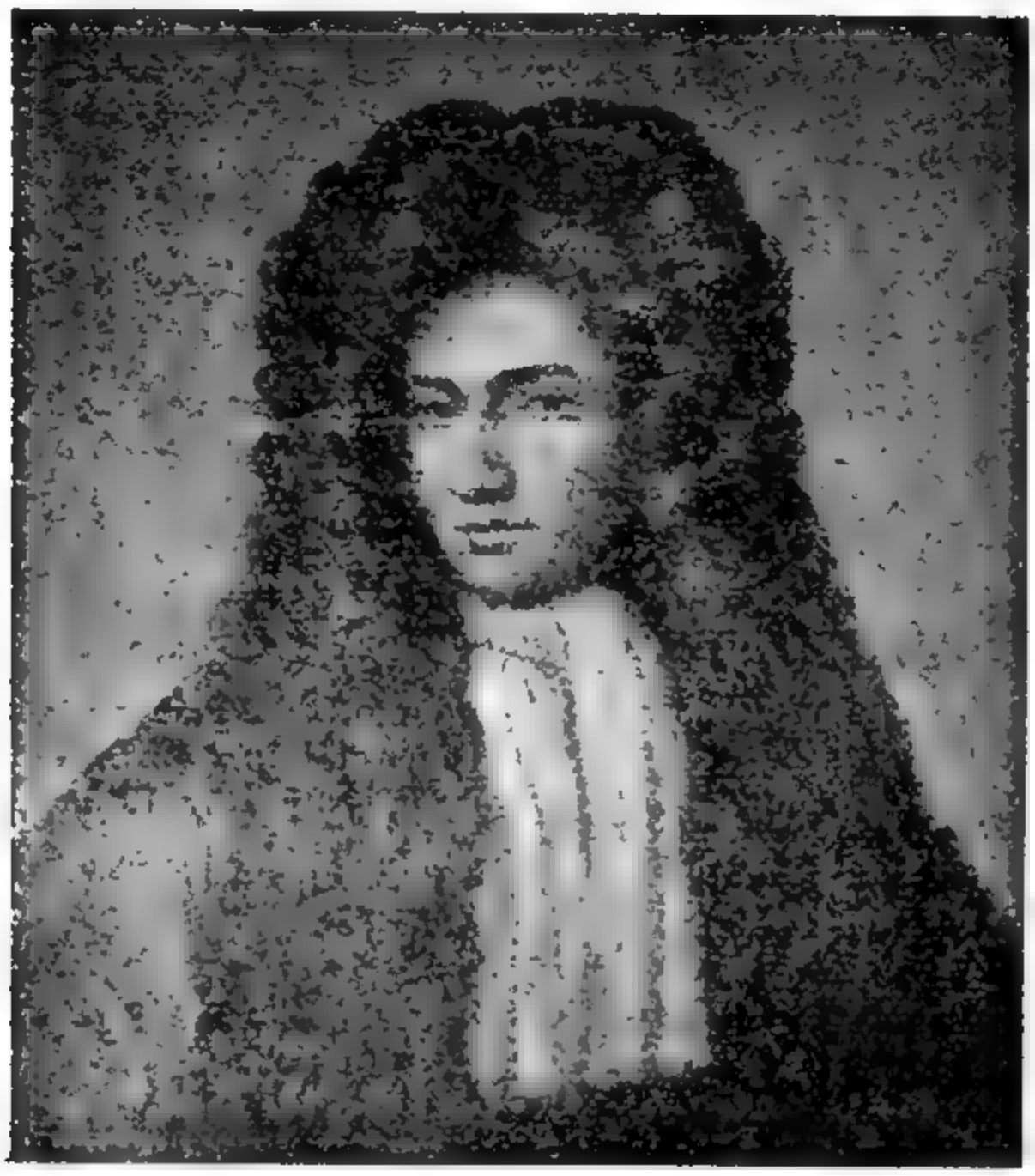


Fig. 191—Robert Boyle

Summary

Atmospheric pressure is the pressure on unit area caused by the weight of the entire column of air standing upon it.

The **existence of atmospheric pressure** can be demonstrated by various experiments of which the **Magdeburg hemispheres experiment** is one.

The **magnitude** of the atmospheric pressure is about 15 lbs. per sq. inch, as can be shown by **Torricelli's experiment.**

Barometers are instrument for measuring the amount of the atmospheric pressure at any time. Barometers can also be used for (*i*) *The determination of height* and (*ii*) *Forecasting of weather.* There are instruments of different types *e.g.*, (*i*) *Cistern barometer,* (*ii*) *Fortin's barometer,* (*iii*) *Siphon barometer,* (*iv*) *Aneroid barometer.*

Boyle's Law.—*The volume* (*v*) *of a given mass of any gas at constant temperature varies inversely as the pressure* (*p*) *to which it is subjected*; ***pv*** **is constant.** Archimedes' principle also applies to gases.

EXERCISES ON CHAPTER XI

Reference

1. Describe carefully what you understand by atmospheric pressure. Describe experiments to prove the existence of atmospheric pressure. (C. U.—1928; U. P. B.—1943) — Art. 127

2. Explain fully the meaning of the statement—the atmosphere exerts a pressure of 15 lbs. per square inch nearly. How would you verify the statement experimentally? (C. U.—1941) — Arts. 128 & 129

3. How will you show experimentally that the atmospheric pressure acts equally from all directions? Though the surface of the human body is large explain why we are not crushed by the pressure of the atmosphere. (Utkal. U.—1952) — Arts. 128 & 134

4. Express the normal pressure of air in absolute units. (Density of mercury = 13·6 and $g = 981$ cms. per sec. per sec.). (C. U.—1947; Del. U.—1943; And. U.—1952; Del. H. S.—1954) — Art. 129

A. 1013961·6 dynes per sq. cm.

5. In a barometric column, mercury stands at 30 inches above the level of the mercury in a trough. Express the atmospheric pressure in C. G. S. units, it being given that 1 inch = 2·54 cm. $g = 981$, sp. gr. of mercury = 13·6. — Art. 129

A. 1016629·9 dynes per sq. cm.

6. The readings of a barometer at the top and ground floor of a building are 29·905 and 29·949 inches respectively. The densities of mercury ond air are 13·58 and ·001293 gm. per c.c. respectively. Find the height of the building. (And. U.—1951) — Art. 182

A. 38·4 ft.

7. Describe a good form of a standard barometer. Can a tube of any diameter be used as a standard barometer tube? Why is a thermometer always attached to a barometer? (All. U.—1939) — Art. 180

8. Describe with a neat diagram the principle and construction of a barometer which you use in your laboratory. (Raj. U.—1953; Pat. U.—1952; Dac. U.—1942, '43; *Cf.* Anna. U.—1950; U. P. B.—1952; *Cf.* C. U.—1954; Utkal. U.—1953; P. U.—1952; *Cf.* And. U.—1951) — Art. 180

9. Give a brief description of Fortin's barometer and explain its adjustments. Calculate the atmospheric pressure in dynes per cm.^{2}, when the corrected barometric height is 750 mm. (given $g = 981$ and $\rho = 13{\cdot}6$). (Dac. U.—1931, '41; All. U.—1940; *Cf.* Nag. U.—1953; U. P. B.—1943) — Art. 180

A. 1000620.

10. Explain the principle of an Aneroid barometer. (Dac. U.—1942; R. P. B.—1945; '49) — Art. 181

11. How does a barometer forecast weather? (C. U.—1954; *Cf.* Dac. U.—1931) — Art. 182

12. Explain why it is necessary to correct the reading of a Fortin's barometer for temperature. Obtain an expression for this correction. (Anna. U.—1950; Pat. U.—1953; Nag. U.—1953) — Art. 180

Reference

Art. 130 13. A barometer has a brass scale which is correct at 60°F. The barometer reads 754 mm. at 40°F. Correct the reading of the barometer to 32°F. The co-efficient of linear expansion of brass is 0·00001 per °F. The co efficient of cubical expansion of mercury is 0·0001 per °F. (Pat. U.—1953)

A. 753·2 mm.

Art. 134 14. A bottle whose volume is 800 c.c. is sunk mouth downward below the surface of a pond. How far must it be sunk for 300 c.c. of water to run into the bottle? The height of the barometer at the surface is 76 cm. and the sp. gr. of mercury is 13·6. (Nag. U.—1950)

A. 6·201 metres.

Art. 134 15. The mercury in a barometer tube stands at 75 cm. and the space above it is 5 cm. in length. 2 c.c. of air at atmospheric pressure are admitted into the tube and the mercury now stands at 60 cm. Find the area of cross-section of the tube. (P. U.—1950)

A. ·5 sq. cm.

Art. 124 16. State Boyle's Law. How is the law verified experimentally? (Mad. U.—1951; Nag. U.—1950; Raj. U.—1949, '52; Del. H. S.—1950; E. P. U.—1952)

Arts. 131 & 135 17. State Boyle's Law and describe an experimental arrangement for verifying it for pressures greater than and less than one atmosphere. (Gau. U.—1952; C. U.—1941, '47, '49; U. P. B.—1944; *Cf.* 1953) Dac. U.—1941; P. U.—1950)

Art. 134 18. The volume of an air bubble increases ten-fold in rising from the bottom of a lake to its surface. If the barometric height is 30 inches what is the depth of the lake? Sp. gr. of mercury = 13·6. (Dac. U.—1941; Raj. U.—1952)

A. 306 ft.

Art. 134 19. At what depth in water will a bubble of air have the same density as water, given that water is incompressible and air obeys Boyle's Law for all pressures? Density of air at ordinary atmospheric pressure is 1·25 gm. per litre. (R. P. B.—1949)

A. 27200 ft. taking water barometer = 34 ft.

Art. 134 20. The length of the Torricellian vacuum in a barometer tube was 25 cm. when the reading of the mercury was 75 cm. When 10 c.c. of air at atmospheric pressure was admitted the reading was 50 cm. Calculate the diameter of the tube. (E. P. U.—1952)

A. ·87 cm.

Art. 134 21. A barometer indicates a pressure of 77 cm. The radius of the tube is 0·5 cm. and the height of the space above the mercury in the tube is 10 cm. A certain volume of air is now introduced into the tube and the barometer records a pressure of 75 cm. What will be the volume of the air at 76 cm.

A. 0·24 c.c.

Art. 134 22. A glass tube closed at one end is 72 cm. long. The inside is covered with a soluble pigment. In sounding the depth of the sea, the tube is lowered vertically below the sea (open end down) till it just touches the bottom. When taken out the pigment was found dissolved

Reference

to within 4 cm. of the top. If the average density of sea water is 1·02, find the depth of the sea at the place. Density of mercury = 13·6. Atmospheric pressure = 76 im. (U. P. B.—1944)

A. 172·88 metres.

23. The space above the mercury in a faulty barometer measures 12 cm. and the mercury column extends 72 cm. above the mercury in the cistern. On depressing the tube in the cistern, the mercury stands at 70 cm. and the space above measures 7·2 cm. What is the atmospheric pressure in centimetres of mercury? Art. 134

A. 75 cm.

24. Describe a Fortin's barometer and show how you will proceed to take a reading. If the barometer tube is lowered into a deep cistern of mercury what changes will be noticed in the length of the mercury column (*a*) if the space above mercury is perfectly vacuous, (*b*) if the space contains a little air and (*c*) if the space contains a little water vapour? Give reasons. Art. 134 (R. P. B.—1945)

25. Explain fully how the height of a barometer affords us a measure of the pressure of the atmosphere. What is the weight of a litre of air collected at the top of a mountain when the barometer stands at 50 cm.? (1 litre of air at a pressure of 76 cm. weighs 1·29 grams). Arts. 129 & 134

A. 0·8487 gm.

26. The height of barometer is 75 cm. of mercury, and the evacuated space over the mercury surface has a volume of 10 cc. One c.c. of air at atmospheric pressure is introduced into the evacuated space. What is the new reading of the barometer? (The cross-section of the tube is unity). Art. 134 (Gau. U.—1952)

A. 70 cm.

27. Mercury is poured into a barometer tube, till a volume of 15 c.c. is left, filled with air at atmospheric pressure. When the tube is inverted over a mercury cistern, the air occupies a volume of 25 c.c. the height of the mercury column being 30·2 cm. Find the atmospheric pressure at the time. Art. 134

A. 75·5 cm.

28. The density of air at N. T. P. is 0·00129 gm. per c.c. Find the alteration in the weight of 15 litres of air when the barometer falls from 76 cm. to 74 cm. Arts. 134

A. The weight decreases by 0·51 gm.

29. On introducing a bubble of air of volume 3 c.c. the mercury in a barometer which originally stood at 76 cm. fell by 12 cm. Find the space above mercury in air-free barometer, assuming the cross-section of the tube to be unity. Art. 134 (R. P. B.—1943)

A. 7 c.c.

30. Find the pressure exerted by a gramme of hydrogen in a vessel of 5·55 litre capacity at 0°C., assuming that the mass of a cubic centimetre of hydrogen at 0°C. and at a pressure of 760 mm. of mercury is 9×10^{-5} gm. Art. 134 (Dac. U.—1930)

A. 152·6 cm.

Reference

Art. 134 31. Due to the presence of a small quantity of air in the empty space above the top of mercury in a Torricellian tube, the height of mercury column is 752 mm. when the atmospheric pressure is 762 mm. and 750 mm. when the atmospheric pressure is 759 mm. What is the true atmospheric pressure when the height of the mercury volume is 740 mm.? (Del. H. S.—1950)

A. 746 mm.

Art. 134 32. A tube 5 ft. in length, closed at one end, is half filled with mercury and then is inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of mercury inside the tube? (Pat. U.—1952)

A. 11·5 in.

Art. 134 33. The length of the tube of a faulty barometer containing a small quantity of air is 31 inches obove the surface of mercury in which it is immersed. It reads 28 inches when the atmospheric pressure is 29 inches. Find its reading when the atmospheric pressure is 30 inches. (Utkal. U.—1953)

A. 28·7 in.

Art. 134 34. A barometer contains some air above mercury whose height is 73 cm. The top of the barometer tube is 80 cm. above mercury level in the cistern and the atmospheric pressure is 75 cm. of mercury. What is the atmospheric pressure when the mercury in the barometer reads 67·5 cm.? (U. P. B.—1952)

A. 71·5 cm.

Art. 136 35. A barometer reads 30 inches and the space above mercury is 1 in. If a quantity of air which at atmospheric pressure would occupy 1 in. of the tube, is introduced, what will be the reading of the barometer?

A. 25 in.

Art. 187 36. The mass of a balloon when empty is 10·454 kgm. When inflated with hydrogen, its volume becomes $35{\cdot}5 \times 10^6$ c.c. If the density of the enclosed hydrogen is 0·000080 gm. per c.c. and that of the surrounding air be 0·0012214 gm. per c.c. find the additional weight which can be lifted by the balloon.

A. 29·85 kgm.

CHAPTER XII

AIR PRESSURE MACHINES

138. Syringe.—It is one of the instruments whose working depends on the atmospheric pressure and is the simplest form of pump to raise a liquid from one vessel and then to transfer it to another. It consists of a hollow glass or metal tube with the lower end tapering to a nozzle. An air-tight piston works within the tube. With the piston at its lowermost position, the nozzle dips under a liquid. When the piston is drawn up, a partial vacuum is formed within the cylinder below the piston and so the atmospheric pressure acting on the liquid surface becomes greater than the inside pressure. The liquid is therefore forced into the cylinder following the piston as the latter is raised up. When sufficient liquid has been drawn in, the syringe is removed. Owing to the pressure of the atmospheric air acting at the nozzle the liquid cannot escape. If the piston is now pushed down, the pressure inside becomes greater than the outside pressure and so the liquid is forced out

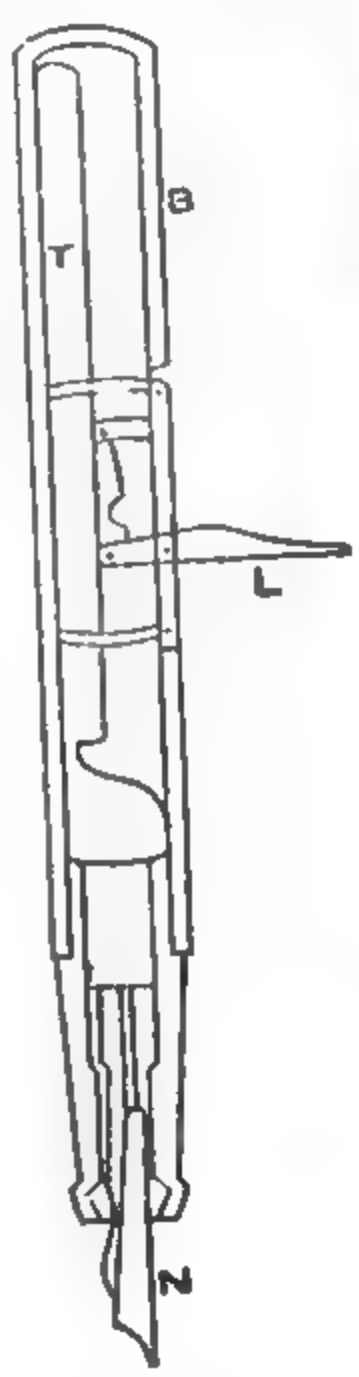


Fig. 192

Fig. 193

An eye dropper or a fountain-pen filler works on the same principle as a syringe The rubber bulb at the top on being pressed forces some air to go out of the tube through the nozzle. If now the nozzle is put under a liquid and pressure on the bulb is released, the bulb regains its shape expanding the inside space The pressure inside is thereby lowered and the requisite quantity of the liquid is sucked in to equalise the pressure. Fig. 192 shows a pen with an automatic filler. Within the barrel B of the pen there is a rubber tube T in communication with the capillary tube leading to the nib. A lever L, fixed to the side, on being raised, presses the tube and forces air out of it. The pen is lowered in ink and the lever is released so as to inflat the tube, whereby the ink is sucked in to fill up the tube as shown is fig. 193.

By the same principle air is taken into the human lungs. During inhalation the muscles of the chest cause the lungs to expand whereby the internal pressure is reduced and so air is drawn in and during exhalation the lungs contract and so the air is forced out.

139. Principle of Pumps.—Pump is a class of machines which transfer fluids from a region of low pressure to a region of high pressure.

Almost every type of pump consists of a large strong cylinder into which a suitable piston works air-tight. A complete to-and-fro motion of the piston is called a *stroke*.

Valves.—The air pressure machines are provided with devices, called **valves**, which act as very light trap doors placed at the openings of discs or in connecting pipes of pumps. A valve allows the fluid to pass in *one direction only* but completly blocks the passage in the opposite direction. Valves differ in forms according to the purposes for which they are meant; one may be a flat disc turning about a hinge on a disc (Fig. 194, A & B). Imagine that the disc serves as the partition wall between two compartments. If the fluid pressure in the lower compartment exceeds that in the upper, there would be an excess force acting from below on the valve, which would then open out and allow some fluid to pass through the hole. But if the fluid pressure in the upper compartment be in excess, the door would be pressed air tight on the disc and no fluid is allowed to pass through. Instead of the form of a flap door, a valve may be of the type of a conical piece moving in a conical slot (Fig. 194, C) or even a light ball moving in a tapering hole. The form of valves, generally used in air pumps, consists of a strip of oiled silk stretched over a hole (Fig. 194, D). A valve should be very light so that it can work for smallest possible difference of pressures.

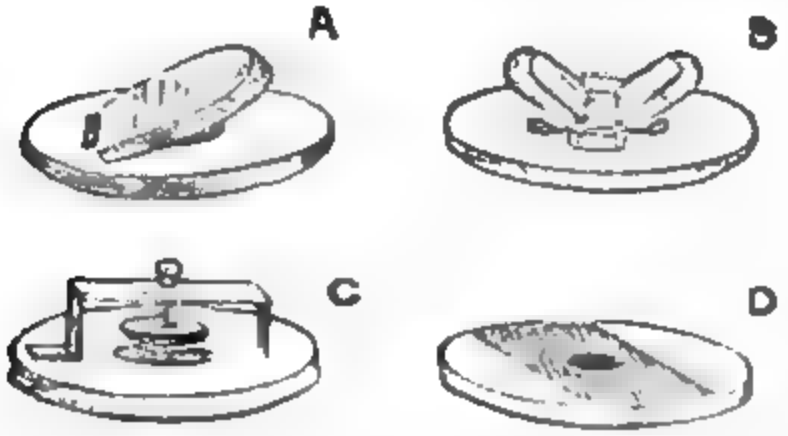


Fig. 194

310. **Common Pump or Suction Pump.**—It consists of cylinder A (Fig. 195), called the *barrel* within which a *piston* P works smoothly. The piston is made air tight within the barrel by a collar of soft leather or rubber strip. The barrel is connected at its lower end with a rising pipe whose end is introduced into a reservoir from which water is to be raised. There are two *valves*, both opening upwards, one of which (V) is at an opening in the piston, while the other (C) is at the junction of the barrel and the pipe. Near the top of the barrel there is a *spout* E through which the water flows out. In the case of a hand pump, the piston is worked by means of a handle which works as a lever and is attached to the piston-rod.

Action.—Let us start with the piston at its lowest position. During the *upward stroke*, the piston is raised against the atmospheric pressure and hence the valve V remains closed. The space below the piston therefore increases and the pressure inside falls. The valve C is opened by the excess of pressure of the air inside the pipe and so this air partly enters into the barrel. The pressure of the air inside the pipe and the barrel becomes less than the atmospheric pressure and so water from the reservoir is sucked up into the pipe. This sucking is continued till the piston reaches the top-most position. During the course of suction some water may even reach the barrel.

During the *down-stroke*, the air within the barrel below the piston is compressed due to which the valve C becomes closed. When the inside pressure becomes greater than the atmospheric pressure, the air inside the barrel pushes the valve V open and escapes through it. Air continues to escape through the upper valve till the piston reaches its lowest position, the valve C remaining closed all the while. If some water has been raised within the barrel during the up stroke, this water finds its way above the piston through the valve V.

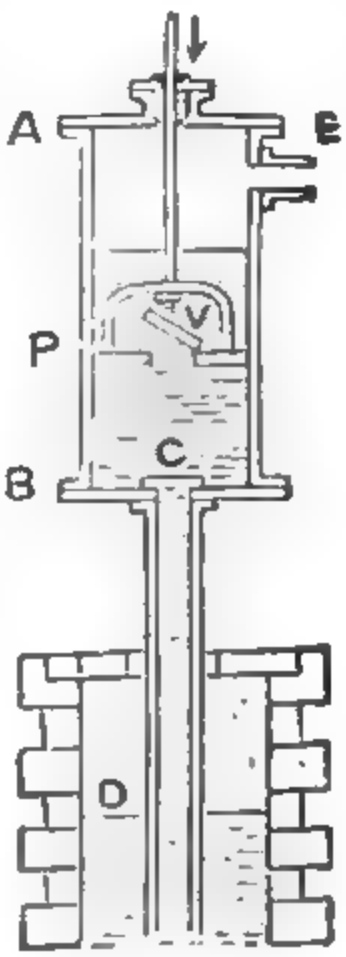


Fig. 195

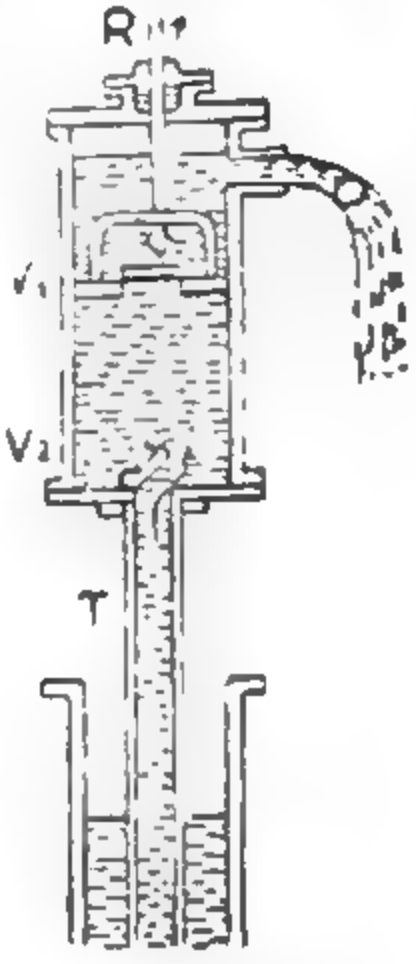


Fig. 196

During the next *up-stroke*, the valve V remains closed due to atmospheric pressure and pressure of any water above it, while due to the vacuum produced above the valve C it opens and water is forced into the barrel. The water above the piston flows out through the spout E (Fig. 196). It may often be necessary to work the piston a *few times* to drive off air from inside the pump and hence to get the *first* supply of water. When once a supply of water is obtained, water is given off at every upward stroke of the piston. It is to be noted that although water is said to be 'sucked in', it is really 'pushed up', by the atmospheric pressure.

Limitation of the Suction Pump.—Since water is raised by the atmospheric pressure, the height of the piston above the surface of of water in the reservoir must not exceed that of the water barometer, *i.e.*, about 34 ft. But as the valve has some weight and the piston is not perfectly air-tight, the difference in height is found to be slightly less. Moreover the pump gives an *intermittent flow* of water, supply being made during every up-stroke.

This pump is now being widely used in the tube wells If not used for sometime the valve in the piston may not remain perfectly air-tight owing to the leather collar being dried up and water would not rise up in the barrel. In this case some water is to be poured into the top of the barrel before using the pump. This would make the piston move air-tight and thus help the quick starting of the pump.

141. Lift Pump.—This pump is *used* to raise water to any desired height. It is only a modification of the common pump having a third valve V_s (Fig. 197) opening upwards at the junction of the delivery pipe and the barrel.

Action.—Suppose the piston is at its lowest position and that water has already reached upto the top of lower pipe T after some strokes.

During the *up-stroke*, as the piston is raised, the valve V_1 opens due to the partial vacuum created within the barrel while the valve V_2 remains closed due to the excess pressure above it. As air within the cylinder above V_2 is compressed, valve V_3 is opened and air escapes through the pipe into atmosphere. The pressure inside falls and water is raised into the barrel. During the *down stroke*, the pressure inside increases and the valve V_1 remains closed, while the valve V_2 opens and the water passes into the space above the piston. During the next *up stroke*, a fresh quantity of water is drawn into the barrel, while the water above the piston opens the valve V_3 and passes into the pipe above. As this process is repeated more and more water is forced up the tube through the valve V_3.

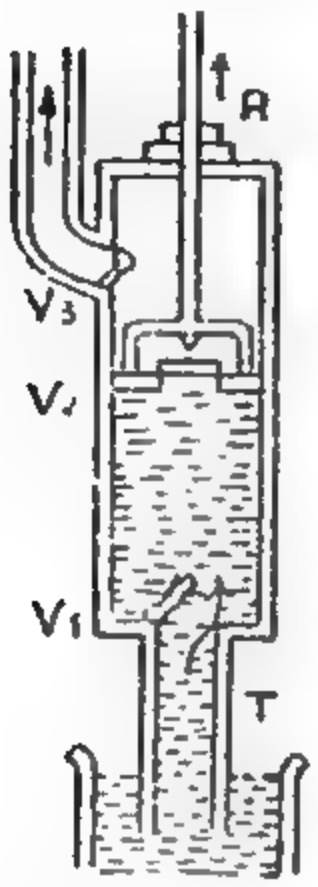


Fig. 197

It is evident that during the down-stroke of the piston, the valve V_3 supports the weight of the column of water within the pipe above and so relieves the valve V_2 of this weight during this period. By this pump water can be raised to any height, provided the different parts of the pump are strong enough to bear the strain caused by the weight of water above. It is important to remember that since the collection of water within the barrel depends upon suction, the barrel must be so placed as to be within 34 ft. from the water surface of the source.

112. **Force Pump.**—It consists of the strong *barrel* AB (Fig. 198) fitted with an air tight *solid piston* P and provided with a pipe at the bottom, which dips into the reservoir of water. There is a strong valve C opening upwards at the junction of the barrel and the lower tube. An outlet pipe E is fitted to a side close to the bottom of the barrel and at the junction there is another strong valve D opening into the pipe. It also works under a similar principle as a lift pump.

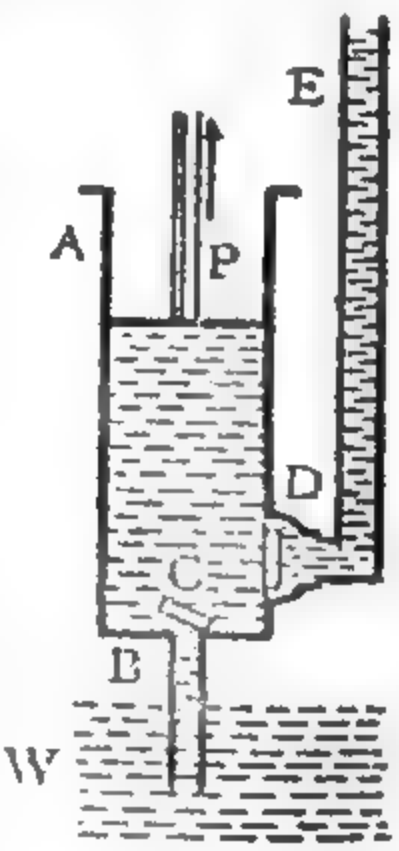


Fig. 198

Action.—As the lower pipe by the first few strokes is exhausted of air, the level of water reaches upto the valve C. During the next *up-stroke* of the piston, pressure inside the barrel falls, the valve D remains closed by the atmospheric pressure while the valve C opens and water is forced into AB. During the *down-stroke* the pressure inside increases, the valve C closes while the valve D opens and water is forced up the tube DE. Thus during each successive up-stroke, a quantity of water rises into the barrel, while the valve D remains closed due to the back pressure of the water within the pipe E ; and during each down-stroke, the water is forced into the outlet tube, the valve C remaining closed. Like the lift pump, this instrument also can raise water to any height, depending on the strength of the machine and the

force applied at the handle. Here also the output is *intermittent*, water flowing out during the down-strokes only.

Arrangements for Continous Flow.—The output can be made *continuous* by having an air-chamber R (Fig. 199) into which the outlet tube directly passes. The delivery tube leads into the chamber at its upper portion and reaches nearly to the bottom, while the upper end of the tube leads up to the required height.

Action.—During the *up-stroke*, the water rises into the barrel while the valve C at the bottom of the barrel remains open, the valve D remaining closed. During the *down-stroke*, the pressure inside increases ; so the valve C remains closed, while the valve D opens and water is forced into the chamber R. If the pump is rapidly worked, then during the downward stroke along with the discharge of water from the delivery tube, some water collects into the chamber, so that the air in R is compressed. During the upward stroke of the piston, the valve D remains closed but the compressed air in the chamber R expands and thus forces the water to flow out of the delivery tube. Thus a *continuous flow* of water ma. be maintained.

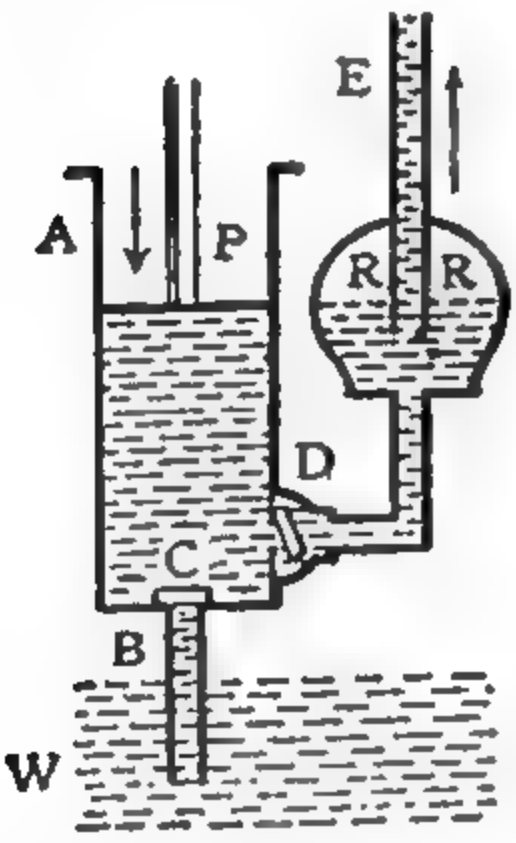


Fig. 199

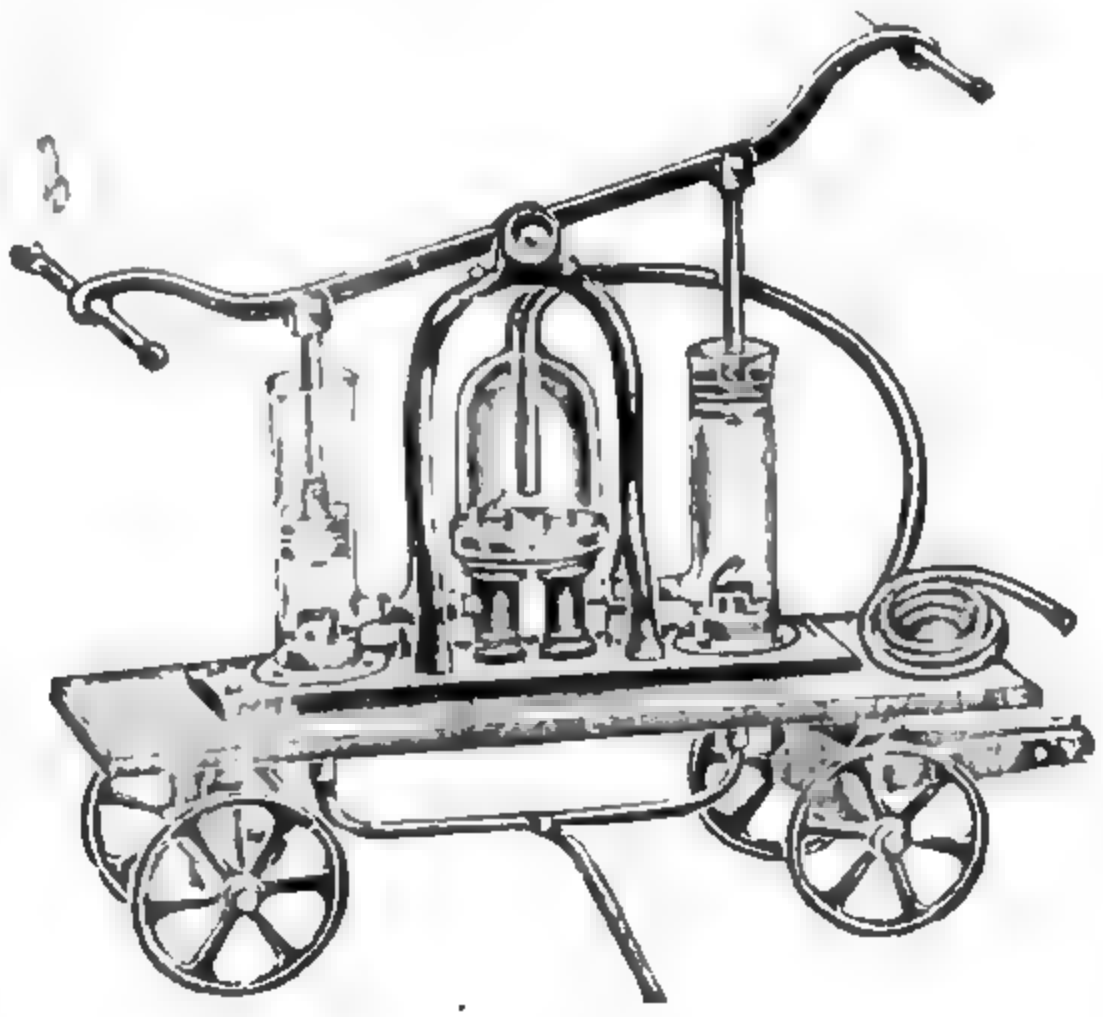


Fig. 200

Fire Engine.—This is *used* to throw a continuous stream of water at a place on fire which may be at a great height or at a large distance from the source of water. In the earlier type, a continuous flow was maintained with the help of an airc-hamber, as just described. In the present form, the fire engine consists of two force pumps, the barrel of which is connected to a common air-chamber (Fig. 200). Both the pumps draw from a common reservoir , when the piston in one moves down, that in other moves up and *vice versa*. The delivery tube is fitted to a lateral aperture in the air chamber. The action of the piston is likely to stop momentarily at the time of reversal but the continuity of

the flow is maintained by the air-chamber. The latest type is operated by a petrol motor or electric power.

Example :—

1. Water is pumped up from a well through a height of 30 ft. by means of 5 H. P. motor. If the efficiency of the pump is 85%, find in gallons the quantity of water pumped up per minute. One gallon of water weighs 10 lb.

(Gau. U.—1952)

A. Since 1 H. P. = 33000 ft. lb. of work per minute, a 5 H. P. motor can do 165000 ft. lb. of work per minute on the pump.

But efficiency of pump $= \frac{\text{work done by pump}}{\text{work done on pump}} = 85\% = \frac{85}{100}$

$\therefore$ Work done by pump per minute $= 165000 \times \frac{85}{100} = 140250$ ft. lb.

Let x be the number of gallons of water raised through 30 ft. in one minute by the pump. Then work done $= 10\,x \times 30$ ft. lb.

or, $300\,x = 140250$ whence $x = 467{\cdot}5$ gallons per minute.

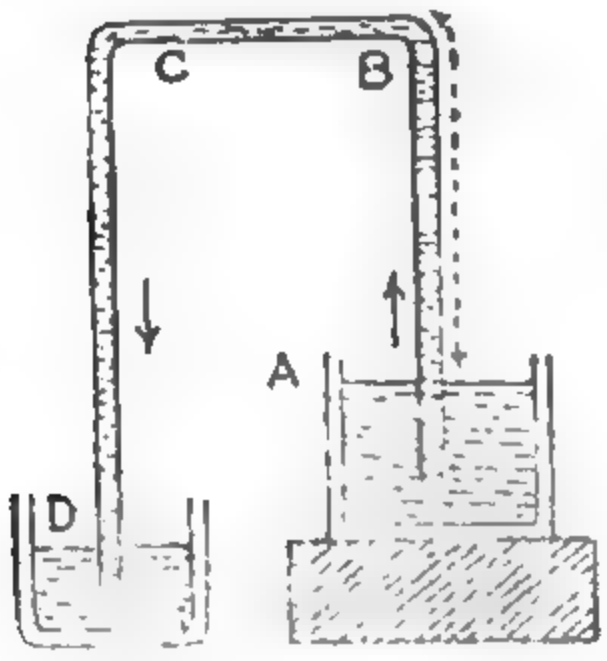


Fig. 201

143. Siphon.—It is *used* for transferring a liquid from one vessel into another when the whole mass of the liquid is not to be disturbed or when the ordinary method of pouring the liquid is not convenient or desirable.

The **siphon** consists of a bent tube CB usually of glass, (Fig. 201) open at both ends and with one limb longer than the other. The tube is first filled with the liquid to be transferred. Having temporarily closed both the ends with fingers, the shorter limb is introduced into the liquid contained in the vessel A, while the longer limb project outside into another vessel with its end below the level D of the liquid. On now opening the ends, the liquid at once begins to flow out of the longer limb in a continuous stream.

Principle of Action.—The start of the flow may be explained in the following way. Let C and B be two points in the upper part of the tube and A, D the levels of liquid in the vessels.

Let the atmospheric pressure at the time $= P$

vertical heights of C and B above D and A respectively $= h, h'$

density of the liquid in the vessel $= \rho$

Then static pressure at C $= P - h\rho g$ and that at B $= P - h'\rho g$

Now as $h > h'$, pressure at B > pressure at C.

Hence the liquid begins to flow from B at a higher pressure to C at a lower pressure. Due to the motion of some liquid from B a partial vacuum is created and fresh liquid is forced up to B by the atmospheric pressure. Thus a continuous flow is maintained, the shorter limb of

course remaining under liquid. In case the end of the longer limb dips below a liquid, h' is measured from the surface of the liquid D in the vessel. When the liquid is in motion, the pressures at B and C change, although the former remains higher than the latter (vide Art. 159).

Conditions for the Working of a Siphon.—

(1) The level of the liquid in the vessel to be emptied must be *higher* than that in the other. For when $h = h'$, the pressure at C = the pressure at B and water would not flow from B to C. Hence the siphon ceases to work when the levels of the liquid in both the vessels becomes the same.

(2) As atmospheric pressure forces up the liquid into the shorter limb, the height h' of the top of the siphon above the liquid in the vessel to be emptied must remain less than the height of the corresponding liquid barometer.

(3) The siphon would not work in vacuum, as the atmospheric pressure can not work therein.

144. Applications of a Siphon.—A siphon may also be used to produce an intermittent flow of a liquid. This is illustrated in the following cases.

Tantalus' Cup.—It consists of a cup in which a siphon is placed concealed within the bust of a man, representing the King Tantalus (Fig.202). The siphon is so placed that its bend is just below the level of the lower lip of the figure. The shorter limb ends near the bottom of cup, while the longer limb just passes out. As water is poured into the cup, the level gradually rises both in the cup and in the shorter limb. As soon as the water reaches the top of the bend and is about to reach the lips, it begins to flow out. If the supply of water is then stopped or if it flows out at a faster rate than it is supplied, the level of water gradually sinks into the cup until it goes below the end of the shorter limb; the siphon now empties itself and the action stops. If water is supplied again, the same process is repeated; so water never reaches the lips of Tantalus, thus keeping him thirsty as ever.

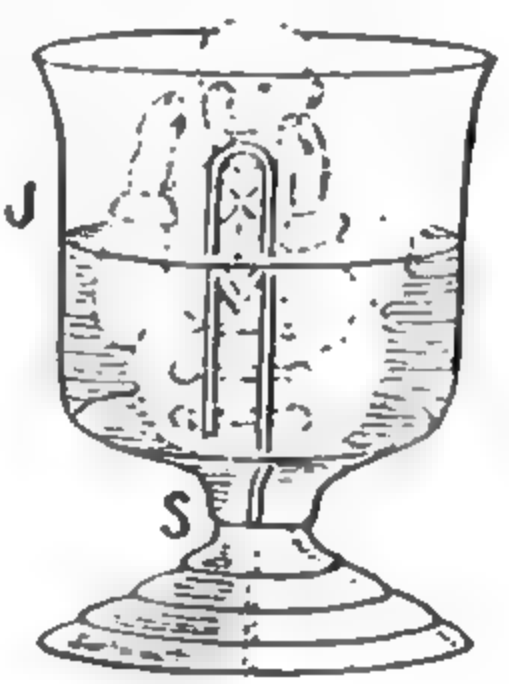


Fig. 202

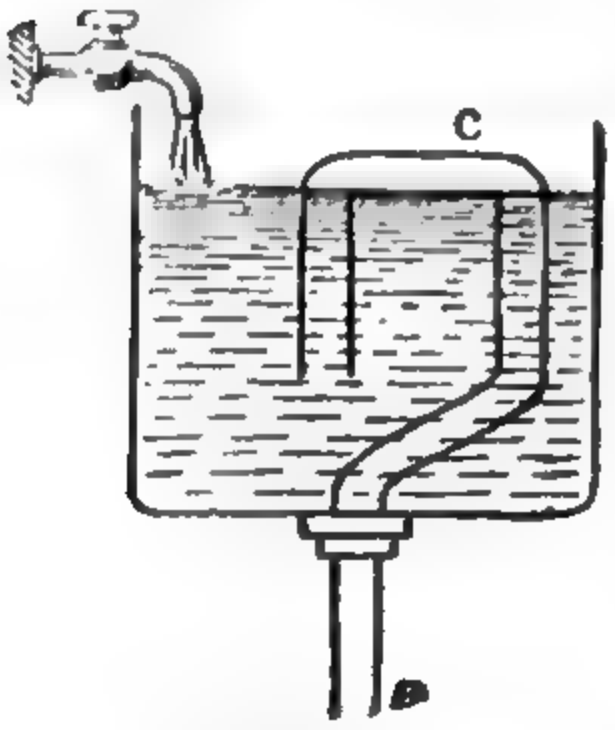


Fig. 203

Automatic Flush.—The principle is also utilised in *automatic flushes* often fitted in public latrines, etc. A siphon is fitted inside a tank and the longer limb passes out (Fig. 203). A stream of water flows into the tank and its level gradually rises. Ultimately when the level of water reaches the bend, the siphon begins to work and the water flows out in a powerful gush till the level goes below the open end of the shorter limb.

Example :—

1. A tall cylinder 90 cm. in length is full of mercury. What is the greatest depth of it which can be emptied out by means of a siphon, the barometric height being 756 mm. of mercury. (P. U.—1924)

A. The siphon can work till the height of the mercury column in the shorter tube above the mercury surface is not greater than 756 mm. Hence placing the bend of the siphon on the upper edge of the cylinder, mercury can be removed from the cylinder to a depth of 756 mm.

145. Water Jet Pump or Filter Pump.—This is one of the simplest forms of an air pump. It is ordinarily made of glass. The side tube C (Fig. 204) is connected with a rubber tubing to the vessel to be exhausted, while the end T is also connected by a rubber tubing to the water mains, the pressure of which should remain constant. As a strong jet of water is forced out of the narrow mouth of the nozzle J at a very high speed, it entangles some of the air surrounding the narrow nozzle and carries it down the tube due to viscous friction. A partial vacuum is thus produced and so air is sucked from the vessel connected to it. The filter pump may produce a pressure only as low as 7 mm. of mercury and it is therefore not used when a good vacuum is required. It is however very simple, economical and convenient, requiring very little attention.

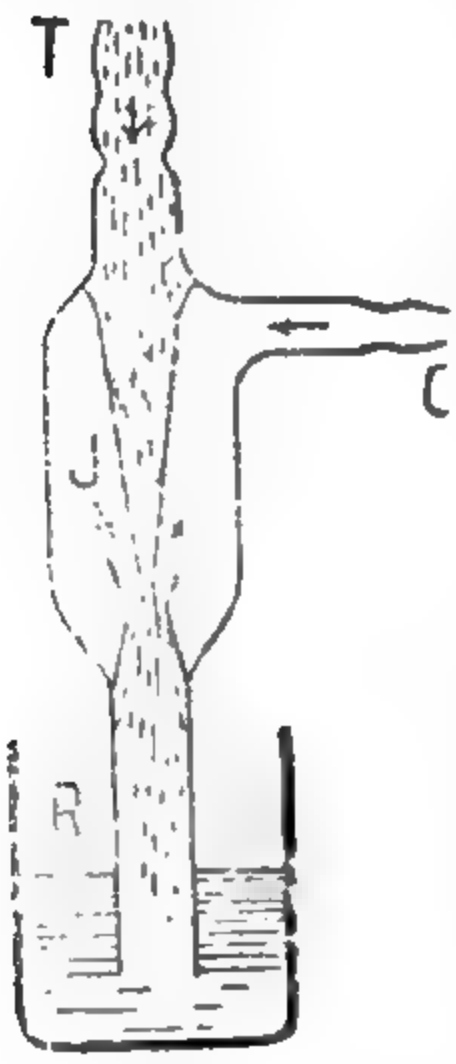


Fig. 204

A modified form of pump, known as Sprengel pump, had its origin from the principle of action of such a jet pump. This modified pump was devised in 1865. Mercury is allowed to fall through a delivery tube in drops being controlled by a pinch cock. From near the top of the fall tube, a side tube is connected to the receiver which is to be evacuated. By this means, as drops of mercury trickle down, small quantities of air entrapped between successive drops, are continuously swept out. With proper care and modification, a Sprengel pump is capable of producing a pressure of 10^{-6} mm. of mercury. Sir William Crookes used such a pump in his memorable investigations on radiometers and discharge tubes.

146. Air Pump (or Exhaust Pump).—The first mechanical air pump was constructed by Otto Von Guericke in 1650. The air pump consists essentially of a cylindrical metal *barrel* AB (Fig. 205), provided with an air-tight *piston* P, which can be moved up and down by means of a suitable handle. The pipe CD at the bottom connects the barrel with a vessel called the *receiver* R from which air is to be exhausted. The receiver may be well greased and fixed air-tight on a flat circular disc EF, called the *plate* of the pump. There is a valve V in the piston and another at the junction of the barrel and the pipe CD, both of which open upwards. To indicate the pressure

within the receiver, a manometer may be connected to the tube CD by a side tube.

Action.—Let us start with the piston at its lowest position. During the *up-stroke*, as the piston is drawn up the valve V is closed due to excess pressure at its top. The air in R in expanding opens the valve *a* and enters the barrel. During the *down-stroke*, the air in the barrel below the piston is compressed. The inside pressure therefore increases, due to which the valve *a* becomes closed When the inside pressure becomes greater than the atmospheric pressure, the valve V is pushed open and the air from inside goes out.

During the next *up-stroke*, the air remaining within the receiver again expands, the valve *a* opens and a fresh quantity of air from the receiver fills the barrel, the valve V remaining closed all the time. Thus during each up-stroke the barrel is filled with some amount of air from the receiver, which is let out during the next down-stroke. As the operation is repeated, the receiver is more and more exhausted. This goes on until the pressure of the air left in the receiver becomes too weak to open the valve *a* ; the limit of exhaustion attained by the instrument is then reached and any further evacuation stops.

The action of the pump with a single barrel is evidently *intermittent*. To make it continuous, a double-barrelled pump, (Fig. 206) for example, *Hawksbee's air pump*, is used. The two pistons P and P′ are simultaneously worked by a rack and pinion arrangement K, connected with a lever rod MN so that when one of them goes down, the other moves up. The barrels are connected to a common tube leading to the

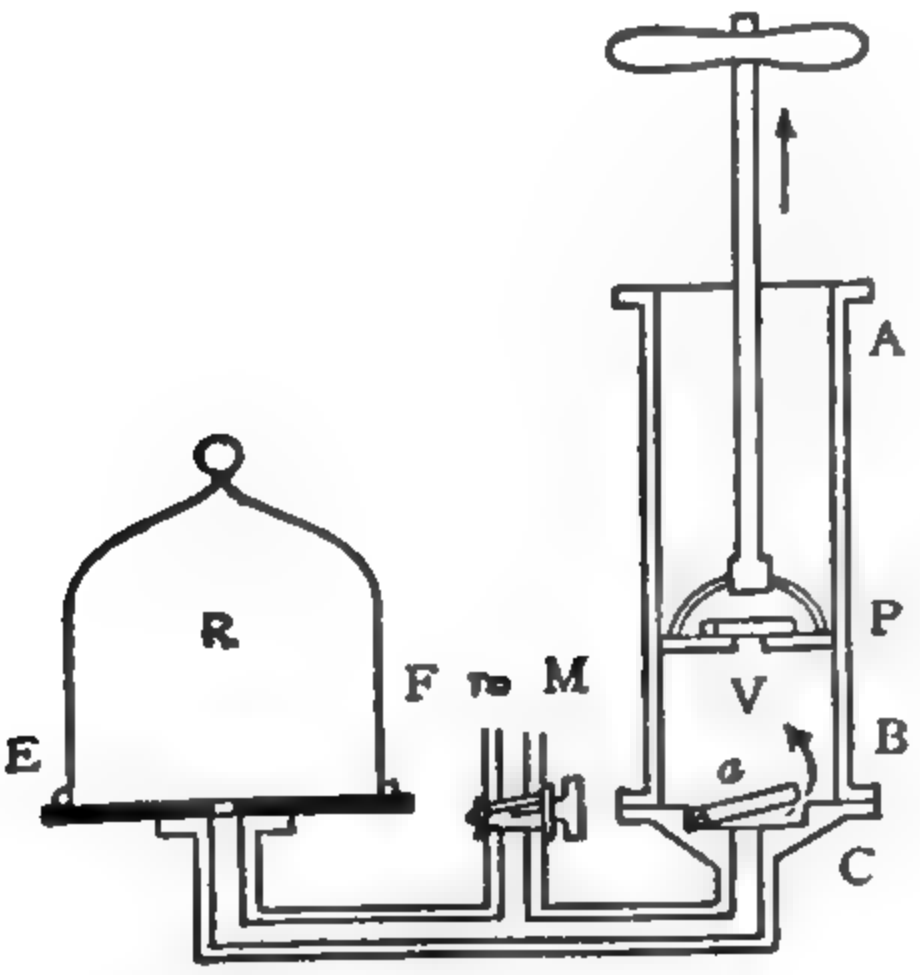


Fig. 205—Air Pump

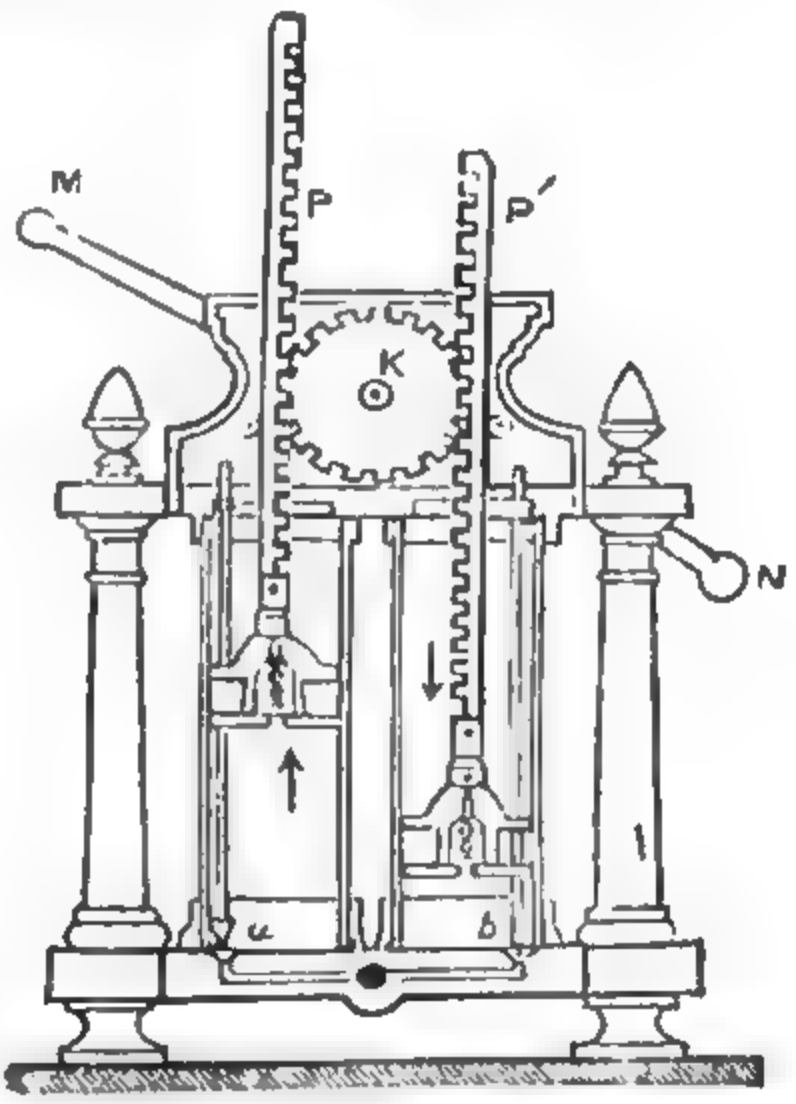


Fig. 206—Double-Barrel Pump

receiver. Thus air is taken out continuously and consequently the rate of evacuation becomes twice as rapid as in a pump with a single barrel.

Degree of Evacuation.—The pressure and the density of air within the receiver continuously fall off as the pump is worked. The following calculation shows how pressure and density change with the number of strokes.—

Let the volume of the receiver, including that of the pipe CD = V
volume of barrel between extreme positions of piston = v
density of air *initially* in the receiver = ρ

At the end of each up-stroke a volume V of air expands to occupy a volume $(V + v)$ and is thus reduced in density. Let the density of the air at the end of the 1st up-stroke be ρ_1.

Then, $V\rho = (V + v)\rho_1$ whence $\rho_1 = \frac{V}{V + v}\rho$

At the end of the next down-stroke, the air in the barrel escapes above the piston through its valve and a quantity of air of volume V and density ρ_1 is left in the receiver. At the end of the 2nd up-stroke, this air of the reciver expands to a volume $(V + v)$ and thereby becomes reduced in density to ρ_2 say,—

Then $V\rho_1 = (V + v)\rho_2$ when $\rho_2 = \frac{V}{V + v}\rho_1 = \left(\frac{V}{V + v}\right)^2 \rho$

Proceeding in this way, it can be shown that the density of air left in the receiver after n strokes is given by

$$\rho_n = \left(\frac{V}{V + v}\right)^n \rho \qquad \ldots(146,1)$$

If the temperature remains constant, Boyle's law holds good and the pressure becomes proportional to density. So if P be the initial pressure of the gas in the receiver and P_n be the pressure of the gas left in the receiver after n complete strokes then,

$$P_n = \left(\frac{V}{V + v}\right)^n P \qquad \ldots(146,2)$$

From mathematical standpoint, since $V/(V + v)$ is a quantity less than unity, the higher is the value of n, the less becomes the right side expression of eqn. (146.2), but it cannot practically attain a zero value. A high degree of evacuation can, however, be attained with a good pump after a sufficient number of strokes. But in fact, with a mechanical air pump, as described above the valves become more or less leaky at lower pressure. Moreover the valves have some weight and at a certain stage of lower pressure, the air pressure of the receiver is unable to open the first valve. It is at this stage that any further evacuation by the pump ceases.

Examples :—

1. After four strokes the density of the air in the receiver of an air pump is found to bear to its original density the ratio of 256 to 625. What is the ratio of the volume of the barrel to that of the receiver? (C. U.—1923)

A. Let the volume of the receiver and that of the barrel be V and v,

Then $\frac{256}{625} = \left(\frac{V}{V + v}\right)$ or $\frac{V}{V + v} = \frac{4}{5}$ whence $\frac{v}{V} = \frac{1}{4}$

2. The barrel of an exhaust pump has half the volume of the receiver to be evacuated. Find the pressure in the receiver after 4 complete strokes, if the original pressure was 76 cm. of mercury. (Mys. U.—1952)

A. If P_n be the pressure of air in the receiver after nth stroke, V and v the volumes of the receiver and the barrel of the pump, then neglecting the volume of the communicating pipe, and assuming the starting pressure to be P.

$$P_n = \left(\frac{V}{V+v}\right)^n P = \left(\frac{V}{V+\frac{1}{2}V}\right)^4 \times 76 = \left(\frac{2}{3}\right)^4 \times 76 = 15 \text{ cm. of Hg.}$$

3. The volume of the receiver of an air pump is 1250 c.c. and that of the barrel is 250 c.c. If the original density of the air in the receiver to 0·001293 gm. per c.c. find the density after 10 complete strokes. Find also the mass of air removed from the receiver.

A. Let the density of air the receiver after 10 storkes $= \rho'$

Then $$\rho' = \left(\frac{1250}{1250+250}\right)^{10} \times 0.001293 = \left(\frac{5}{6}\right)^{10} \times 0.001293$$

Taking logarithms, $\log \rho' = 10\ (\log 5 - \log 6) + \log 0{\cdot}001293$

$\therefore \log\rho' = -0{\cdot}792 + \bar{3}{\cdot}1116 = \bar{1}{\cdot}2080 + \bar{3}{\cdot}1116 = \bar{4}{\cdot}8196$ whence $\rho' = 0{\cdot}0002087$

So mass of air removed $= 1250\ (0{\cdot}001293 - 0{\cdot}000208)$ gm. $= 1{\cdot}356$ gm.

147. Guericke Pump.—An improved piston type of an air pump consists of a barrel C with a side tube E to be connected with he receiver (Fig. 207). The tube communicates with the barrel through

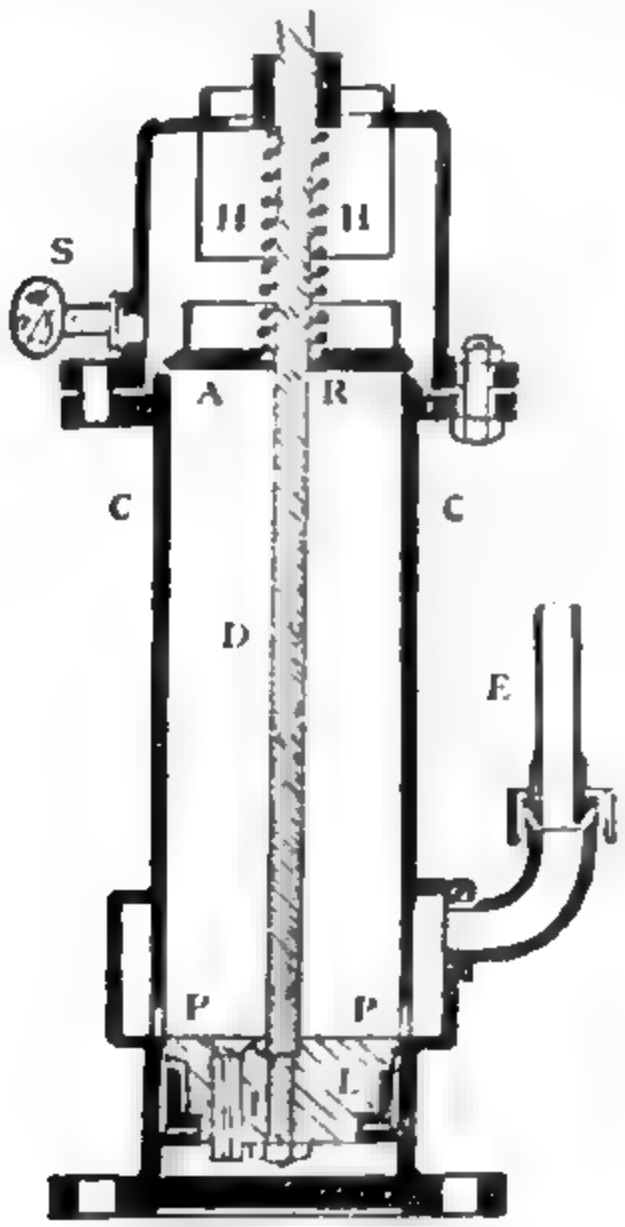


Fig. 207

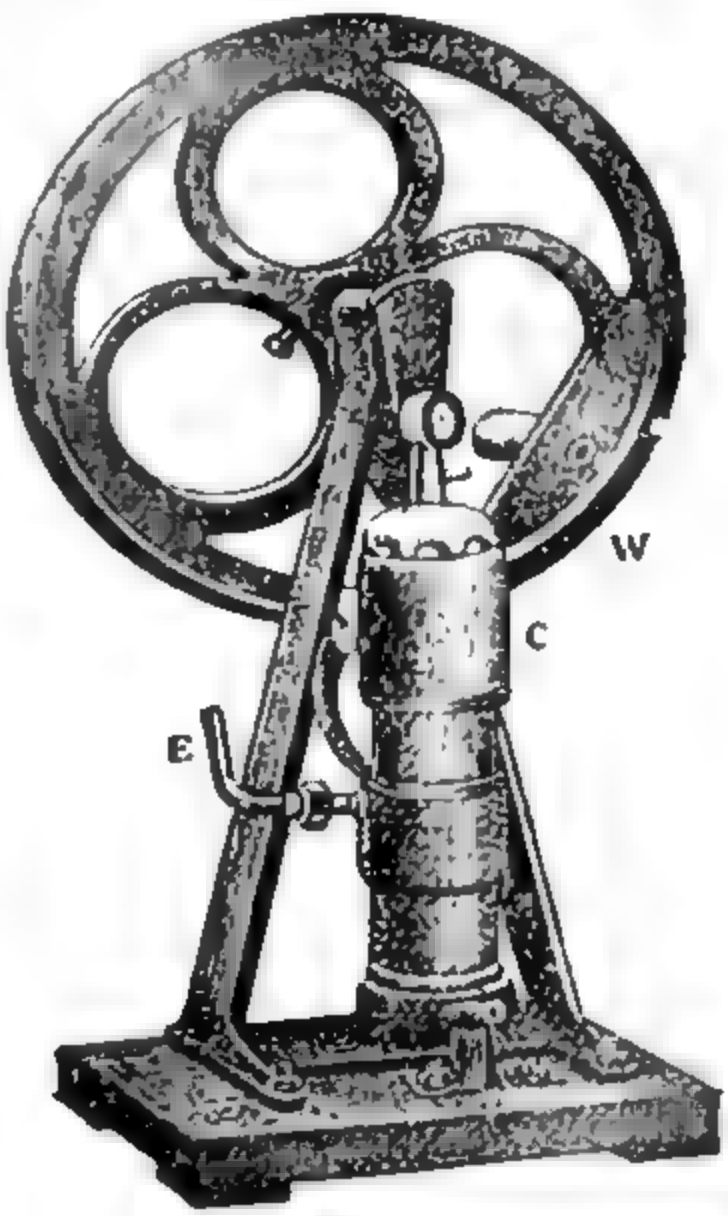


Fig. 208

a number of port-holes PP. The piston L is provided with an oil valve at the bottom and is screwed rigidly with a rod D. This rod passes through a valve R and is connected to a lever by which motion may be

imparted to the piston. The actual apparatus is shown in Fig 208. The piston rod is sometimes connected externally to the flywheel W and by rotating the wheel a continuous up and down motion of the piston may be obtained. To make the junctions air tight, a thick layer of heavy oil is preserved on the piston and on the valve R. The valve R is kept pressed downwards by a spring HH.

The receiver is connected to the inlet pipe E. When the piston is at the bottom, the air within the receiver expands into the barrel through the port-holes But when the piston is raised up, the air within the barrel is isolated from the receiver, and this air is gradually compressed till at the top-most position the piston, pushes the valve up to make space for air to escape. When the piston is moved down a bit, the spring closes the valve R and the space within the barrel is cut off from atmosphere. When the piston is at the lowest position, the air within the receiver again finds its way into the barrel through the port-holes. In this way by the continuous up and down motion of the piston, air is intermittently taken out of the receiver.

148. Töepler Pump.—The apparatus, originally designed by Töepler in 1862, consists of a glass bulb B to the ends of which two vertical barometer tubes F and G, each about 32 inches long are connected (Fig. 209). The tube F is narrow and of about 1 millimetre bore and its lower end, which is usually bent upwards, is dipped into mercury contained in a vessel K. The wider tube G has got a bend at its lower part and is connected to a short piece of glass tubing T. One end of stout pressure tubing is connected to the base of this short tube while its other end is connected to a reservoir M containing mercury. A side tube connected with the bulb B at C leads to the receiver R through an enlarged part in which a small glass beed L acts as a valve by not allowing mercury to run into the receiver. In original Töepler's machine, there was no valve but the part CE was a vertical tube of glass of length of about 32 inches. This attachment made the apparatus about 8 ft in height, but the provision of the valve has made the apparatus about 5 ft in height with a greater ease of operating the machine. The receiver is connected very carefully with leak tight pressure tubing.

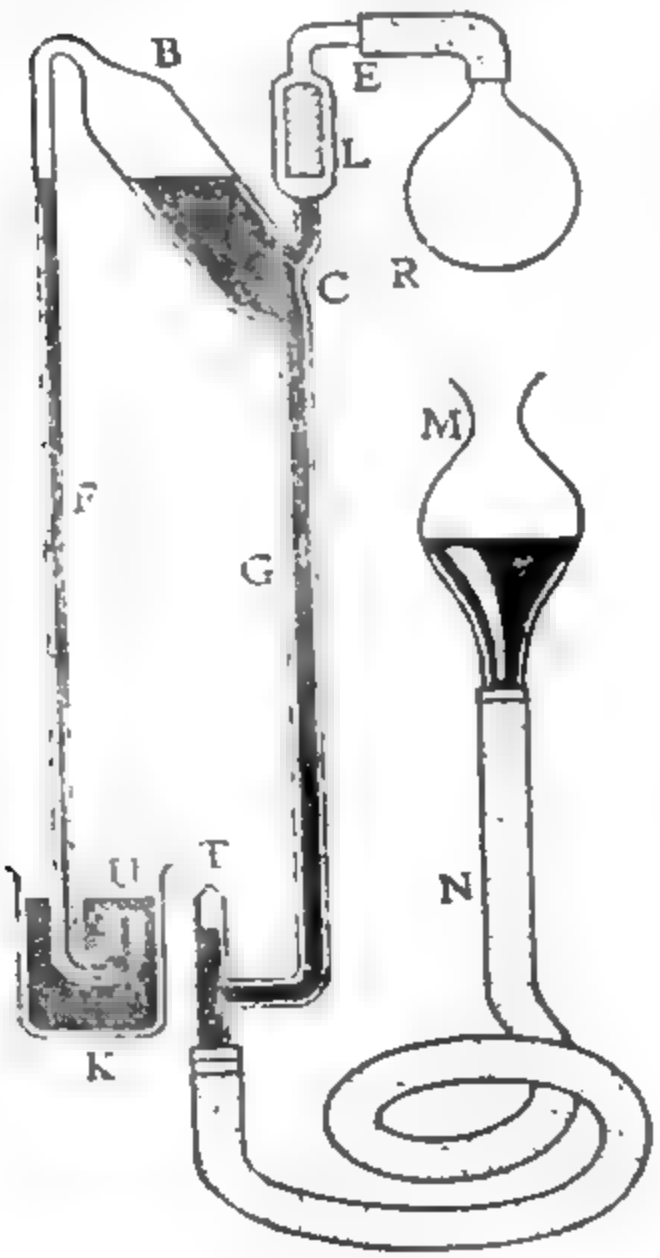


Fig. 209

At the start the receiver, a part of the tube G, the bulb B and the whole of the tube F are full of air at atmospheric pressure. The reservoir M is gradually raised when the mercury level in G gradually rises slowly compressing air at differ-

ent parts of the apparatus. When M is raised to such a height that the mercury level enters the bulb B and is just above the point C, then the air within the receiver R is cut off from the air within R and F. On now raising the receivoir still higher, the glass valve L floating on mercury closes the passage to the receiver while mercury level in B rises and forces air to pass to the atmosphere through the mouth U. Actually mercury is allowed to run down through F to drive off last traces of air in this region.

The reservoir is now lowered and the mercury level in B comes down to leave a vacuous space in it. When the level of mercury comes below C, the air within the receiver expands to fill the vacuous space in B. Again the reeervoir is raised to cut off air in B and ultimately to drive air out of U. Thus lowering of mercury level below C, establishes a communication between B and R, and a part of the air expands into B and raising of the level of mercury to the extent that mercury falls through the tube F, expels this air to the atmosphere. Thus with a continued operation more and more air from the receiver is discharged to the atmosphere and so the pressure within the receiver lowers down. The mercury in G plays the part of air tight piston and mercury in the fall tube F plays the part of the valve in not allowing atmospheric air to enter the bulb B. As there is an intermittent stream of mercury running down through the fall tube, mercury gradually collects within the vessel K and has to be poured back into M from time to time. It is a very satisfactory type of pump as we can get a good vacuum with it. But the action of the pump is very slow. Lord Rayleigh states that he required the whole of the morning to evacuate a receiver of 2 litres capacity. However slow might be the operation, the pump is capable of producing a pressure of the order of 10^{-4} mm. of mercury.

149. Rotary Hyvac Pump.—This type of the pump, originally designed by Gaede in 1907 depends for its working upon a continuous rotation of a cylinder within a suitable chamber (Fig. 210). The instrument consists of a hollow cylindrical body of iron having two openings near each other at the top. Of these, the one called the inlet port I, is provided with a pipe projected vertically upwards which can be connected with a rubber tubing to the vessel to be evacuated. The other called the outlet port O, is provided with a valve such that any gas contained within the cylinder may pass out but nothing can come within. The inlet and the outlet ports are located on either side of a movable, air-tight vane S kept pressed by a lever L and a spring. The solid cylinder within the hollow cylinder is rigidly fixed to a shaft and can be rotated eccentrically about an axis. With the revolution of the inner cylinder, a rotatory contact is maintained between the solid and hollow cylinders and the vane is always kept pressed on the inner cylinder. The complete picture of a rotary pump is given in Fig. 211, in which there are two pumps side by side, working on the same shaft and enclosed within a rectangular metal box.

The principle of action of the instrument is as follows. When the

solid cylinder in course of rotation occupies the position as shown in Fig. 212 it closes the outlet port and the inlet becomes just open. The gas from the receiver expands within the crescent shaped space M. The next stage is shown in Fig. 213 when the expanded gas is cut off from the receiver into the space M. At the third stage, the cylinder gradually compresses the gas within the diminishing space M and also

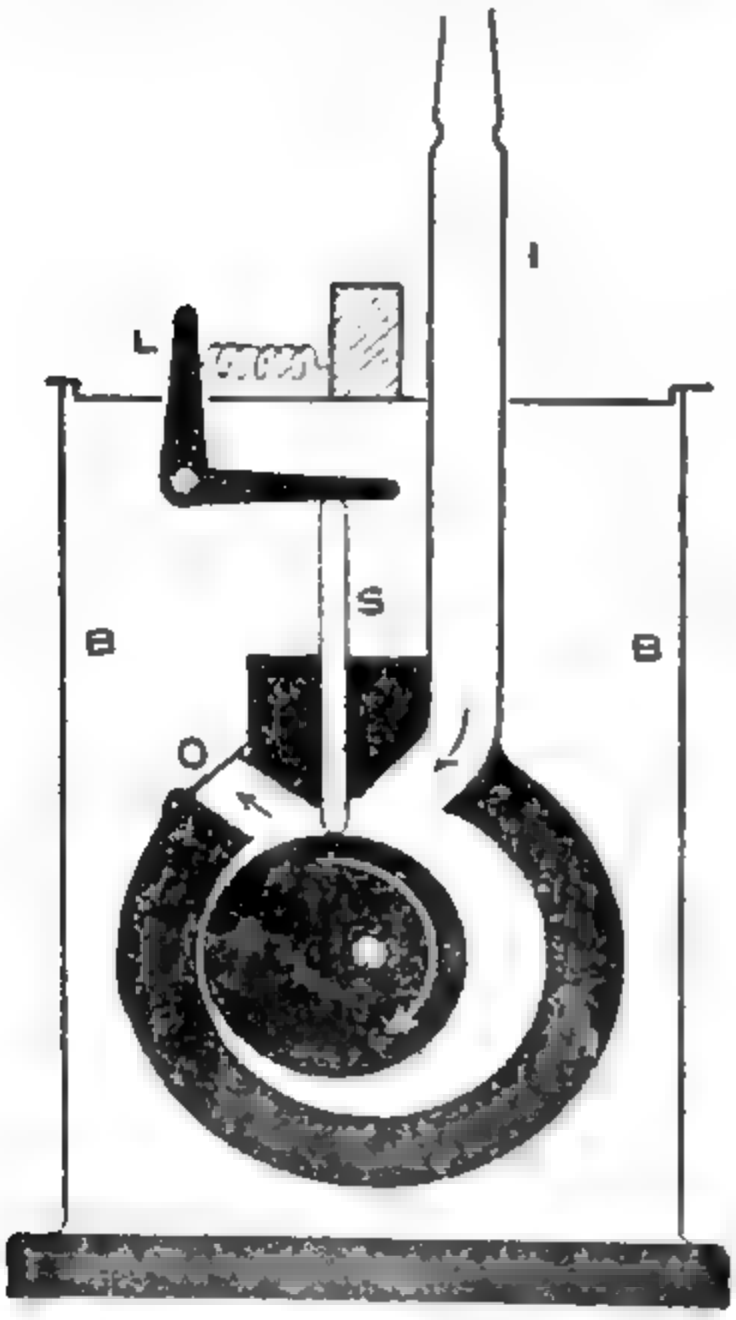


Fig. 210

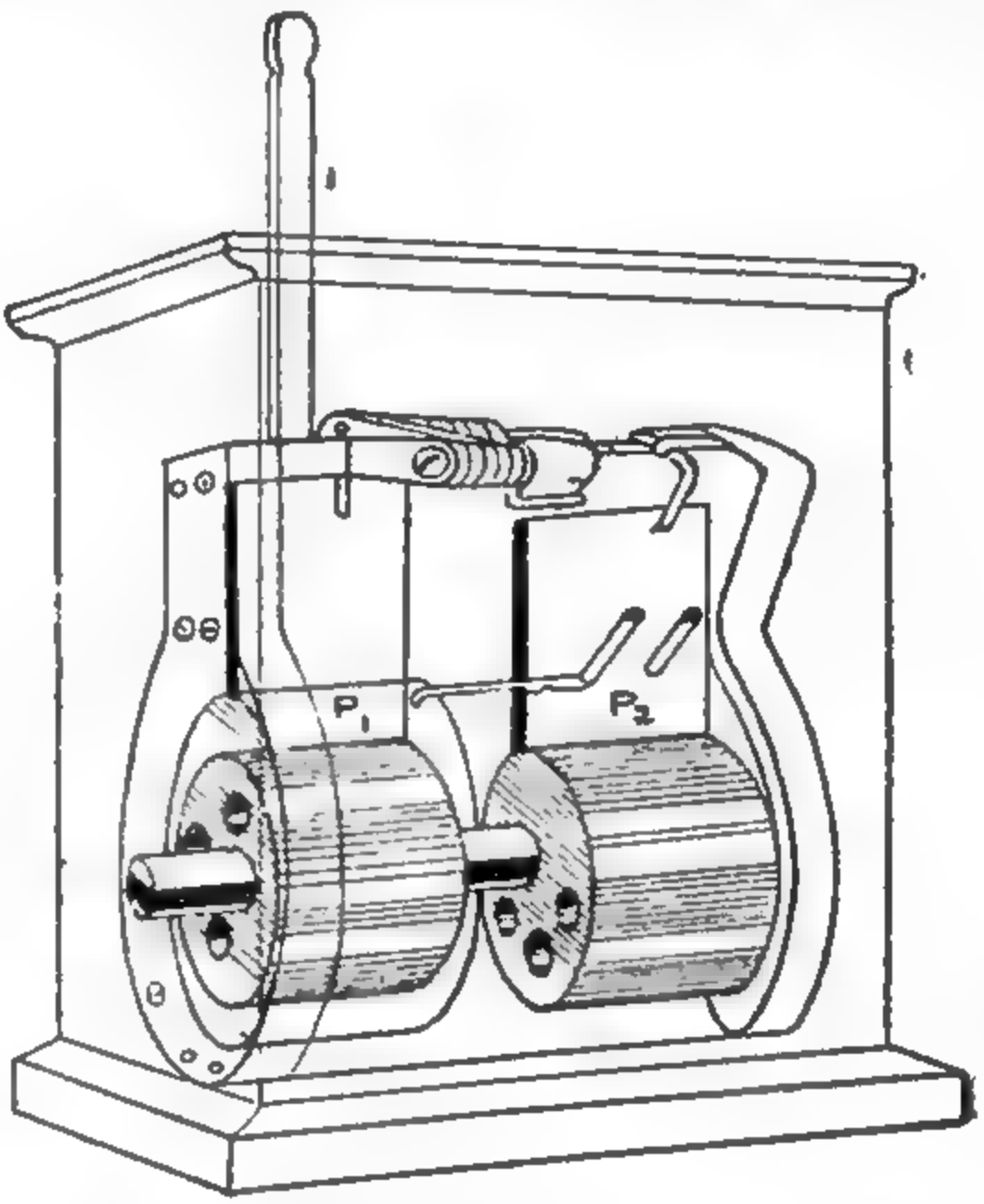


Fig. 211—Rotary Hyvac Pump

allows air from the receiver to fill in the space N (Fig. 214). At the fourth stage of operation, all entrapped gas is pressed out through the outlet port (Fig. 215). Thus with continuous retation the gas is gradually transferred from the receiver to the outside atmosphere. The whole apparatus is immersed in machine oil in a rectangular metal box B so as to make it air-tight and isolate from atmospheric air.

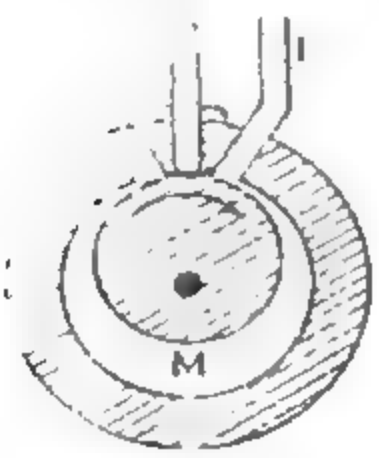


Fig. 212

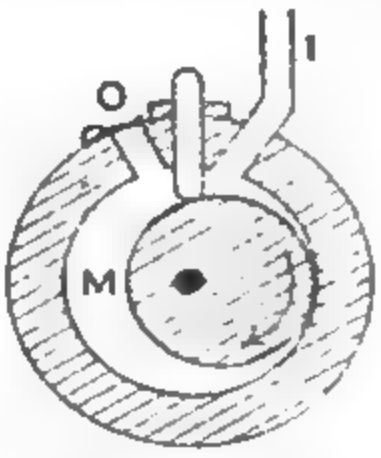


Fig. 213

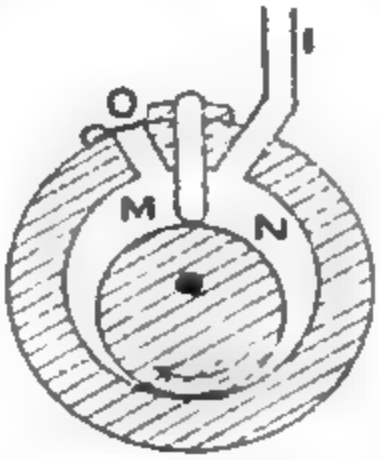


Fig. 214

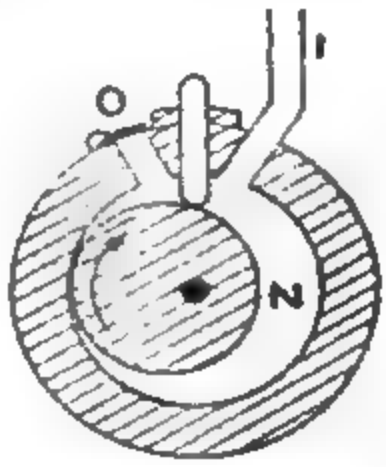


Fig. 215

It is evident from the principle of pumping that with each revolution

of the shaft, the amount of gas occupying the volume M is forced out into atmosphere. So it is not possible to make the receiver *absolutely vacuous*, but the limitation of pumping depends upon the difference of pressures acting on the valve at the outlet. If the pressure outside the valve is the atmospheric pressure, as in the case when a single pump is worked, it can evacuate a gas upto a pressure of 10^{-2} mm. of mercury. But if the pressure outside the valve is kept lower than the atmosphere, the degree of evacuation is much higher. For this reason a 'two stage' pump is frequently used. The first pump or the fore-vacuum pump creates a pressure of 10^{-2} mm. of mercury just before the outlet of the second pump which is, connected to the receiver. Such a two-stage pump as shown in fig. 211, can evacuate down to a pressure of 10^{-4} mm. of mercury.

Units for Low Pressure.—Since any rough suction pump is able to evacuate a receiver to a pressure less than that due to a column of mercury of height of 1 cm., it is customary to denote such a pressure in terms of height of mercury in millimetres. Thus one millimetre height of mercury corresponds to a pressure of 1333 dynes per sq. cm. taking density of mercury to be 13·59 at 0°C and $g = 980·6$ at latitude 45°. Moderately low pressure is measured in terms *of micron* (μ) of mercury which is 10^{-3} mm. of mercury (1.33 dynes sq cm.) and very low pressures are measured in terms of a *millimicron* ($\mu\mu$) which is 10^{-6} mm. of mercury. For example, a pressure of ·000031 cm. is equivalent to a pressure of 31 $\mu\mu$ of mercury.

***150. Vacuum Brake System.**—Air pressure is conveniently utilised to arrest the motion of automobiles whenever necessary. Fig. 216 represents a "vacuum brake" system sometimes employed in railways and tram cars. Stout curved pieces of steel called "brake shoes" S are fixed near the rim of every wheel with a very small clearance between them under normal condition. These shoes can be operated by a system of levers L and M and can be pressed hard on the wheels. The mechanism of action of the levers is as follows.

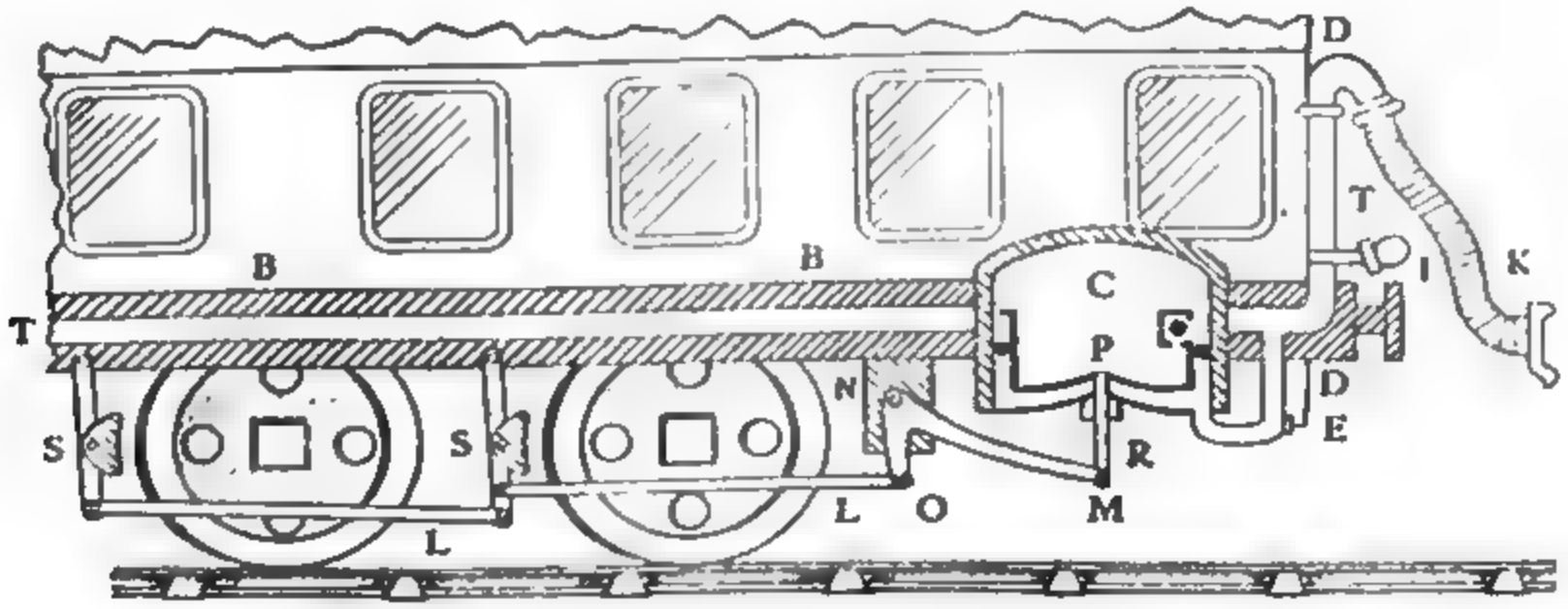


Fig. 216—Vacuum Brake System.

Within the chamber C, known as the vacuum chamber placed under the carriage, there is a piston P dividing C into two compartments. A ball valve V fitted at one corner of the piston (shown enlarged in

Fig. 217) allows air to pass from the upper to the lower compartment but not in the reverse direction. The piston moves over a rubber ring G and is so air-tight. The lower compartment is connected, to a system of pipes TT running through every bogey composing the train. This pipe is called the *train pipe*. One end of this system of pipes is closed and the other end is connected to a suction air pump fitted at the engine. When the pump of the engine is worked continuously, a partial vacuum is produced within both the compartments of the vacuum chamber of every bogey. The weight of the piston and its accessories is sufficient to lower the piston P down. A downward displacement of the end M of the piston rod causes the end O of the lever to move to the left side and to make the wheels free from brake pressure.

The valve E connected with the train pipe can be opened by pulling a chain provided within the bogey, called the *alarm chain*. When the alarm chain is pulled, the valve E is forced open and immediately the lower compartment of the vacuum chamber is filled with air pressing the piston up. The upward motion of the piston works the lever in the opposite direction forcing the brake shoes on the wheels. For an indication, a flat bar is coupled with the chain in every bogey and this bar projects out when the alarm chain has been pulled from it. Once the alarm chain is pulled from a train, it takes some time to re-evacuate the chambers before the engine can start again. A small displacement of the end of the lever O to the right pulling the lever rod L for brakes to work is shown in Fig. 217.

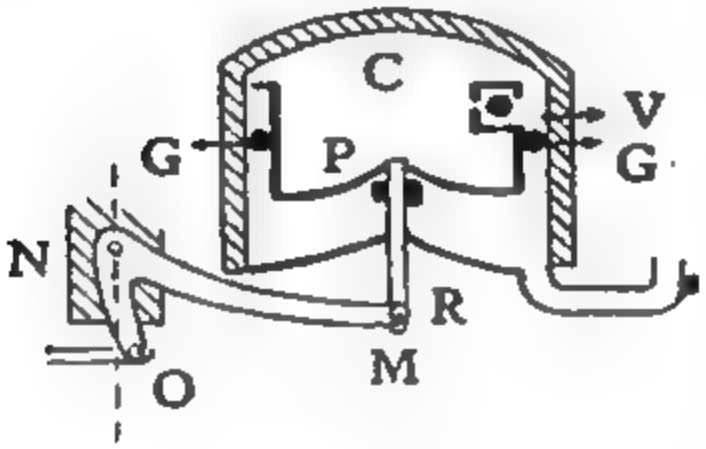


Fig. 217

The 'Westing house' automatic brake, similar in principle works by compressed air which pushes the piston up when a valve is released. Air is compressed under high pressure in a reservoir under the engine and this reservoir is connected to the train pipe through a valve. The opening of the valve fills the lower compartment of the chamber C with air at high pressure raising the piston and pressing the brake shoes on the wheels. To release the brake another valve is opened and this compressed air escapes to atmosphere lowering the piston. The usual 'hissing' of the electric tram cars at the time of start is due to the escape of compressed air to the atmosphere through the release valve.

151. Condensing or Compression Pump.—This is used for compressing air into a closed vessel called the receiver. It consists of a

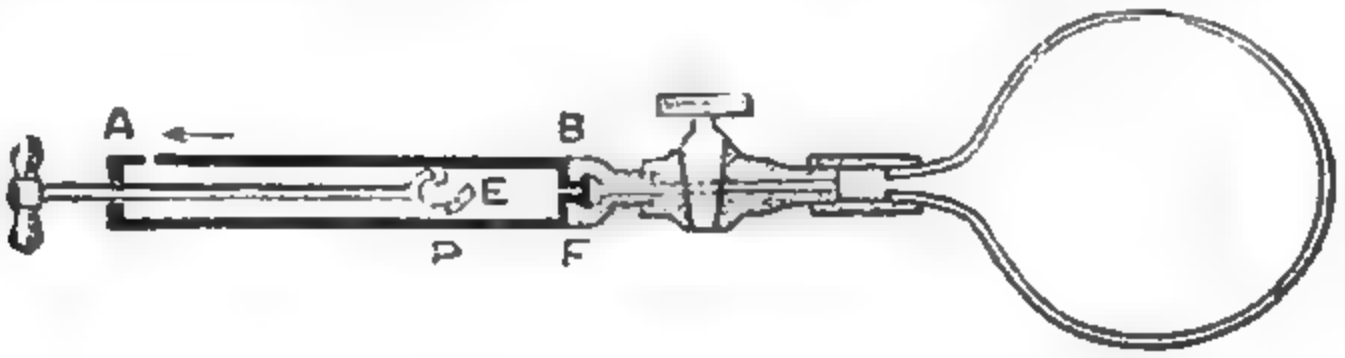


Fig. 218—Compression Pump

barrel AB (Fig. 218), provided with an air-tight piston P, which can be moved in or out by a suitable handle. The end of a barrel may be connected through a stop-cock to the vessel into which air is to be compressed. The stop-cock is closed when the required degree of compression has been attained The piston and the end of the barrel are provided with valves E and F both of which open towards the receiver.

Action.—Let us start with piston at the end of the barrel towards the receiver. As the piston is drawn out during the *backward* stroke, the space between the valves E and F increases so that the pressure therein falls. The valve F is immediately closed owing to the excess of pressure of the air in the receiver. The atmospheric pressure opens the valve E and the barrel is filled with air at the atmospheric pressure.

As the piston moves inward during the next *forward* stroke, the air within the barrel between the valves is compressed. The inside pressure becomes higher than the atmospheric pressure when the valve E is at once closed. When the pressure within the barrel becomes greater than that of air in the receiver, F is pushed open and the air in the barrel is forced into the receiver. Thus at the end of each outward stroke, the barrel is filled with some air at the atmospheric pressure, which during the next inward stroke, is forced into the receiver. Therefore the density of air within the receiver becomes higher and higher with number of strokes.

Degree of Compression.—The density and pressure of air in the receiver after a number of strokes can be calculated in the following way.—

Let the volume of the receiver with that of the connecting tube $= V$
volume of barrel between extreme positions of the piston $= v$
density of atmospheric air at the time $= \rho$
density of air within the receiver after n strokes $= \rho_n$

Suppose the air in the receiver has initially the same pressure and density as those of the atmospheric air. It is evident that at the end of each forward stroke a quantity of air of volume v and density ρ and so of mass $v\rho$ is forced into the receiver.

Now mass of air initially within the receiver $= V\rho$
Mass of air forced into the receiver by n strokes $= nv\rho$
∴ Total mass of air in the receiver after n strokes $= V\rho + nv\rho$

As the volume of the receiver remains equal to V,—

$$V\rho_n = V\rho + nv\rho \text{ whence } \rho_n = \left(1 + \frac{nv}{V}\right)\rho \quad \ldots \quad (151,1)$$

Again if the temperature remains constant during the time, Boyle's law holds good and so the pressure is proportional to the density. Hence if the pressure of air initially in the receiver be P and that of the air inside after n strokes be P_n then,

$$P_n = \left(1 + \frac{nv}{V}\right)P \quad \ldots \quad (151,2)$$

If the air in the receiver was initially at the atmospheric pressure, then the pressure of the air inside after n strokes is

$$P_n = \left(1 + \frac{nv}{V}\right) \text{ atmospheres.}$$

152. Different uses of a Condensing Pump.—The ordinary foot-ball inflator (Fig. 219) is a common example of *condensing pump*. Within a barrel works a piston N connected with the rod R to the handle H. The valve at the piston head is replaced by a cup-shaped

Fig. 219

Fig. 220

disc of leather, called *cup valve* or washer. The piston disc is a little smaller than the barrel of the pump. During the *inward* stroke, the washer expands due to the compressive reaction of inside air and fits air-tight within the barrel. When the pressure of the compressed air within the barrel becomes higher than the pressure within the receiver, a light ball valve V at the nozzle of the pump moves to the right to make room for air to pass into the receiver. A perforated diaphragm D does not allow the ball valve to move too far. During all the time of the inward stroke, air from outside enters into the left part of the barrel through the port-hole of the handle. During the *outward* stroke (Fig. 220), the volume of the enclosed air within the receiver in trying to expand forces the ball valve to sit air-tight into the conical hole and so entrapped air cannot leak out. The space between the piston and the nozzle becomes partly vacuous. The atmospheric air then forces its way through the corners of the washer filling up the space. For the next inward stroke, some air is again forced into the receiver.

Fig. 221 shows a bicycle inflator, which is essentially the same as a foot ball inflator, but instead of a valve at the nozzle O of the pump there is a separate attachment of an air-tight valve-tube as shown in Fig. 222. The compressor as usual consists of a cylindrical barrel B, in

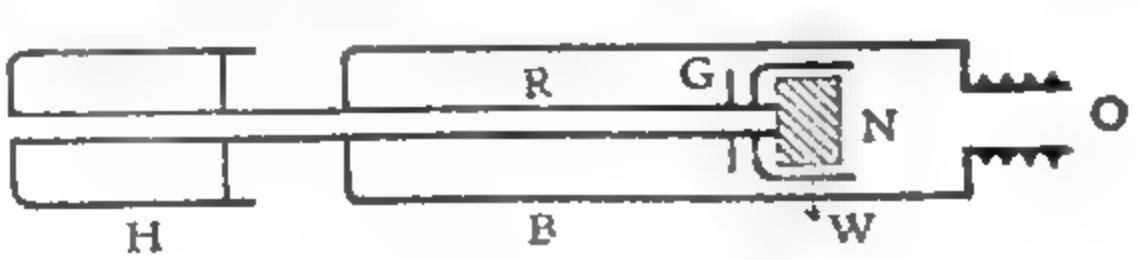


Fig. 221

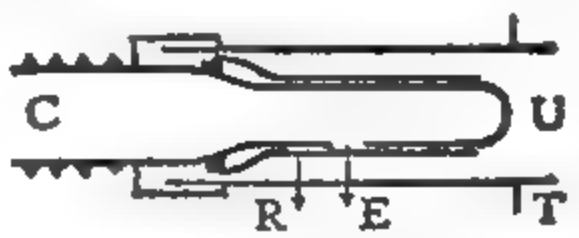


Fig. 222

which works a piston N with its leather washer W, a fixing disc G and the piston rod R with its handle H The nozzle is provided with a screw thread to which the screw cap of a short rubber tubing can be fixed air-tight. The other end of this tubing is screwed to the end C of the valve tube permanently attached to the tube of the wheel. The valve tube (Fig. 222) consists of a narrow metal tube U closed at the extreme

end and having an opening E at its side. It is preserved in an outer metal casing T. There is a rubber cylindrical jacket R covering the valve tube.

When the pressure of the compressed air forced by the pump into the space around C exceeds that on the other side of the rubber cylinder, the flap of rubber at the opening E is raised up and air is admitted into the tube. But when the handle H is drawn out, the pressure around C falls and air from the tube of the wheel, in an attempt to force its way through E, tightly presses the flap on the hole and closes the opening. Hence with a continued operation of the handle, more and more air is pumped into the tube. This inflated tube rests within a thick India-rubber tyre called, the **pneumatic tyre.** The compressibility of the enclosed air helps the wheels to pass undeviated over obstacles ; while due to the expansibility of air, the tyre regains its original form after being depressed.

Air Cushion.—This consists of a hollow bag of rubber sheet with a small mouth at one side fitted with a stop-cock By means of a *condensing pump* air is compressed into the bag to the required extent when the stop-cock is closed. Owing to its compressibility and expansibility the enclosed air serves as the cushion. **An air-gun** may be regarded as a *condensing pump* without any valve. By a mechanism, the air in the barrel is suddenly compressed. This air when released works upon a spring which throws out the shot.

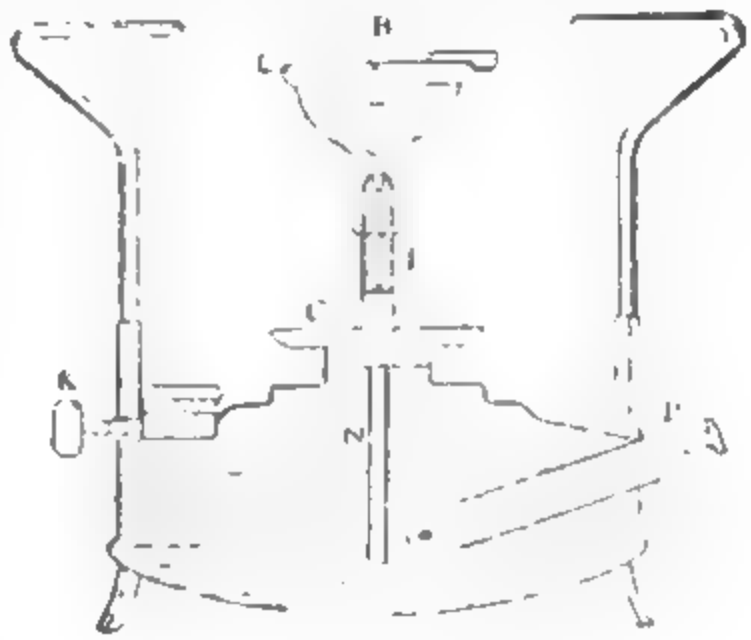


Fig. 223

Oil Stove.—In stoves and various lamps in which oil vapour burns, the oil pot is designed to leave some space above the oil level. A small *condensing pump* P is fitted with the pot (Fig. 223). There is an air tight key K, which can keep air entrapped within the pot when necessary. Air being pumped into the pot bubbles through the oil and is stored at its upper part. The pressure of air forces the oil to rise along the pipe N to the jet-tube J. There is a coiled wire gauge within the tube, through which the oil has to run up. If this wire gauge is sufficiently heated before the first circulation of oil, then the oil in course of reaching the jet is converted into vapour. The oil vapour rebounds at the burner B and circulates within the funnel. The heat of the burner is sufficient to ignite the vapour which burns at its mouth.

Compressed air is used in spray painting, in driving boring machines, propelling torpedoes etc. The **soda-water machine** is practically a *condensing pump* Carbon dioxide gas is taken from the reservoir in which it is generated and then forced into a bottle containing water. The gas is absorbed by the water, which is said to be *aerated* and this water is called *soda water.* All the machines described in this Article work on the principle of utilisation of the power of air.

153. Diving Bell.—It is used to enable a diver to go down to a great depth under water to lay the foundation of pires or do some such work there. It consists of a heavy cylindrical or bell-shaped iron vessel A with rigid walls, closed at the top and open at the bottom (Fig. 224). There is a platform attached inside the bell on which the worker can take his seal. It is lowered into water by means of chains C with the open end downwards, and it sinks under its own weight. The air inside is compressed and water rises slowly inside. Thus taking the height of the water barometer as 34 ft., the air inside the bell would be under a pressure of 2 atmospheres at a depth of 34 ft. and so its volume would be reduced to half. Evidently this will be inconvenient for the workmen inside.

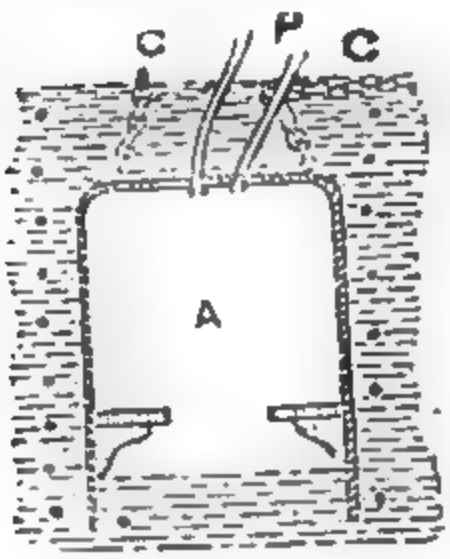


Fig. 224

Now-a-days to keep down the water to a convenient level inside the bell and to supply the workmen freely with fresh air, air is pumped into the bell through the tube P at its top by a *condensing pump* above. A second tube leading from the bell to the surface carries away the foul air from inside.

Examples :—

1. The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1000 c.c. respectively. How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres? (C. U.—1925; Gau. U.—1932).

A. Let the required number of strokes be n.

Then $$4 = \left(1 + \frac{75n}{1000}\right) \text{ whence } n = 40$$

2. A bottle whose volume is 500 c.c. is sunk mouth downwards below the surface of a tank containing water. How far must it be sunk for 100 c.c. of water to run up into the bottle? The height of the barometer at the surface of the tank is 760 mm., and the specific gravity of mercury is 13·6. (Pat. U.—1928)

A. The volume of air inside the bottle when 100 c.c. of water rushes in, is 400 c.c. If the pressure of the air within the bottle be now P then from Boyle's law, $P \times 400 = 76 \times 500$ whence $P = 95$ cm. of mercury.

Hence pressure due to water $= (95 - 76) = 19$ cm. of mercury.

$= 19 \times 13{\cdot}6 = 258{\cdot}4$ cm. of water

∴ The bottle must be sunk to a depth of 258·4 cm. under water

2 The volume of the receiver in a condensing air pump is 10 times that of the barrel. Calculate the number of strokes necessary to raise the pressure of air in the receiver to 5 times that of external air. (Utkal. U.—1953)

A. If P_n be the pressure in the receiver after nth stroke, the atmospheric pressure P, then $P_n/P = 5$. Again the ratio of the volume of the receiver to that of the barrel $= V/v = 10$. Then from eqn. (151.2),

$$\frac{P_n}{P} = 5 = 1 + \frac{nv}{V} = 1 + \frac{n}{10} \quad \text{whence } n = \text{number of strokes} = 40.$$

4. A cylindrical diving bell 6 ft. high, is lowered into water until the top of the bell is 31 ft. below the surface of water. If the height of the water barometer at the time be 34 ft. find to what height water will rise into the bell. If the cross-section of the bell be 10 sq. ft. find how much air at atmospheric pressure must have to be introduced into the bell so as to keep away water from entering into it.

Let the water rise into the bell upto x ft. from the top.

Then the pressure of the air inside is equal to that of a column of water of height equal to $(34 + 31 + x)$ ft. $= (x + 65)$ ft.

The original and final volumes of air in the bell are proportional to 6 and x respectively.

$\therefore \quad 6 \times 34 = x\,(x + 65)$ or $x^2 + 65x - 204 = 0$ whence $x = 3$ ft. or -68 ft Since a negative value of x is absurd, we have to accept the positive value of x to be 3 ft. Original volume of air in the bell $= 60$ cu. ft. initial pressure $= 34$ ft.

When the top of bell is 31 ft. under water and all water entering it is kept away, pressure of air inside $= (34 + 31 + 6) = 71$ ft. water.

Let the volume of air introduced under atmospheric pressure $= v$ cu. ft.

Then $34\ (6 + v) = 71 \times 60$ whence $v = 65{\cdot}3$ cu. ft.

154. Pressure Gauges.—In order to be able to know how much a particular type of the air pump can evacuate or compress air into a receiver, we have to use a class of apparatus which indicates the pressure of the residual gas within the receiver. Such a pressure measuring device is called a *pressure gauge*. There are various types of the instruments designed for different ranges of pressures.

Siphon Gauge.—It consists of a glass tube CBD, one limb CA of which is straight while the other limb D is bent at the top and is connected to the receiver (Fig 225). The U-tube is about half-filled with a suitable liquid such as mercury, oil or water and is fixed vertically to a stand fitted with a scale. When the pressure to be measured is either *a little above or below the atmospheric pressure*, the end A is kept open. If the pressure within the receiver is equal to the atmospheric pressure, the column of liquid has at its ends the same pressure and hence the free surfaces of the liquid within the U-tube are at the same level. But if the pressure within the receiver becomes higher, it would push down the level at D causing the other level at C to rise up. If, in the steady condition, the difference of levels in C and D be h cm, as recorded by the scale attached, then from the law of hydrostatic pressure,—

Pressure within receiver $= h\rho g$ + atmospheric pressure ;

The density of liquid used is ρ and acceleration due to gravity is g. Thus the difference of gas pressure and atmospheric pressure is proportional to the difference of levels. When the gas pressure falls below the atmospheric pressure, the level at D goes up and C down, the difference of the pressures being also $h\rho g$. For a given difference of pressures, $h\rho g$ would be constant and hence lighter is the liquid used, the larger would be the difference of levels. Thus a lighter liquid would be more convenient to find a small difference of pressures.

When the pressure to be measured is *high*, say upto 5 or 10 atmospheres, the limb C is closed at the top and the part AC contains some dry air. As the pressure within the receiver is increased, the liquid level in D is depressed and that in C is forced up, so that air enclosed is gradually compressed. When the temperature remains fairly uniform and assuming that the bore of the part AC is uniform, we can apply Boyle's law to find the resulting pressure. If the length of the air column in AC is l_1 when the two levels are same under atmospheric pressure P_1 and if the length of the column contracts to l_2 under a preassure P_2, then from Boyle's law.

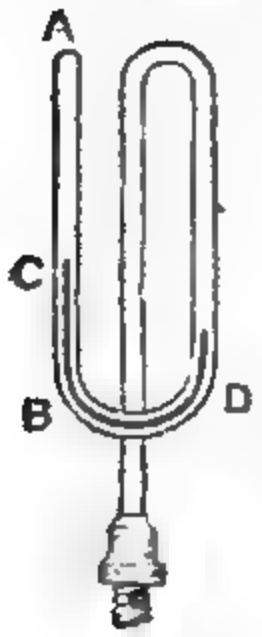


Fig. 225

$$P_2 l_2 = P_1 l_1 \quad \text{whence} \quad P_2 = P_1 \frac{l_1}{l_2} \qquad \ldots(154,1)$$

If at this stage, the difference of levels of the liquid be h, then the gas pressure within the receiver is $P_2 + h\rho g$. Suppose that mercury of density 13 59 gm/c c., is used as the gauge liquid and l_1 is 30 cm. If on connecting the receiver, l_2 becomes 8 cm., then the pressure P_2 within the closed end is $30 \div 8 = 3{\cdot}75$ atmospheres. If the barometer shows the atmospheric pressure to be 75·14 cm. of mercury, then $P_2 = 3{\cdot}75 \times 75{\cdot}14$ cm. $= 281{\cdot}775$ cm. of mercury. If now the difference of levels of the mercury column be 43·12 cm., then the gas pressure within the receiver is $281{\cdot}775 + 43{\cdot}12 = 324{\cdot}895$ cm. of mercury.

McLeod Gauge.—This pressure gauge was designed by McLeod in 1874 and is used extensively in measuring very low-pressures which are far beyond the scope of a Siphon gauge. The apparatus consists of a closed top calibrated glass capillary tube B of about ·5 mm. bore and of length about 50 cm. fixed at the top of a cylindrical glass bulb of capacity 200 to 500 c.c. (Fig. 226). The bulb at its lower end is connected by a uniform glass tube extending upto E. Another capillary glass tube with open end at D is also sealed just below the cylindrical glass bulb and is about a metre in length. The two capillaries run *vertically* along side a metre scale (not shown in the diagram). A rubber tubing is connected at E and its other end is in communication with a mercury reservoir F. This reservoir can be placed at any height by suitable clamping arrangement.

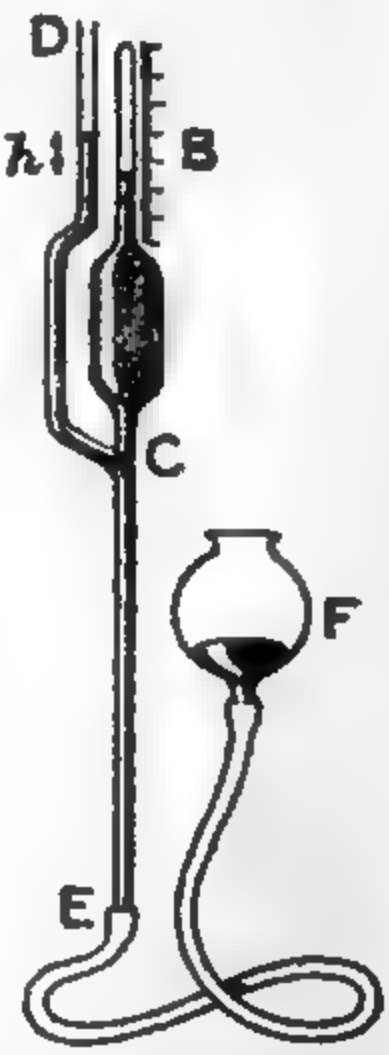


Fig. 226

The end D is connected to the receiver whose pressure is to be measured and so long as the receiver is not *attaining a uniformly low pressure*, the reservoir F is lowered sufficiently so that mercury level in the tube CE is below the point C. At this stage there is a free communication of pressure of the receiver to the bulb and the capillaries. Now the reservoir is slowly raised and when the level of mercury is above the point C, the

reservoir is cut off from the bulb. The reservoir is raised more and more till mercury fills the bulb and reaches a point B in the capillary. The mercury in D would remain at a higher level. This happens because after the bulb is cut off from the receiver by the mercury column, any compression of the residual gas in the closed tube obeys Boyle's law,—pressure inside increasing as volume is compressed. But mercury within the other capillary rises against a constant pressure of the source.

If V represents the volume of the bulb and the closed capillary, v_o the volume of 1 mm. length of the capillary, and a the length of the enclosed air column and h the difference of levels in mm. of mercury within two capillaries, p the receiver pressure in mm. of mercury, then remembering that at the cut off point volume of the gas is V and the pressure p,

$$pV = av_o(p+h) \text{ or } \frac{p+h}{p} = \frac{V}{av_o} \quad \text{or} \quad 1+\frac{h}{p} = \frac{V}{av_o}$$

$$\text{whence } p = h\frac{av_o}{V-av_o} = \frac{h\,a\,v_o}{V} \text{ very approximately} \qquad \ldots (154,2)$$

Suppose that $h = 1$ mm., $av_o = 5$ cu. mm. and $V = 500$ c.c. $= 5 \times 10^5$ cu. mm.; then substituting in the equation the pressure recorded is 10^{-5} mm. of mercury. Pfund has constructed a most sensitive form of a McLeod gauge capable of measuring a pressure of 10^{-7}mm. of mercury. Very lately the gauge has been further modified to read a pressure of 10^{-9}mm.

Limitation of so-called Vacuum.—It is very difficult to say which is the highest possible vacuum reached till the present time. With a very best type of pump and pressure gauge, the lowest pressure recorded so far is about 10^{-8}mm. with a flickering value of 10^{-10}mm. of mercury. For a steady and continuous working of a pump, a receiver shows a constant pressure of 10^{-6}mm. So it is safe to say that a pressure of 10^{-8}mm. is always under our control. We may now calculate how many molecules of the gas are still present per unit volume of the receiver at such a low pressure. From Avogadro's law of gaseous volume and from other evidences, we now know that at normal temperature and pressure 1 cubic cm. of a gas contains 3×10^{19} molecules. From this if we calculate the number of molecules in unit volume still present at a pressure of 10^{-8} mm. the number comes out to be nearly 1000 million, which is about the present population of Asian continent. Thus 'perfect vacuum' as speculated by early philosophers is still remote.

*155. **Molecular Motion.**—That there is molecular motion in solid, liquid and gaseous substances is borne out by the following facts.

Diffusion of Solids.—It has been shown by Sir W. Roberts Smith that if a layer of lead is kept on a layer of gold at ordinary temperature for some months, minute traces of gold are found in different layers of

lead. At moderately high temperatures above 500°C many metals show this property of diffusion to a remarkable degree.

Sublimation.—In case of *sublimation*, due to evaporation of molecules a solid directly passes into gaseous state.

Evaporation.—The molecules in the liquid state of a substance possess kinetic energy and some of them proceeding towards the surface may escape into the space above. With rise of temperature, the motion of the molecules increases and so the rate of evaporation increases.

Diffusion.—Take a tall jar partly filled with water. Introduce a strong solution of copper sulphate carefully into the bottom of the water column by means of thistle funnel. If the cylinder is now left undisturbed, the blue colour moves upwards and ultimately after a few hours the whole mass assumes a uniform colour. Hence the molecules of copper sulphate *diffuse* upwards against the attraction of gravity while those of water pass downwards in the solution.

Osmosis.—Diffusion may take place in liquids, even if they are separated by porous partitions, such as a sheet of parchment, the bladder of an animal or a piece of unglazed porcelain. The phenomenon is known as **osmosis.**

Brownian Motion.—A small amount of very fine powder of insoluble carmine is thrown into water. If observed with a very powerful microscope, the particles are found to be in a state of continuous agitation, as if they are continuously colliding against the neighbouring molecules. This phenomenon is known as Brownian Motion and can be explained on the supposition that the molecules are in a state of rapid motion. If very minute droplets of oil be suspended in stagnant air, the oil drops are found to be continuously in motion in all possible directions, being bombarded by the gas molecules from different directions.

Expansibility of Gases.—In the case of exhausting a closed space by an air pump, as soon as a quantity of the air is taken out, the remaining air fills up the vessel. This continues even when a trace of air is left, indicating that the molecules must be in a state of *rapid motion.*

Pressure of a Gas.—Owing to their continuous motion, the molecules of gas contained in a vessel strike against the walls. The change of momentum of the molecules due to such collision causes an outward pressure on the walls, which is the pressure of the enclosed gas (vide Heat).

Diffusion of Gases.—If two gases be contained in two closed vessels with a common partition wall, then on making a small hole in this wall, the gases will diffuse into each other. After a short time there will be a uniform mixture within the two compartments. This mixing takes place whatever be the nature and the pressure of the gases.

156. Absorption of Gases in Liquids and Solids.—If a quantity of water is heated slowly in a glass beaker, small bubbles of air

appear on the walls which escape to the surface. This shows that water contains dissolved air. Soda water is ordinary water which has been made to absorb carbon dioxide at a high pressure. The amount of gas absorbed by liquid depends on the nature of the gas and with a particular gas it increases with pressure and decreases with a rise of temperature.

Occlusion of Gases.—Even solids, particularly when they are porous, absorb gases to some extent within their pores. This phenomenon is known as *occlusion* or absorption. The absorption is here *selective* ; thus, a freshly heated piece of charcoal absorbs 10 times its volume of oxygen and 90 times of ammonia. Platinum black may absorb 250 times its volume of oxygen. The occluded gases may be driven out by heating. Take a barometer tube and fill it *partly* with mercury and then closing the open end by thumb invert it over a mercury trough. The space above mercury column now contains some air. Now introduce a small piece of freshly heated charcoal into the tube. The charcoal will absorb some of the air above mercury in the tube, the level of which will be found to rise.

Glass and metals also exhibit absorption of gases to a certain extent. It is often found that a little quantity of some gas has found its way into the Torricellian vacuum of most carefully prepared barometers kept for some time under observation under an ideal leak-tight condition. This is due to slow release of gas from the walls of the vessel. Gases may be given off from the *inside surface* of the walls or they may be forced out from the *interior* of the walls. Such occluded gases are a source of trouble to the production of low vacuum since they cannot be driven by chemical processes. Gases which come from the surface of the walls, called *absorbed gases*, are only a few molecules thick and are held very tenaciously by the solid surface in ordinary circumstances. They are probably held fast by the forces of surface tension and adhesion. These absorbed gases may be removed easily by moderately heating the solid in vacuum or by subjecting it to electric discharge.

During the process of manufacture of glass and metals, they require to be melted. In such a liquefied state gases are absorbed. These gases are prevented from escaping by the surface tension of the liquefied mass and its viscosity. When solidified such gases remain entrapped within its body and are called *absorbed gases*. Such absorbed gases can be driven off by raising the temperature of the solid much above its melting point in a vacuum furnace. The gases released from glass are largely water vapour and carbon dioxide and those from metals are mainly hydrogen and carbon monoxide.

OTTO VON GUERICKE (1602—1686)

Guericke was born at Magdeburg in Germany on October 20, 1602. He first studied law in Germany and then went to Holland to learn mathematics. After finishing his career there, he went to France and

England for higher studies. After return Guericke made some contributions in frictional electricity and is said to have constructed a form of friction machine. He was made a senater and later burgomaster of his native town.

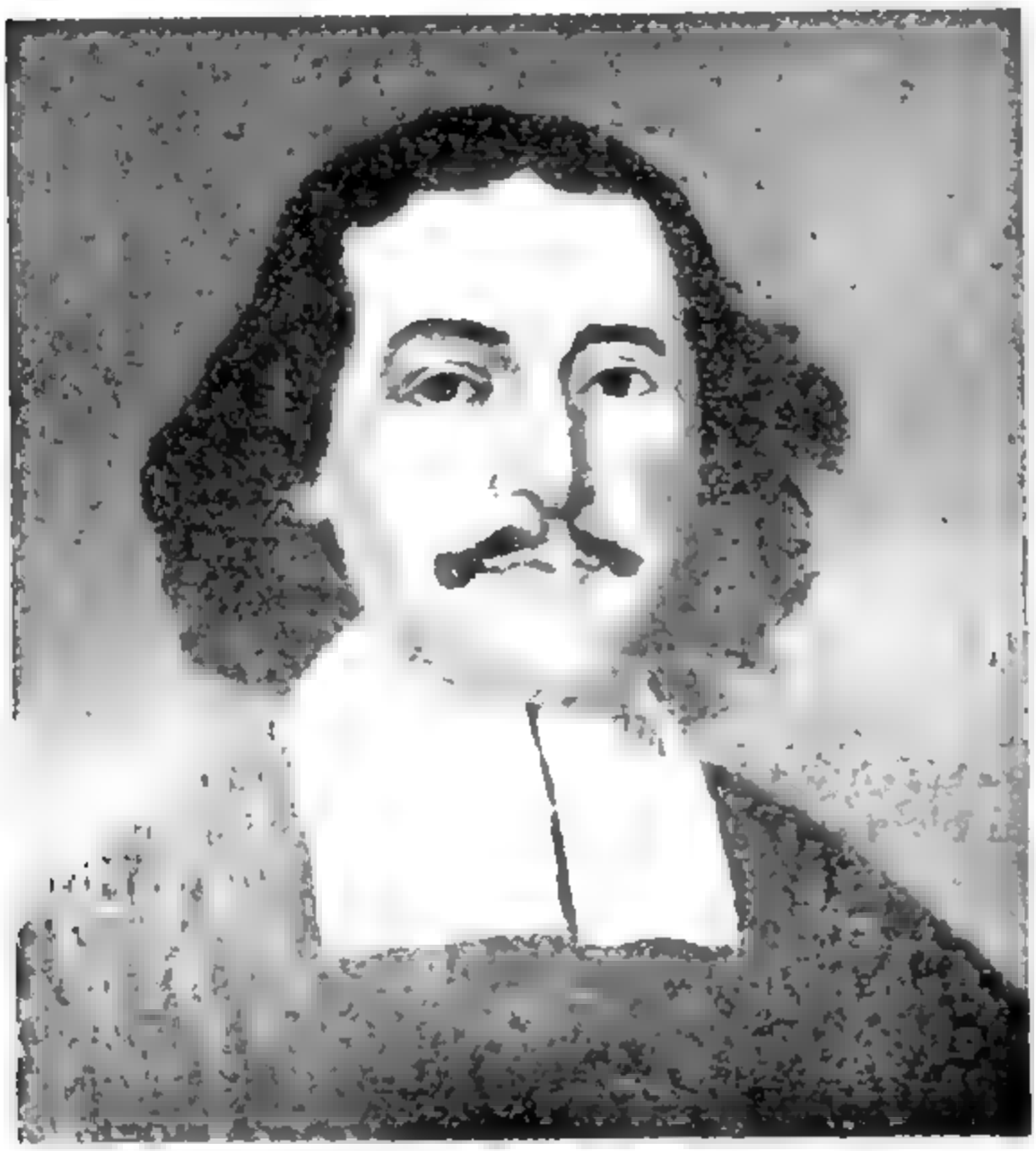


Fig. 27—Otto Von Guericke

He invented air pump in 1650 and proved that air has weight. Then he devised the famous "Magdeburg hemispheres" and by evacuating the hemispheres by an air pump, he proved before a large concourse of people that air exerts pressure. It is said that in one of his experiments, the strength of 25 horses was necessary to separate the evacuated hemispheres. He also demonstrated that the mass of a globe filled with air was always greater than the mass of the same globe when air was taken

[illegible] proved that air has weight. He devised a form [illegible], which could indicate an approaching storm. He [illegible] death. Boyle used his air-pump and discovered [illegible] pressures

Summary

[illegible] pump [illegible] to raise water from a lower level, the difference [illegible] than the height of the water barometer.

[illegible] pump is used to raise water to any desired height above the pump.

[illegible] **pump** with the arrangement for *continuous flow* is used to **transfer** [illegible] either at a great distance or at a great height from the source.

[illegible]

The **siphon** is used for transferring a **liquid** from one vessel into another [illegible] the whole [illegible] liquid.

The air **pump** is used for taking air out of a closed vessel. The *density of* [illegible] gradually [illegible] successive strokes.

The **condensing pump** is used to compress air into a closed vessel. A **pressure gauge** is used to indicate the pressure of air in a closed space.

EXERCISES ON CHAPTER XII

Reference

1. Why it is not possible to raise water more than 34 ft. by means of a suction pump. State reasons and give calculations. (U. P. B.—1951 ; Dac. U.—1932) — Art. 141

2. The spout of common pump is 7 ft. above the level of water in a well. The diameter of the piston is 4 inches. What is the force on the piston rod when the pump is working? Weight of 1 cu. ft. of water is 62·5 lbs. (All. U.—1931) — Art. 141

A. 38·2 lb. wt.

3. Draw the sectional diagram of a force pump capable of delivering a continuous supply of water. Explain how it works. (Utkal. U.—1952) — Art. 143

4. Describe a pump suitable for irrigating the fields by lifting water from a well. What mechanism would you suggest to lift water from a well which is deeper than 35 ft.? (U. P. B.—1941) — Art. 143

5. A siphon is used to empty a cylindrical vessel filled with mercury. The shorter limb of the siphon reaches the bottom of vessel which is 45 inches deep, but it is found that the mercury ceases to run before the vessel is empty. Explain this. (Dac. U.—1930) — Art. 144

6. Describe a siphon and explain the principle of its operation. State the conditions for its working. For what purpose is the siphon used? (C. U.—1948) — Art. 144

7. Explain the principle and action of a siphon. (Utkal. U.—1952 ; Pat. U.—1953 ; C. U.—1937 ; U. P. B.—1946 ; Del. H. S.—1950) — Arts. 144 & 146

8. Describe a filter pump and explain its mode of action. (U. P. B.—1946 ; Raj. U.—1949) — Art. 146

9. Describe an air pump and explain its action. (*Cf.* Raj. U.—1952 ; Gau. U.—1953 ; C. U.—1917 ; Pat. U.—1952 ; U. P. B.—1950 ; *cf.* E. P. U.—1953 ; Mys. U.—1951, '52) — Art. 146

10. Describe with the help of a clear diagram a pump for lifting water from a well about 15 ft. deep. What trouble will be experienced if the depth of the well exceeds 30 ft. (U. P. B.—1945) — Art. 141

11. If the barrel of an air pump is one-third the size of the receiver, what fractional part of the original air will be left after 5 strokes? What will a barometer within the receiver read, the outside pressure being 76 cm.? (Gau. U.—1953) — Art. 146

A. $\frac{243}{1024}$; 18 cm. nearly.

12. If the pressure in a pump were reduced to $\frac{1}{3}$ of the atmospheric pressure in 4 strokes, to what it would be reduced in 6 strokes? — Art. 146

A. $\frac{\sqrt{3}}{9}$ of the initial pressure.

13. The volume of the barrel of an air pump is 1/5th that of the receiver. Find after how many strokes the density of air in the receiver will be reduced in the proportion 125 : 216. — Art. 147

A. 3.

Reference

Art. 146 14. The contents of the receiver of an exhausting air pump is 6 times that of a barrel. Find the pressure of the air in the receiver at the end of the eighth stroke of the piston, when the atmospheric pressure is 15 lb. to the square inch. (Utkal U.—1951)

A. 4·87 lb./sq. in.

Art. 146 15. Describe a double-barrelled air pump and explain its action. (C. U.—1938, '53)

Arts. 146 & 151 16. Compare the pressures in the receiver of condensing and exhausting air pump after the same number of strokes and account for the fundamental difference in form of the two expressions. (Pat. U.—1931)

Art. 149 17. Explain the action of a rotary oil pump. (E. P. U.—1952)

Art. 151 18. Describe in detail, with a diagram, a condensing pump and its mode of action. (Utkal U.—1951; C. U.—1934)

Art. 151 19. The volume of the receiver in a condensing pump is 25 times that of the barrel. Find after how many strokes the pressure inside the receiver will be increased from one to three atmospheres.

A. 50.

Art. 152 20. Explain the mode of action of a foot-ball inflating pump. (P. U.—1929; Utkal. U.—1953)

Arts. 151 & 152 21. What do you mean by a compression pump? Cite two common examples. Describe with a diagram, the working of an ordinary bicycle pump and the action of the valve in the bicycle tube. (Gau. U.—1952)

Art. 153 22. Write a note on Diving Bell? (Utkal. U.—1952)

23. Explain how would you produce and measure a high vacuum. Describe some uses of high vacuum. (And. U.—1951)

24. Write short notes on various types of pressure gauges for the measurement of pressures greater and less than one atmosphere. (Utkal. U.—1952; *cf.* And. U.—1952)

CHAPTER XIII

ELEMENTS OF HYDRODYNAMICS AND AERONAUTICS

157. Hydrodynamics.—The branch of Hydrodynamics deals with the properties of fluids in motion. During the eighteenth century Euler and Bernoulli founded the classical hydrodynamics, which deals with the motion and properties of an 'ideal fluid'. The subject became highly mathematical without adequate experimental background, so that it formed a part of 'pure mathematics'. Experimentation with water or any other real fluid did not yield results consistent with the forecast of classical hydrodynamics. Engineers made the experimental observation in a systematic way and called this Book of collected data as Hydraulics.

Within the last fifty years, many theoretical and experimental researches have been carried on regarding the properties of liquid and gaseous fluids in motion. Scientists have now obtained a real picture of these and have been able to get a link between the classical hydrodynamic and hydraulics. The properties of gaseous fluids in motion have found a separate subject of study known as Aerodynamics. A part of Aerodynamics dealing only with the motion and properties of air in relation to an air-craft is called Aeronautics.

158. Ideal Fluid.—An ideal fluid is defined to be one, which can flow without offering any internal resistance. If two adjacent layers of an ideal fluid move with different velocities, there would be no viscous or frictional force of one layer on the other.

Stream Line Motion.—A fluid is always to be preserved in a particular vessel, whose shape it automatically takes up. Hence a fluid is incapable of withstanding any shearing force. In order that a mass of fluid may flow in a particular direction, there must be a difference of pressures acting at the ends of the fluid in question. For example, water flows from a higher to a lower pressure, wind blows from a region of higher atmospheric pressure to another at a lower pressure.

When an ideal fluid is supposed to flow through a finite container such as a pipe (Fig. 228), different layers of the fluid move parallel to each other. Let AB and CD be the walls of a pipe through which an ideal fluid is passing under a pressure $P_2 - P_1$. Then all the layers of the fluid would move parallel to AB and CD. This type of fluid motion is called stream line motion. In case of a pipe of unequal cross-section or a bent pipe, stream lines may be curved (Fig. 229 & 230). A stream line is defined to be a curve which is always tangential to the flow of the fluid, so that a fluid cannot cross a stream line but can only

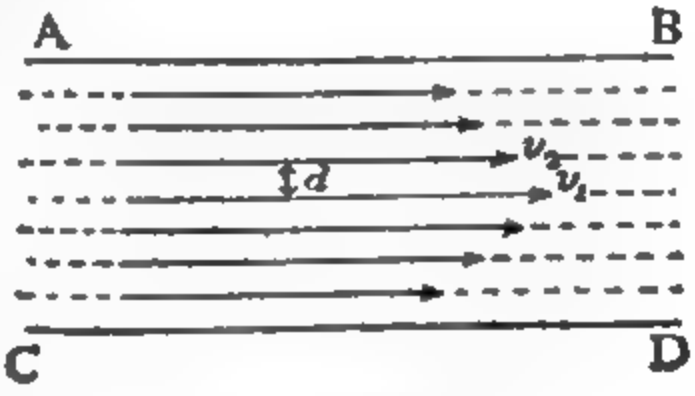


Fig. 228

flow along it. We can thus imagine stream lines to form a series of tubes through which the fluid can pass. Such imaginary tubes, through which a fluid is supposed to flow, are called stream tubes.

Equation of Continuity.—Whenever a fluid flows with a uniform velocity at any region, the pressure and density of the fluid at that part also assume steady values. This is one of the most fundamental conditions of a stream line motion of a fluid. Imagine a pipe of unequal cross-section through which a fluid is flowing (Fig. 229). The fluid enters through one end of the pipe and emerges out through the other end. Further more, since there is no variation of density at any region, *the total mass* of the fluid contained within the pipe *is always constant.* It follows therefore that the mass of the fluid entering through one end would be equal to the mass of the fluid leaving the other end in a given interval.

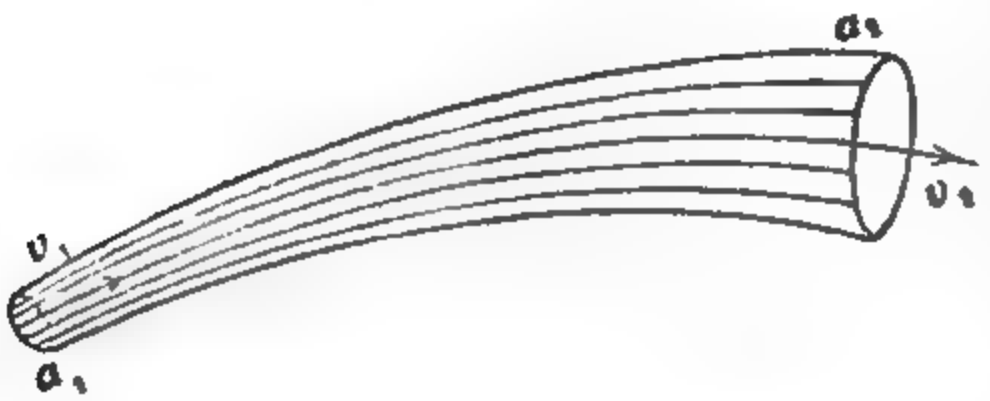


Fig. 229

If m be the quantity of a fluid entering the tube per second through one end of a cross-sectional area a_1 with a velocity v_1 and if ρ_1 be the density of the fluid at this region, then

$$m = a_1 v_1 \rho_1$$

If a_2, v_2 and ρ_2 denote the corresponding area of cross-section, fluid velocity and density at the other end through which the fluid is passing out, then

$$m = a_2 v_2 \rho_2 = a_1 v_1 \rho_1 \quad \text{or} \quad av\rho = \text{constant.} \qquad \ldots(158,1)$$

where a, v and ρ denote the general values of the cross-section, velocity and density of the fluid at any region of the pipe. Hence we can say that *the continued product of the density of the fluid, its velocity and cross-sectional area at any region of the pipe is constant.* If the fluid is almost incompressible, such as a liquid, then the density does not vary sensibly with a moderate difference of pressures. Hence $\rho_1 = \rho_2$.

$$\text{Therefore } av = \text{constant.} \qquad \ldots (158,2)$$

In case of a stream line flow in a gaseous medium, the stream tubes behave as conducting pipes through which fluid moves and there is generally a variation of density of the medium from point to point along a stream tube. Hence eqn. (158,1) holds good. It has been found that for air, which is a compressible fluid, the density at various points of a stream tube remains fairly constant upto a speed limit of 200 to 250 miles per hour.

It may be noted that in a stream line flow, the number of stream tubes would be equal at any section of the pipe. If n be the number of stream tubes within the pipe, α_1 and α_2 the cross-sections of the stream tubes at the two regions of the pipe whose cross-sectional areas are a_1 and a_2, then $a_1 = n\alpha_1$, and $a_2 = n\alpha_2$. Now if density of the fluid does not change, then according to eqn. (158,2)

$$n\alpha_1 v_1 = n\alpha_2 v_2 \quad \text{or} \quad \alpha_1 v_1 = \alpha_2 v_2 \qquad \ldots (158,3)$$

This shows that in a stream line flow the product of the velocity of the fluid and the corresponding area of cross-section of the stream tube is constant. Thus where the fluid has a higher speed, the stream tubes would narrow down.

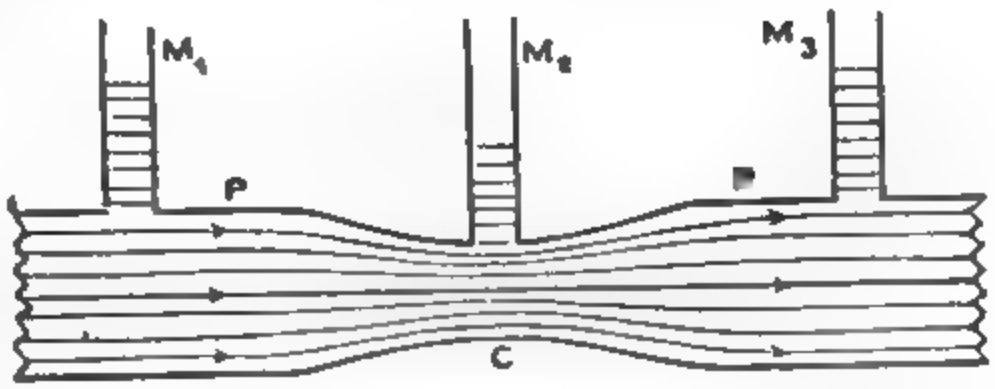


Fig. 230

Fig. 230 shows a pipe narrowed at the middle part and through which an ideal fluid is passing. Since the cross-sections at the ends are larger than that at the middle, then according to the relation given in eqn. (158,2), the velocity of fluid at the middle must be higher than that at ends. The stream tubes at the constricted part are much narrower than at the ends.

If the tube is provided with a number of pipes projecting vertically upwards, then while the *liquid is flowing through the tube*, it would exert some pressure at every point due to which liquid would rise to some fixed level in every pipe. The height of the liquid level within the vertical pipe at any region of the tube indicates the fluid pressure at that region.

159. Bernoulli's Theorem.—This theorem states that for any ideal incompressible fluid, free from any viscous resistance and flowing steadily along a pipe of non-uniform cross-section, the total change in energy per unit mass *taken along a stream line* between two levels is equal to the work done by the fluid in falling through the pressure difference. It is a statement of the principle of conservation of energy for a special case of a fluid.

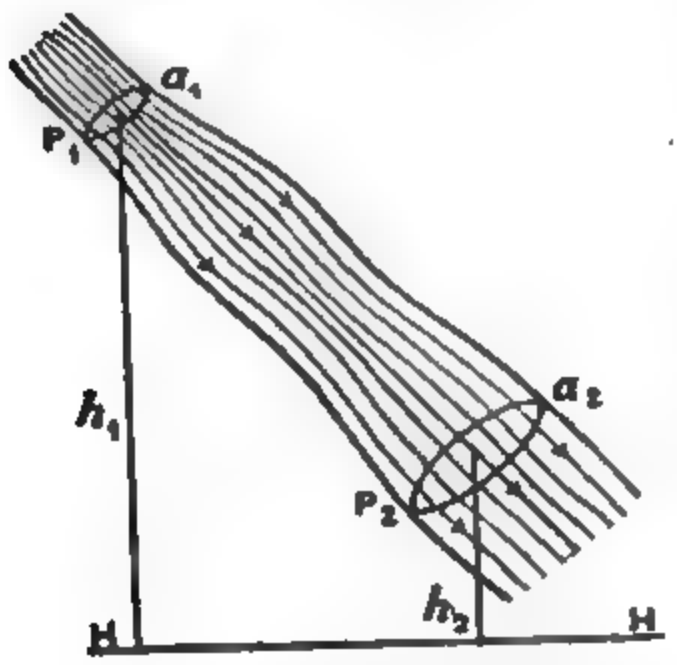


Fig. 231

Consider a pipe as shown in Fig. 231, through which a fluid is moving in a stream line motion. Let a_1 and a_2 be two cross-sectional areas taken at *right angles* to the stream at two regions of the pipe at heights h_1 and h_2 from some reference level HH. Let v_1 and v_2 be the speeds of the fluid at two regions. Let m be the mass of the fluid passing per second across any section of the tube. The kinetic energy of this mass of fluid over the section a_1 is $\frac{1}{2}mv_1^2$ and its potential energy is mgh_1. Hence the total energy over this section is $\frac{1}{2}mv_1^2 + mgh_1$. From a similar consideration the total energy of a mass m over the section a_2 is $\frac{1}{2}mv_2^2 + mgh_2$.

If V be the volume of the liquid having a mass m, then the work done when it moves from a pressure P_1 to a pressure P_2 is $(P_2 - P_1)V$. Then from the principle of conservation of energy,—

$$(\tfrac{1}{2}mv_1^2 + mgh_1) - (\tfrac{1}{2}mv_2^2 + mgh_2) = (P_2 - P_1)\,V$$

$$\therefore \quad \tfrac{1}{2}mv_2^2 + mgh_2 + P_2V = \tfrac{1}{2}mv_1^2 + mgh_1 + P_1V$$

If ρ be the density of the fluid, then $\rho = m/V$ and hence the equation may be written in the form,

$$\tfrac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 = \tfrac{1}{2}\rho v_2^2 + \rho g h_2 + P_2 \qquad \ldots(159,1)$$

Since the two sections have been taken at any two regions *along a stream tube*, the sum of the kinetic and potential energy-density together with the pressure is constant along stream line,

With reference to Fig. 230, it may be remarked, that when the pipe is kept horizontal, every part of it is at the same height from a reference level. Hence the potential energy density ($\rho g h$) is the same throughout. Thus eqn. (159,1) simplifies to $\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$. Let the velocity of the fluid and its pressure at the wider end be v_1 and P_1 ; and those at the constricted part be v_2 and P_2. Since the velocity at the wider part is less, the corresponding pressure at this region, as recorded by the manometer M_1 or M_2 would be higher than that at the middle.

Air in some respects behaves as an ideal fluid. Hence this theorem can be applied very approximately in case of air in a stream line motion. It would be seen subsequently in wind tunnel experiments (Art. 167) that there is no marked variation of air density for moderate air speeds along a stream tube. Let us assume that the friction between layers of air is so small that when a volume of air is set into motion relative to its neighbour, the frictional loss of energy is negligibly small.

When an air-craft moves through air, different parts of it may be considered to be at same height. Hence in so far as the distribution of the stream tubes round a flying aeroplane is concerned $\rho g h_1 = \rho g h_2$ and Bernoulli's theorem simplifies to

$$\tfrac{1}{2}\rho v_2^2 + P_2 = \tfrac{1}{2}\rho v_1^2 + P_1 \qquad \text{or} \qquad \tfrac{1}{2}\rho v^2 + P = \text{constant.} \qquad \ldots(159,2)$$

When the *static pressure* P is expressed in poundals per. sq. ft ρ is expressed in pounds per cubic foot ; but when the pressure is expressed as pounds per sq. ft.. the density has a technical name in *slugs* per cubic foot. The quantity $\tfrac{1}{2}\rho v^2$, which is really the kinetic energy per unit volume of the fluid, is sometimes called the *dynamic pressure* of the flow. It is the pressure felt at the nose of a projectile where the impinging stream is brought to rest.

Examples :—

1. Water is flowing steadily along a horizontal round pipe of varying cross-section. At a certain region the speed of the stream is found to be 10 ft. per second while the pressure is 35 lb. per sq. inch. The pressure at other place is 25 lb. sq. in. What is the speed there? What is the ratio of the radii of these sections?

A. Let $P_1 = 35$ lb./sq. in. $= 35 \times 32 \times 144$ poundals/sq. ft.
$P_2 = 25$ lb./sq. in. $= 25 \times 32 \times 144$ poundals/sq. ft.
speed $= x$ ft./sec.

Then $\tfrac{1}{2}\rho v_1^2 + P_1 = \tfrac{1}{2}\rho v_2^2 + P_2$.

or $\tfrac{1}{2} \times 62{\cdot}5 \times 10^2 + 35 \times 32 \times 144 = \tfrac{1}{2} \times 62{\cdot}5 \times x^2 + 25 \times 32 \times 144$

whence $x = 39{\cdot}7$ ft./sec.

From the law of continuity, if a_1 = area of cross section where speed is 10 ft./sec. and a_2 that where speed is 39·7 ft./sec.

$$10a_1 = 39{\cdot}7a_2 \quad \therefore \quad \frac{a_1}{a_2} = 4 \text{ approx. hence } \frac{r_1}{r_2} = 2.$$

2. Compressed air of density ·07 lb. per cubic foot is forced with a velocity of 40 ft. per second against an object in an wind tunnel. It is found that the stream tube under the body at a particular region has expanded to double its cross-section while that above it has reduced to one-third its original value. What are the pressures at these parts?

A. Let a be the cross-section of the stream-tube of velocity 40 ft./sec., v_1 and v_2 the velocities of upper and lower streams. Then from the equation of continuity, assuming the density to remain unchanged, $40a = \frac{1}{3}\ a\ v_1$ and $40a = 2v_2a$, so,

v_1 = velocity of upper stream = $3 \times 40 = 120$ ft./sec.

v_2 = velocity of lower stream = $40/2 = 20$ ft./sec.

If P = pressure of air far from the body and P_1 = pressure within the upper stream line then, from Bernoulli's Theorem,

$P_1 - P = \frac{1}{2} \times {\cdot}07\ (40^2 - 120^2) = -448$ poundals/sq. ft.

Hence at the upper surface there is a suction of 448 poundals per sq. ft normal to the surface.

Again if P_2 be the pressure within the lower stream line, then

$P_2 - P = \frac{1}{2} \times {\cdot}07\ \ (40^2 - 20^2) = 42$ poundals/sq. ft.

Hence at the lower surface there is a pressure of 42 poundals per square foot normal to the surface.

160. Torricelli's Theorem.—When a liquid flows out through a jet, the velocity with which the liquid escapes is equal to that which a body would acquire in falling freely from rest under the action of the gravity through a height equal to the depth of the jet below the free surface of the liquid.

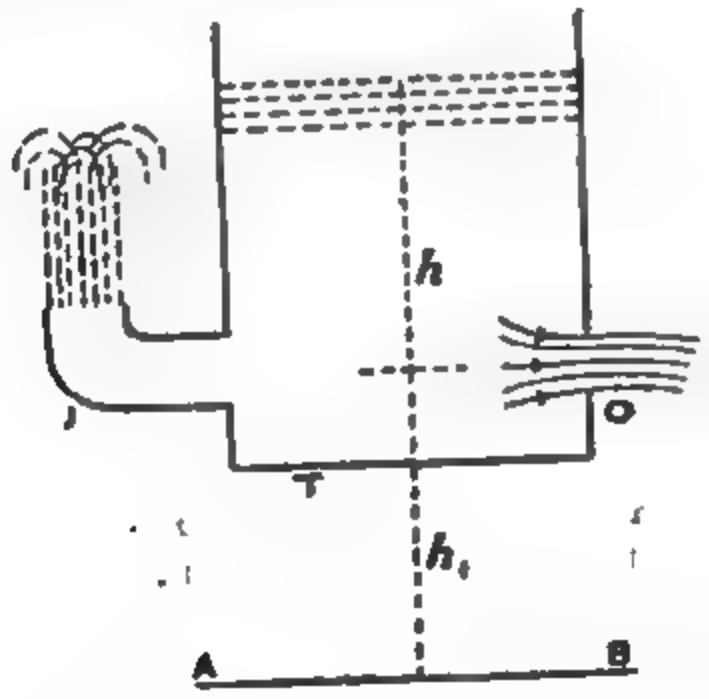


Fig. 232

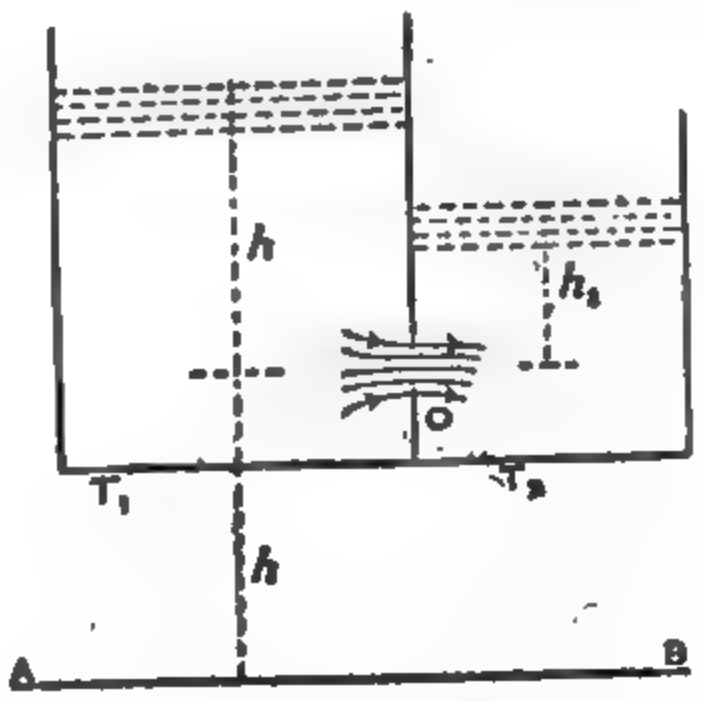


Fig. 233

Let T represent a reservoir containing a liquid (Fig. 232) and J and O two jets at a depth h below the free surface of the liquid within the reservoir and at a height h_1 from some reference level AB. The liquid escapes through the openings, but suppose that with an equivalent supply of liquid from outside, the level of liquid within the reservoir is kept constant. At the free surface of the liquid the potential energy per unit volume of the fluid is $\rho g\ (h_1 + h)$ and the pressure energy is P

where P is the atmospheric pressure. The kinetic energy is nil, since the liquid is stationary at the free surface. Again the pressure at the jet is P being open to atmosphere. Let the velocity of issuing water be v there. Hence kinetic energy per unit volume at the jet is $\frac{1}{2}\rho v^2$ and the potential energy is $\rho g h_1$. If the loss of energy due to friction be neglected, then applying Bernoullis theorem,—

$$\rho g(h_1 + h) + P = \rho g h_1 + \tfrac{1}{2} v\rho^2 + P$$

$$\text{whence } \rho g h = \tfrac{1}{2}\rho v^2 \text{ or } v^2 = 2gh \qquad \ldots(160,1)$$

This equation gives the velocity acquired by a body falling from rest under the action of the gravity through a distance h. If the jet be directed vertically upwards, the height to which water would be sprayed up would be equal to the height of the free surface of water within the reservior. The spray in fact rises to a height a little below on account of friction within the jet and with air.

If the jet enters into a second tank T_2 in which the liquid stands at a height h_2 above the jet (Fig. 233). then the eqn. (160,1) reduces to $v^2 = 2g(h - h_2)$

161. Real Fluid.—A real fluid is one which offers a viscous friction when two adjacent layers of it move with different velocities. The flow of a real fluid is obverved to be of two types as given below.

Stream **Line and Turbulent Flows.**—It is experimentally observed that when a liquid flows through a pipe, then for a moderate speed of a liquid through the pipe, any portion of the liquid flows paral-

Fig. 234—Stream line Motion Fig. 235—Turbulent Motion

lel to the axis of the pipe The flow of the liquid then may be divided into concentric cylindrical layers moving parallel to the axis of the tube Such type of flow is called the *stream line* or laminar flow (Fig. 234).

If the speed of the liquid through the pipe be continuously increased by increasing the pressure at the inlet, then upto a certain speed limit of the liquid, the stream line motion persists. The speed limit, which depends upon the nature of the liquid, the pressure causing the flow, smoothness of the walls and in short tubes upon the length of the tube is sometimes called the *critical velocity*. Beyond this speed limit, any portion of the liquid begins to move irratically in various directions—sometime towards and sometimes away from the walls in a "turbulent" manner producing *eddies* or *vortices* (Fig. 235). The transformation from a stream line to a turbulent flow may be observed through the glass walls when dust particles are mixed with the flowing liquid. Hence a critical velocity marks a transition from a stream line

to a turbulent flow. Such a turbulent motion of fluids was first observed and studied by Osborne Reynolds in 1883.

162. Utilisation of Water Power.—Nature supplies us with an inexhaustible source of energy from water-power. Rivers and canals carrying current are sources wherein we get vast quantities of water in motion and this huge kinetic energy may be utilised in very many useful purposes. In hilly countries streams of water are occassionally found to come down speedily from a higher to a lower level in form of 'falls' and in such process of a water-fall, the potential energy of water at the higher level is converted during its fall into kinetic and other forms of energy. These two forms of energy of water have found application to industry in two different ways,—

Overshot wheel.—This is a device to convert potential energy of water in a 'fall' into a regular power-supply. It is well-known that stream of a fall becames very violent during the rains and meek during winter; consequently to have a constant and regular supply of water at all seasons a large reservoir of water is constructed somewhere at the up-stream to store excess water during the rains and to use it during the dry season. Water from this reservoir is supplied by pipe lines or channels, known as *penstocks*, to a large water wheel through an adjustible aperture S called the sluice (Fig. 236). The diameter of this wheel is made nearly as large as the height of the fall. The wheel is provided with a large number of buckets B shaped in such a way that they can hold water rushing out from the sluice and retain it almost upto the bottom where it is discharged. Thus at any time nearly half the total number of buckets is full of water and it is the weight of this water that rotates the wheel.

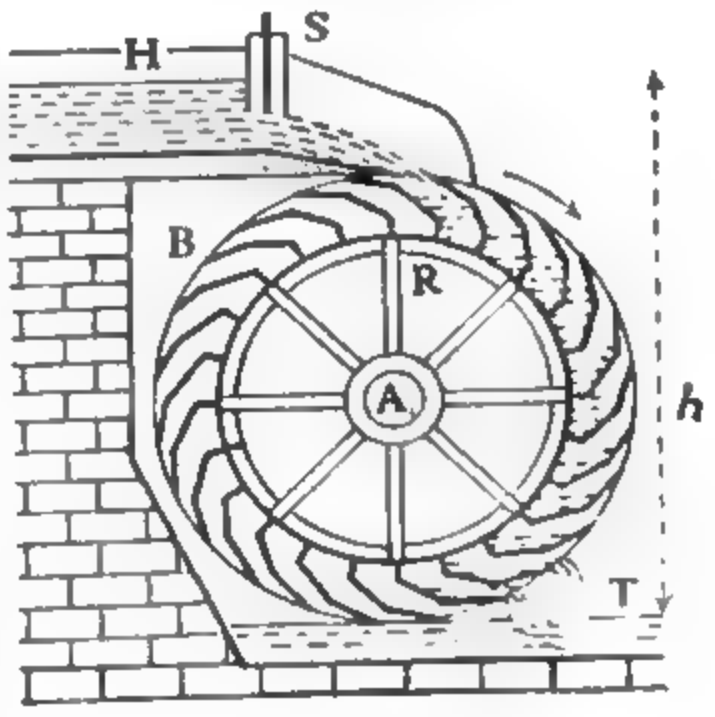


Fig. 236

If h is the total fall of water in feet and Q the number of cubic foot of water entering the buckets per second and w the weight on one cubic foot of water in pounds, then the available horse-power H. P. is given by the expression.

$$\text{H. P.} = \frac{Qwh}{550} \qquad \ldots (162,1)$$

It is evident from eqn. (162,1) that the larger is h, the greater would be the available power. Hence the diameter of the wheel is made nearly equal to the height of the fall. To give a proper shape, the size and number of the buckets cannot be indefinitely increased. Wheels having over 70 ft. in diameter are already in use. The efficiency of an overshot wheel is between 70 to 80%. The sluice is generally operated by an automatic control device known as a governor.

The power is led from the main shaft A or from a pinion attached to the underside of the wheel. Some times such an appliance is called an **overshot water turbine,** which can be used to run a factory or to generate an electric power. In Indian Republic there is a dearth of a powerful water fall where an overshot turbine might be profitably used.

Undershot wheel.—This device is taken where the kinetic energy of water is to be utilised in a power transformation. By means of a barrier called a *dam*, the width of the stream is narrowed down except for a small opening through which water is made to rush out. Since all the water of the stream has to find its way through this gate, the current at this region is considerably strengthened. This gives a high kinetic energy of water. The volume of water discharged is regulated by an adjustable door S, called a *sluice* (Fig. 237). Just beyond the sluice, there is a wheel provided with a number of fixed radial vanes V and mounted in such a way that water on rushing through the sluice gate impinges against the lowermost vanes and delivers its kinetic energy at the rim of the wheel in a forward direction. The wheel thus rotates uniformly since for all its positions equal number of vanes are acted on by water. The difference of bed level on the two sides of the sluice may be kept 3 or 4 ft. The power is taken from the main shaft A or any gearing arrangement. The kinetic energy of water is not very well taken up by the straight vane type of undershot turbine, as a good part of water within the vanes rebounds backwards producing sprays and vortices and these are generated at the cost af energy of water doing no useful work. The efficiency of such type of weeels scarcely exceeds 30%.

Poncelet designed an undershot wheel with curved vanes as shown in fig. 238. Water at a high speed is ejected through the sluice S into the curved blades at the lowermost position of the wheel. During the short interval of stay of water within the blades, it exerts an im-

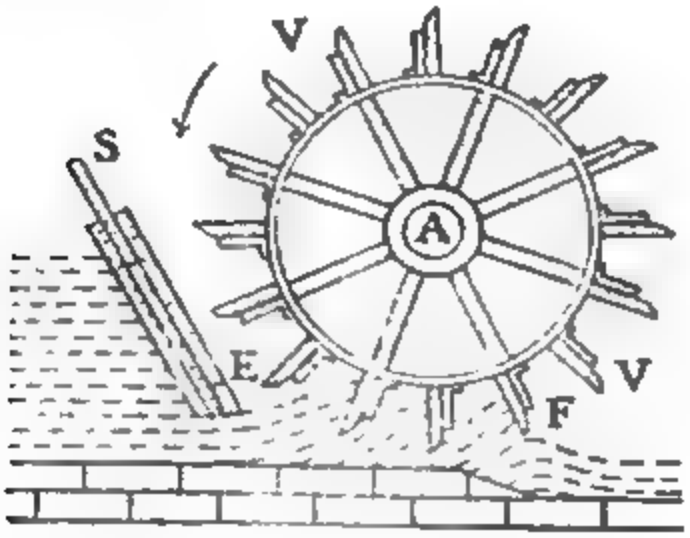


Fig. 237

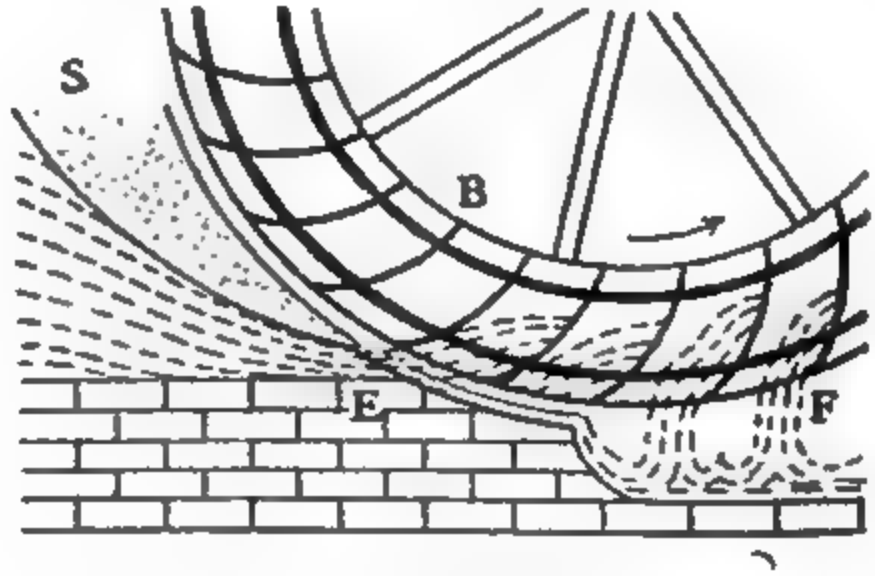


Fig. 238

pulse on the rim of the wheel in a forward direction and it leaves the vanes with a small velocity in a vertical direction, Little eddies or sprays are produced in this process. The efficiency of such wheel is about 60%. The diameter of a Poncelet wheel is between 15 to 20 ft. If Q be the number of cubic feet of water discharged by the sluice per second, m the mass of one cubic foot of water in pounds and v the

velocity of water at the sluice, then the available horse-power H. P. of curved vane undershot turbine is given by the expression.

$$\text{H. P.} = \frac{Qmv^2}{2 \times 550} \qquad \ldots (162,2)$$

But such wheels cannot deliver power satisfactorily in enormous quantities needed by modern plants. They have been replaced by newer types of wheels, known as *impulse turbine* operating an electric generator. The biggest of the impulse turbines is at Niagara falls generating about $1'2 \times 10^6$ kilowatts of power. Other uses of water power are to be found in hydraulic presses and rams, hydraulic brakes and lifts etc.

163. Viscosity of Fluids.—If the flow of a liquid through a pipe in stream line motion be studied, it is found that different layers of liquid move with different velocities. The velocity is highest along the axis of the tube and it continuously decreases in a direction towards the walls. The liquid in contact with the walls has no velocity due to adhesion (Fig. 239). There is thus a change of velocity or *relative motion* as we move from layer to layer from the walls towards the axis.

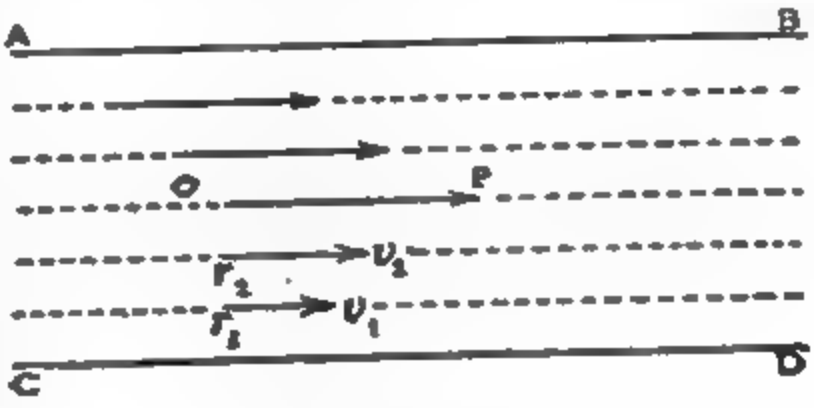


Fig. 289

In order to maintain a relative motion between different parts of the fluid, it is necessary to apply an external force and when this force is removed, the relative motion soon disappears. If, for example, we stir a fluid a relative motion between different parts of the fluid starts but such a motion ceases soon after stirring is stopped. In a similar way waves on a stormy sea subside when high wind abates. These facts prove that a fluid offers resistance to a relative motion of its parts or that a fluid possesses internal friction or *viscosity*.

Let v_1 be the velocity of a layer of liquid at a distance r_1 from the axis of the tube and v_2 that at a distance r_2, the difference between r_1 and r_2 being supposed small. The relative velocity between two layers is $v_2 - v_1$. The ratio of the change of velocity to the corresponding change of distance is called the velocity gradient at a mean distance of r_1 and r_2.

$$\text{Hence the velocity gradient} = \frac{v_2 - v_1}{r_1 - r_2} \qquad \ldots (163,1)$$

In the C. G. S. system, the unit of velocity gradient is one cm. per sec. per cm. or simply sec. $^{-1}$.

When adjacent layers of a liquid flow with some difference of velocities, one layer would move past its neighbouring layer with some relative velocity. We know when a solid body slides over the other, there is a frictional force called into play between the surfaces in contact. The effect of this frictional force is to retard the motion

of the faster layer. In the liquid also some force akin to friction plays parallel to the layers in contact. To have a uniform motion of some liquid an external force is necessary just to overcome such a frictional or viscous force.

The viscous force is proportional to the surface areas in contact and the velocity gradient at the region in question. If a be the surface area and v the difference in velocities over a very small distance r across the moving surfaces in contact, and if F be the viscous force called into play,

$$\text{Then } F \propto a\frac{v}{r} \quad \text{or } F = \eta\frac{av}{r} \qquad \ldots (163,2)$$

The constant η (pronunciated as *eta*) is called the coefficient of viscosity of the liquid at the observed temperature and may be defined as the tangential force acting per unit area over two adjacent layers of a fluid for a unit velocity gradient. In the C. G. S. system the unit of this coefficient is gm. per cm. per sec. or dynes. sec. per sq. cm. Hence the dimension of η is $ML^{-1}T^{-1}$.

164. Reynolds Number.—Reynolds studied the conditions under which a transition takes place from a stream line to a turbulent motion in a fluid. He found that the transition takes place not only when the velocity exceeds a certain critical value in a pipe of fixed diameter, but also when the velocity is kept constant and the diameter is increased beyond a certain value. He then took pipes of various diameters and passed through these pipes different fluids with different velocities. He observed that the product of the diameter and limiting velocity for a stream line flow increases more and more with fluids having lesser coefficient of viscosity. Thus the transition from the stream line to the turbulent flow is directly proportional to the limiting or critical velocity u of the fluid, the diameter d of the pipe and inversely proportional to the co-efficient of viscosity η of the fluid. The quantity ud/η is called the Reynolds number. It is a non-dimensional quantity or a pure number, but it has a great significance in Aeronautics. It is found that for a straight pipe the stream line flow is transformed into a turbulent flow, if the quantity exceeds about a value 2,200. A ball of diameter $\frac{1}{4}$ inches moving in air with a speed of 30 miles per hour has a Reynolds number of nearly 30,000; while an aircraft operates at Reynolds number of the order of a million.

165. Properties of Atmospheric Air.—A systematic record and observation have given out the following properties of atmospheric air—

(1) *Extent of the atmosphere.*—There is no definite knowledge regarding the height to which the atmospheric air extends. The highest altitude to which an aeroplane has risen till the present day is about 79,000 ft. Self-registering balloons carrying apparatus rose upto 35 miles. Echoes of wireless waves have been obtained from various heights ranging upto 200 miles indicating the presence of traces of atmospheric air at that height.

(2) *Pressure of the Atmosphere*—The average pressure at the sea

level is about 14·7 lbs. per sq. inch, which changes slightly with different localities and weather conditions. But with increasing altitude pressure continuously diminishes The following table supplies the variation of pressure with altitude.

Altitude in ft.	Pressure in lbs./inch²	Altitude in ft.	Pressure in lbs./inch²
0	14·7	15,000	8·31
1,000	14·1	20,000	6·82
5,000	12·2	30,000	4·37
10,000	10·5	40,000	2·72

(3) *Density of the Atmosphere.*—Due to the continuous decrease of pressure with altitude, the density of air grows less as we go up. At sea level the density of the air is about ·08 lb. per cu. ft. Taking this value of density as unity, the density ratio at higher altitudes is found from the following table.—

Altitude in ft.	Density ratio	Altitude in ft.	Density Ratio
0	1	15,000	·629
1,000	·971	20,000	·582
5,000	·861	30,000	·373
10,000	·737	40,000	·246

Inertia is dependent on density and is the property of air to resist any tendency to change its state of rest or motion.

(4) *Temperature of the Atmosphere.*—The temperature continuously falls with altitude mainly due to two reasons. As the distance from the surface of the earth increases with height, the heat radiation from earth diminishes. Again air moves continuously vertically upwards from a region of higher atmospheric pressure to a lower one due to a difference of pressures. In doing so a certain volume of air in moving to a region of lower pressure expands and the work done or energy required for such expansion is taken from the mass of air in question. This loss of energy is mainly responsible for the lowering of temperature. The following table supplies the rate of fall of temperature with altitude under normal conditions, taking the sea level temperature to be t°C.

Altitude in ft.	Temperature in °C	Altitude in ft.	Temperature in °C
1	t°C	15,000	t − 29·7
1,000	t − 2	20,000	t − 39·6
5,000	t − 9·9	30,000	t − 59·4
10,000	t − 19·8	40,000	t − 70

(5) *Volume Elasticity of Atmosphere.*—Whenever a difference of pressure is set up at a certain region of the atmosphere, a volume of the atmospheric air between these two pressure limits is compressed or rarefied. A volume of air which has been compresed due to excess pressure tends to expand to release this pressure and vice versa. This property is called the volume elasticity. This property is intimately connected with the average atmospheric pressure at the region and consequently the volume elasticity decreases with the altitude.

(6) *Viscosity of Atmosphere.*—Whenever there is flow of air, such that the different layers of air taken parallel to the direction of flow move with different velocities, some sort of frictional forces parallel to the surfaces in contact, are called into play in between any two adjacent layers. This is due to a property known as viscosity of air. The effect of this viscous force is to retard the motion of the faster layer and accelerate the slower one.

(7) *Humidity of Atmosphere.*—The atmospheric air always contains water-vapour which varies in proportion to different weather conditions and altitude. Since the density of water vapour under the same conditions of temperature and pressure is nearly three-fifths of the density of air, the presence of water vapour in atmosphere diminishes the density of the moist air.

(8) *Turbulence in Atmosphere.*—The motion of air near the Earth's surface is very irregular,—sometimes laminar, sometimes turbulent. This is evident from a simple fact that the smoke from a cigarette held steadily in the hand at first rises up in a regular stream. But a bit higher up the stream of hot air loses direction and exhibits curls showing features of turbulence.

The motion of air constituting the first few hundred feet of the atmosphere is extremely irregular having frequent vortices and up and down currents. These irregular motion is responsible for the formation of low pressure regions or 'air pockets'. A flying air-craft while passing through such regions experiences a sudden fall in level producing discomfiture of passengers. The region of turbulence extends to about 2000 ft. above the sea level and this is also the region in which the earth exerts the viscous drag on the atmospheric air due to its diurnal motion.

Above 2000 ft. the effect of the viscous drag is not appreciable and at such levels the wind is almost non-turbulent. Thus an air-craft normally flies within the non-turbulent air stream. The low level turbulence of the air has also some merits in as much as it mixes up hot and cold layers of air or dry and humid air in a comfortable proportion.

166. Relative Wind.—Whenever a body moves through air, then due to the inertial property of air, there is some sort of resistance offered by air to the motion of the body. This resistance depends upon the *relative velocity* of the air and body. Hence the wind force upon a body travelling north with a velocity of V miles per hour in still air is.

the same as when the body is fixed in space and wind is blowing southwards past the body with a velocity of V miles per hour. The same pressure would result when the body is moving north with a velocity of v miles per hour and the wind is blowing south with a velocity of $V - v$ miles per hour.

In the lower atmosphere wind blows very irregularly with regard to direction and speed, consequently a flying air-craft has got to overcome air resistances of various magnitudes from various directions. The actual speed of an air-craft is its speed relative to the ground, which is called the *ground speed*. The speed relative to air, which might be at rest or in motion, is called *air speed*. When the air-craft moves in any direction with respect to the wind, we can find the relative velocity between the two in the following way :—

Let O be the position of the air-craft moving in the direction OA at any instant with a velocity represented by the length OA, and let

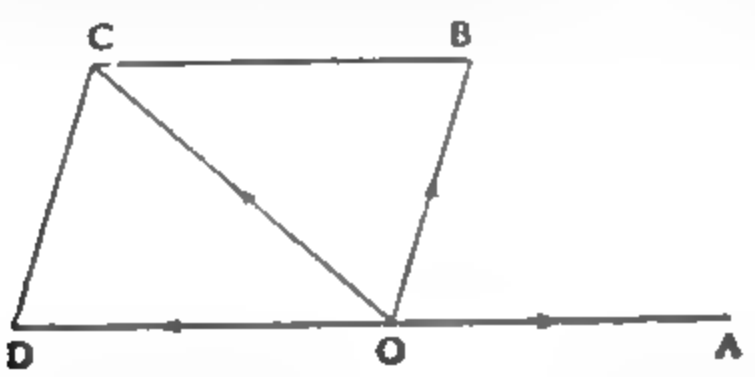


Fig. 240

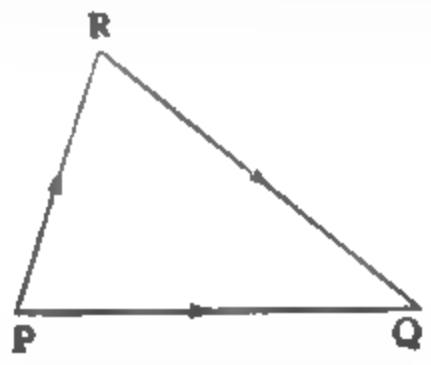


Fig. 241

OB be the direction and velocity of the wind (Fig. 240). To realise the direction and velocity of the wind *appearing* at O, imagine the air-craft at O to be brought to rest by imposing an equal and opposite velocity OD. The resultant of OB and OD is the diagonal OC of the parallelogrm. Hence to an observer at O, wind appears to blow in the direction OC and of magnitude equal to its length. Actually the direction of motion of the air-craft with respect to air would appear to be in the reverse direction CO, which is called the air speed while OA is the ground speed. The direction OC is called *relative wind*. Hence relative wind is equal and opposite to the air speed. Fig. 241 represents the directions and magnitudes of the ground speed PQ, wind velocity PR and the air speed RQ of an air-craft situated at P. The resultant of wind velocity and the air speed is the ground speed.

An air-craft during flight has on seldom occasion a speed of less than 100 miles per hour, while a normal breeze has scarcely a speed of more than 20 miles an hour. Hence air speed under normal conditions is almost equal to the ground speed in magnitude and direction, unless the air-craft is encountered by a storm.

Example :—

1. An aeroplane travelling at a level flight with a speed of 100 miles an hour runs into a vertical up current of air of 20 miles an hour. Find the angle between the horizontal course and the relative wind and the strength of the relative wind at the instant of entering the current.

A. The angle between the actual direction of flight and the wind $= 90°$

Hence from the parallelogram law of velocities the strength of the relative wind $= \sqrt{200^2 + 20^2} = 101{\cdot}98$ miles/hour.

Also $\tan\theta = \frac{20}{100} = {\cdot}2$ whence $\theta = 11°19'$

2. A bomber is spotted at a distance of 10 miles to the north east and travelling in a north-easterly direction with a ground speed of 150 miles per hour. A fighter plane pursues the bomber in a north wind of 30 miles an hour and intercepts it in 5 minutes from the start. Find the air speed of the fighter and the distance at which interception takes place.

A. Since the fighter has got to cover the distance of 10 miles in 5 minutes, the relative gain of speed is 120 miles/hour. Hence the ground speed of the fighter is $150 + 120 = 270$ miles/hour.

$\therefore$ distance travelled by fighter in 5 minutes $= \frac{5}{60} \times 270 = 22{\cdot}5$ miles.

The angle between the direction of wind and the actual motion of the fighter is 45°.

Hence from the parallelogram law.

The air speed $= \sqrt{270^2 + 30^2 + 2 \times 270 \times 30 \times \cos 45°} = 292$ miles/hour.

167. Wind Tunnel Experiments.—The direction and speed of the wind actually set up in the atmosphere have frequent variations. Hence it is very difficult to study the effect of the wind force on any body in actual flight. To attain a steady condition of wind velocity and direction, an artificial air current is produced in an Aeronautical Laboratory within a chamber known as an *wind tunnel.*

There are various models of an wind tunnel, of which the one described here is used at the National Physical Laboratory. It consists of a large wooden pipe T, of rectangular cross-section supported on steel framing (Fig. 242). A part of the tunnel is made circular in section

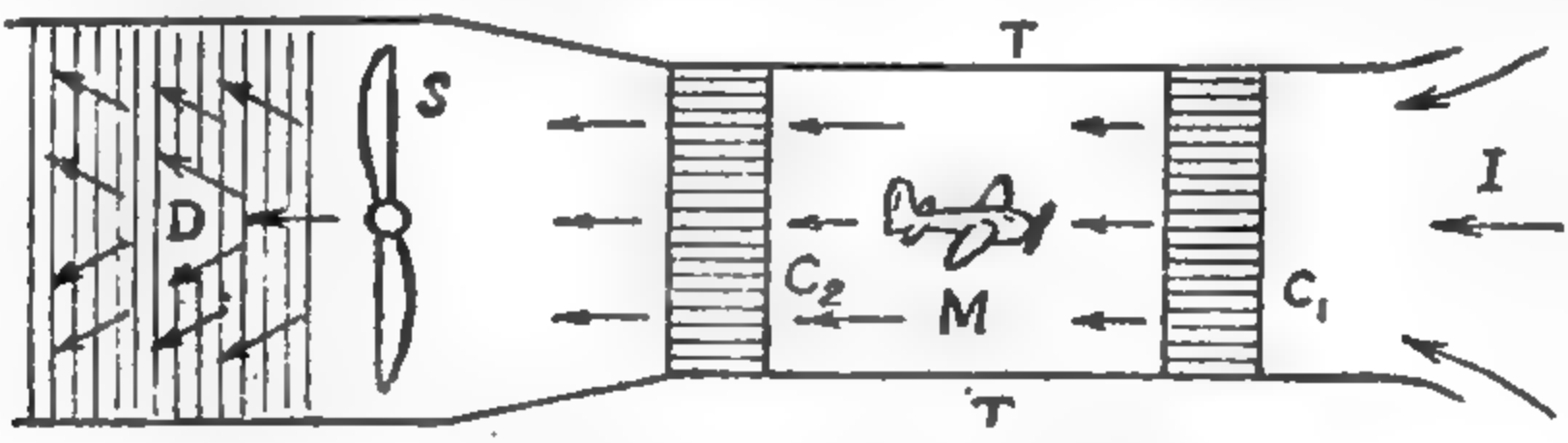


Fig. 242—An Wind Tunnel

where a large air screw S like an electric fan works. The fan can be driven at any desirable speed to suck out air from the chamber. Behind the air screw is the diffuser or distributor D, whose function is to cause the air to return to the in-take at low velocity over a large outlet surface. To avoid the twisting motion of the air current through the tunnel, two honey combs C_1 and C , one near the intake and the other immediately in front of the expanding section, are set up. When the propeller is driven air is forced out of the tunnel, and air from the other end I flows into it to fill up the partial vacuum in a regular stream the

speed of incoming air depends upon the rate of suction or speed of revolution of the propeller, the size and shape of its blades and the dimension of the tunnel. To eliminate the irregularities of flow, the tunnel is widened towards the inlet I something like the horn of a gramophone. Only the central part M of the tunnel, where the flow of air is very regular, is utilised for actual experiments. To investigate the distribution of air pressure over an air-craft in flight, a small model of it is made in the laboratory and is placed at the central part of the tunnel. To attain lower temperatures, as perceived at higher altitudes, special refrigerating plant cools the air of the tunnel at any desired temperature.

The largest of the wind tunnels constructed as yet, is at Seattle, which has a cross-sectional area of 1500 sq. ft. at its narrowest part. The electric generator driving a twin double-blade propellers is of 13,000 horse power. At the highest speed of the propellers, the wind velocity through the central part of the tunnel is about 700 miles per hour, while the severest hurricane sweeps over the earth with a speed not exceeding 120 miles per hour. The refrigerating plant attached to this tunnel cools this unimaginable blast to a temperature of $-67°F$ which is the constant temperature of stratosphere beginning from an altitude of 36,000 ft. above the sea-level.

168. Problem of Air Resistance.—It has already been stated that an ideal fluid is one which in motion does not offer any viscous resistance nor the fluid has any tendency to adhere to a solid body which might be moving through it. Fig. 243 represents the model of

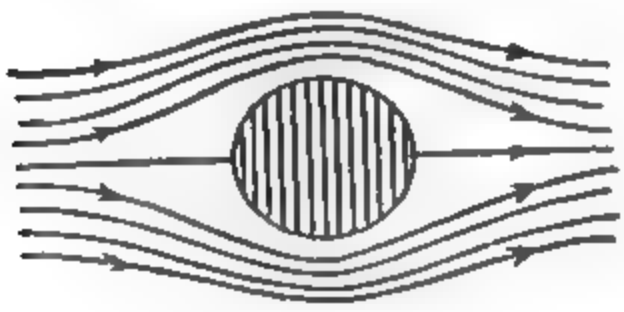


Fig. 243

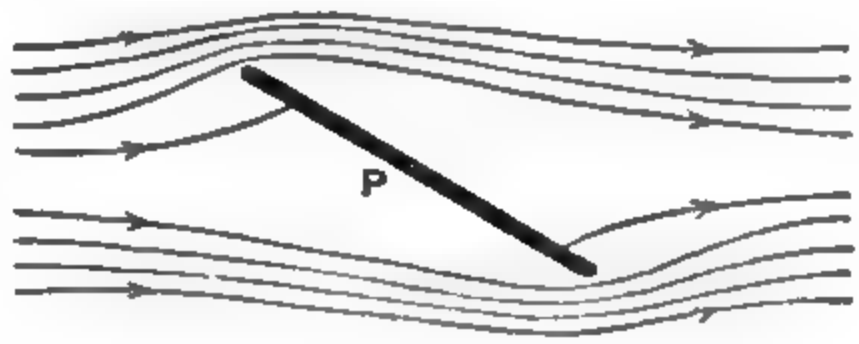


Fig. 244

stream line flow of an ideal fluid across the section of a cylinder of infinite length. Note that at a considerable distance at the front and back of the cylinder, the stream lines are equidistant, which shows that the medium is not disturbed by motion and the pressure at these regions is the normal pressure. But as the stream tubes pass by the edges of the cylinder, they narrow down. The medium here moves with a higher velocity and according to Bernoulli's theorem, the pressure at such a region is lower. The effect of this lowering of pressure is some sort of suction on the edges of the cylinder. Since there is suction both on the upper and lower edges in opposite directions, there is no resultant effect. The cylinder can thus move within the ideal fluid without any resistance. Fig. 244 shows the picture of a stream line flow of an ideal fluid across an inclined plate P. The stream lines narrow down at the edges producing a couple on the plate tending to

make it flat at right angles to the direction of motion. When once made flat there is no resistance to the motion. This was what the laws of Hydrodynamics predicted one hundred years ago.

But it has been an observed fact for a long time that any body moving in air experiences a force tending to resist its motion. Air then

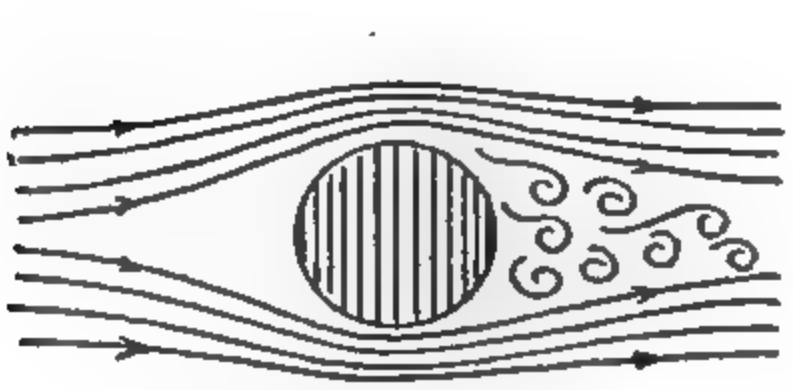


Fig. 245

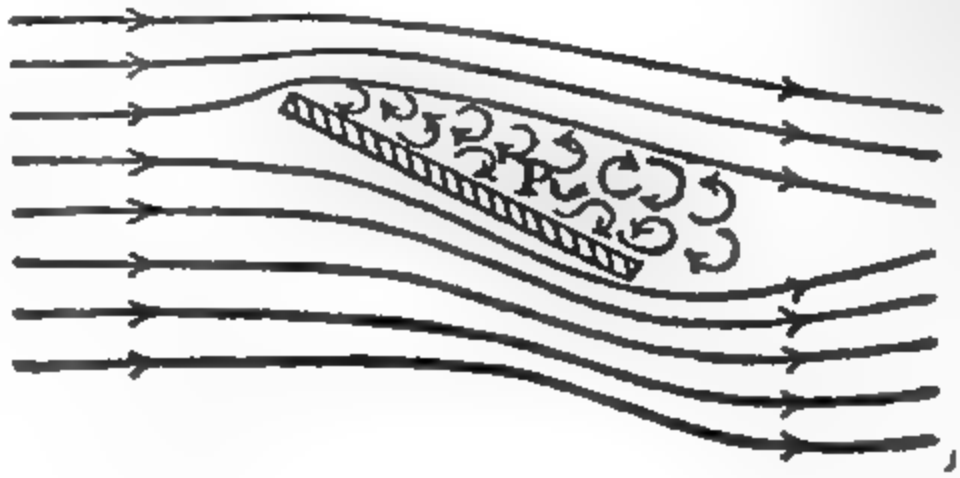


Fig. 246

does not behave as an ideal fluid. When air flows past a body then due to the obstruction of the body, the air deviates its original straight course not only along the surface of the body but also at a considerable distance from the body. So long as different layers of air move side by side within fixed tracks, the motion is stream-lined.

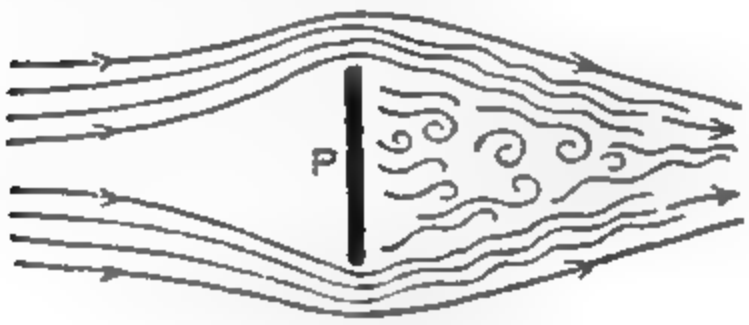


Fig. 247

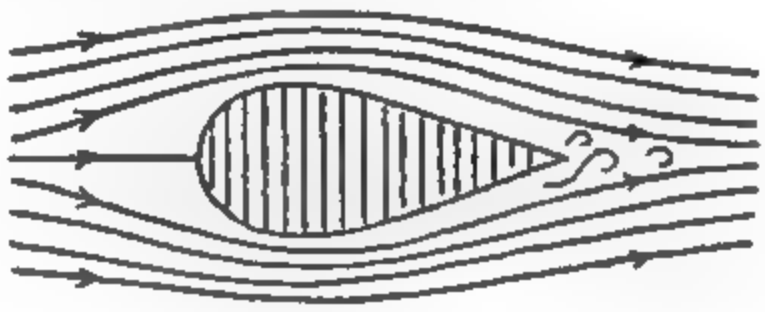


Fig. 248

The nature and distribution of stream lines of air flow around a body depends mainly upon the shape of the body. Small models of the body may be suspended in an wind tunnel and air may be forced through the tunnel with a known speed. Jets of smoke may be introduced within the stream and the course of the smoke may be studied. Fig. 245 & 246 represent the stream line flow around a cylinder and an inclined plate due to air. Observe the formation of eddies behind them. Some quantity of air sticking to the backside is responsible for the formation of eddies and resistance to motion.

Fig. 247 represents a plate P with its surface placed at right angles to the stream of air in a tunnel. The lines of flow bend round the edges and eddies are formed behind the plate as shown by short and irregular lines. There is a considerable fall of pressure within the region where eddies are formed, due to which there is a suction over the plate from behind. This suction pulls the plate from behind and so the plate experiences a large resistance to motion in air at this state. If the plate be inclined as in Fig. 246, eddy resistance becomes less.

Fig. 248 shows a body which has been so shaped that *minimum*

eddies are formed, when placed in a stream of air. Lines of flow are regular and do not break down into vortices. Hence there is minimum resistance to the motion of such a body. A body, so shaped as to exert minimum resistance to the medium through which it moves, is called a *stream lined body*. The conditions determining a stream line flow round a body depend upon the shape and the nature of surface of the body as well as the relative speed of the fluid moving past the body.

169. Wind Force on a Flat Plate.—When a flat plate of area A moves with a velocity v at right angles to its surface in air, we may determine the wind force upon it from a simple consideration neglecting the suction effect due to eddy formation.

We may suppose that the plate pushes before it some quantity of air and in doing so sets that quantity into motion every second. If ρ be the density of air, then the mass of air trapped per second is $Av\rho$ provided no air is deflected and the change of velocity of this mass per second is v. Hence according to Newton's law, the force delivered by the plate is the product of the mass and the rate of change of momentum, that is $Av^2\rho$. Since some air leaks by the edges of the plate, the actual force P is found to be,

$$P = 0{\cdot}64Av^2\rho = 1{\cdot}28(\tfrac{1}{2}\rho v^2)A. \quad \ldots \quad (169,1)$$

The quantity 1 28 is called the *force co-efficient* of the plate for perpendicular incidence of wind. The expression $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume of air and is sometimes called the *kinetic energy density* of the medium.

Example :—

1. A glass pane of a window can withstand a pressure of 25 lb. per square foot. Find the speed of the air blast which will break the pane given that the density of air is ·08 lb. per cubic foot and $g = 32$.

A. Let v = velocity of the blast producing a pressure of 25 lb./sq. ft.

Now 25 lb./ft.^{2} = 25×32 poundals/ft.^{2} = $1{\cdot}28 \times \frac{1}{2} \times {\cdot}08 \times v^2$

whence $v = 125$ ft. per second.

170. Resistances in Aerial Flight.—When a body moves in air or in any other real fluid, some resistances are always offered by the medium at any part of the body. Let a plate AB be moving in air in a direction BM (Fig. 249). The wind moves relatively in the direction W and impinges on the lower surface. Some quantity of air is stopped by this surface. Hence a change of momentum of that quantity of air is taking place every second. This is equivalent to a force R_1 acting at a point C on the under side at right angles to the surface. Some other quantity of air slides past the surface of the body along S in a direction parallel to the surface. Behind the body some quantity of air is thrown into the whirls and eddies. There is a reduction of pressure over this region, due to which the upper surface of the plate is sucked up in a direction CT. This is equivalent to another force R_2 acting at the point C and along the same direction of the force

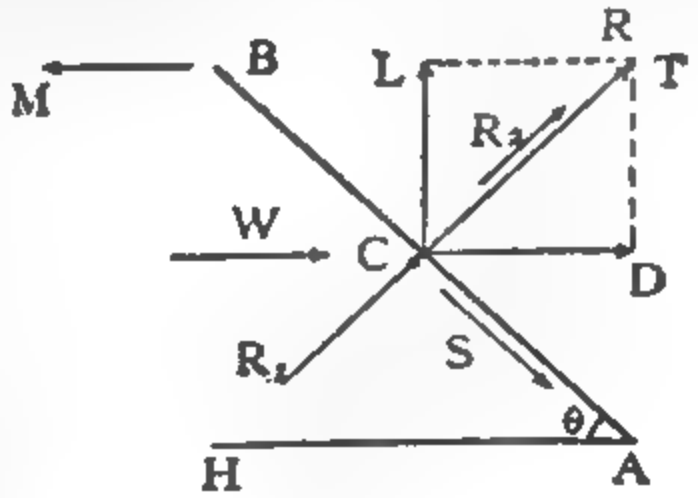


Fig. 249

R_1. Let the sum of these two forces be R which is called the resultant re-action. The different types of resistances to motion may be classified under the following heads.—

(i) Drag.—Let the resultant reaction R be resolved parallel and perpendicular to the direction of the wind. The component D of the force R, measured along the wind is called the *drag*. Thus $D = R \cos \theta$, where θ is the angle between the section AB and its direction of motion. A part of the energy of motion of the body is uselessly wasted in dragging the medium behind it. Moreover due to the formation of eddies there is a large increase of wind resistance. The space immediately behind the body where there is no stream of air is called the *dead space*.

(ii) Skin Friction.—Whatever might be the speed of a moving body, the layer of air in contact with the body moves with the body due to adhesive property. The air at a small distance from the body is practically at rest. If we consider different layers of air parallel to the surface of the body within this region, there would be a relative motion amongst such layers. A relative motion between any two layers of air causes viscous resistance to play and retard the motion of the body. This is called the *skin friction* This theory was put forward by Ludwig Prandtl, a German mathematician in 1904. This is called the Boundary Layer Theory.

(iii) Lift.—The component L of the force R perpendicular to the direction of the wind velocity is called the lift. Thus $L = R \sin \theta$. When the body is moving *in a horizontal direction*, the lift component acts against the weight of the body and is equal to it.

It can be proved that the resistance R is proportional to the frontal area A of the body, square of the relative velocitiy V between the body and air and density ρ of the air. Hence $R \propto AV^2\rho$. Let us put $R = (K\frac{1}{2}\rho V^2)A$ in terms of kinetic energy density of the medium. This constant K is called the *force coefficient* of the body, since, it depends mainly on the shape of the body as well as on the angle θ between wind velocity and surface of the body.

The efficiency of the flight of an object depends upon reducing the drag component and increasing the lift component. The drag is minimised to a great extent by reducing formation of eddies and this is only possible when the medium round of the body relatively moves in a regular or stream line fashion. The stream line shape of the body satisfying such a condition has the maximum lift with a minimum drag.

171. Air Speed Indicators.—The earliest method of measuring the speed of an aircraft was based upon an observation of the speed of revolution of the blades of a small fan fixed with the aircraft due to the thrust of the wind upon it. It is in fact a sensitive form of an anemometer ordinarily used for meteorological purposes. Of late this instrument is replaced by other more sensitive types of measuring apparatus.

The Venturi tube (Fig. 250) consists of a metal pipe wider at both ends and constricted at the middle part B. The axis of the tube is kept horizontal. Air can be forced through the pipe and the speeds of air at different parts of the tube can be examined. The tube is provided with a number of aide tubes at different positions such as A, B and C. These tubes may be connected to sensitive manometers M_1, M_2 and M_3 by rubber tubing. When there is no motion of air inside the pipe, the liquid stands at the same level within the manometer, indicating that the pressure distribution at all parts of the Venturi tube is equal to the atmospheric pressure.

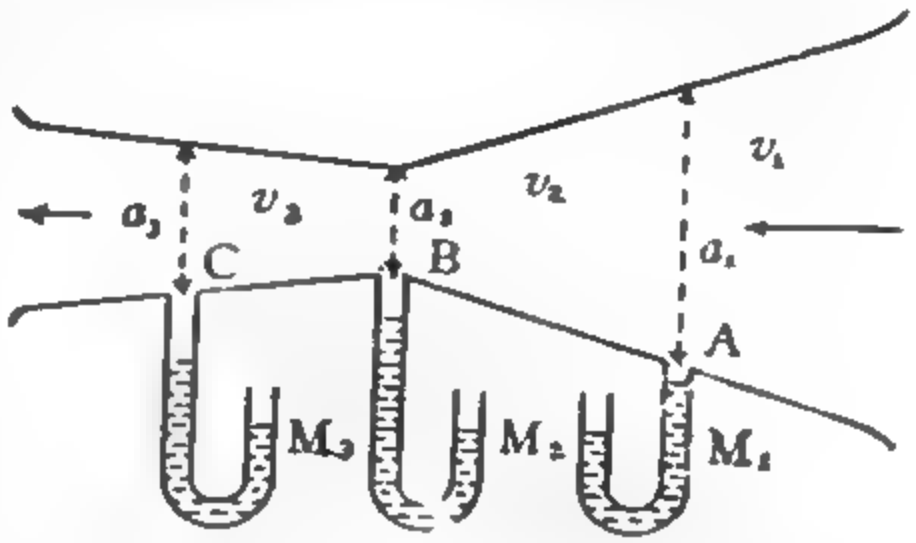


Fig. 250—Venturi tube

When the air is forced through the tube so as to attain a regular motion, then according to the theorem of continuity, the product of the velocity of the air stream at any region of the pipe and the area of cross-section at that region must be constant.

If v_1 = velocity of air on entrance
v_2 = velocity through the constriction
a_1 = cross-sectional area of entrance of air
a_2 = area of constriction

Then, $a_1 v_1 = a_2 v_2$ or $v_1 = \frac{a_2}{a_1} v_2$

Since $a_1 > a_2$, v_2 must be greater than v_1 It is found that the liquid within the tube connected to the constricted part of the pipe rises to the highest point while tubes connected at the wider parts rise less. This shows that where speed of air is high pressure is less.

Now, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 = P_2 + \frac{1}{2}\rho v_1^2 (a_1/a_2)^2$

whence $$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho} \cdot \frac{a_2^2}{a_1^2 - a_2^2}} \quad \ldots \quad (171,1)$$

Thus speed of the air at the entrance of the Venturi tube may be found from an observation of pressures as recorded by manometers. The Venturi meter is widely used to measure the quantity of water passing through a pipe in a given interval.

Pitot Tubes.—These consist of two tubes mounted vertically side by side (Fig. 251). One of them, called the *static* tube S, has a pointed end and a number of small holes are drilled through its walls at a short distance down from the end. The other one P, open at the top, projects a little above the level of the holes of the static tube. It is called the *dynamic* tube. The lower ends of the two tubes are connected to opposite sides of an air tight chamber C having a thin diaphragm D stretched across the middle of the chamber. If there is a difference of pressures at the two halves of the chamber, the diaphragm would be bent by the excess of pressures and the amount of bending

is proprotional to it. The bending is mechanically magnified by the movement of a pointer over a dial R calibrated in miles per hour at the sea level density of air.

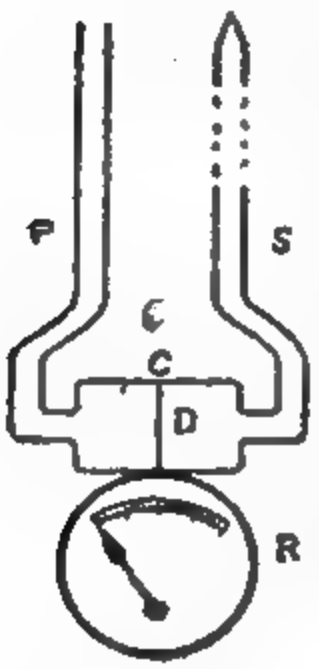


Fig. 251

The holes of the static tube are within such a short distance from the body of the aeroplane so as to be within the adhesive region of the air, where there is practically no relative velocity between the air and the body of the aeroplane. Hence the pressure as recorded by the static tube is the undisturbed atmospheric pressure P. The top of the dynamic tube projects into the stream line flow having a velocity v, say. If the pressure recorded by the dynamic tube be P_1 and if ρ be the densiy of air, then according to Bernoulli's theorem,

$$P = P_1 + \tfrac{1}{2}\rho v^2 \quad \text{or} \quad P - P_1 = \tfrac{1}{2}\rho v^2.$$

Hence the difference of pressures is proportional to the square of the relative velocity. Taking ρ at sea level to be unity,

$$\sqrt{\frac{2(P - P_1)}{\rho}} = v \qquad (171,2)$$

Consequently if the dial is calibrated in proportion to the square root of the difference of pressures, the air speed may be directly found from the reading of the pointer. At any altitude if σ be the relative density of air, the instrument records $v\sqrt{\sigma}$ where v is the relative speed of air at that altitude.

172. An Aerofoil.—An aerofoil is a suitably shaped plate which can move through air with the least possible resistance. It generally means the wing of an aeroplane. A transverse section of an aerofoil,

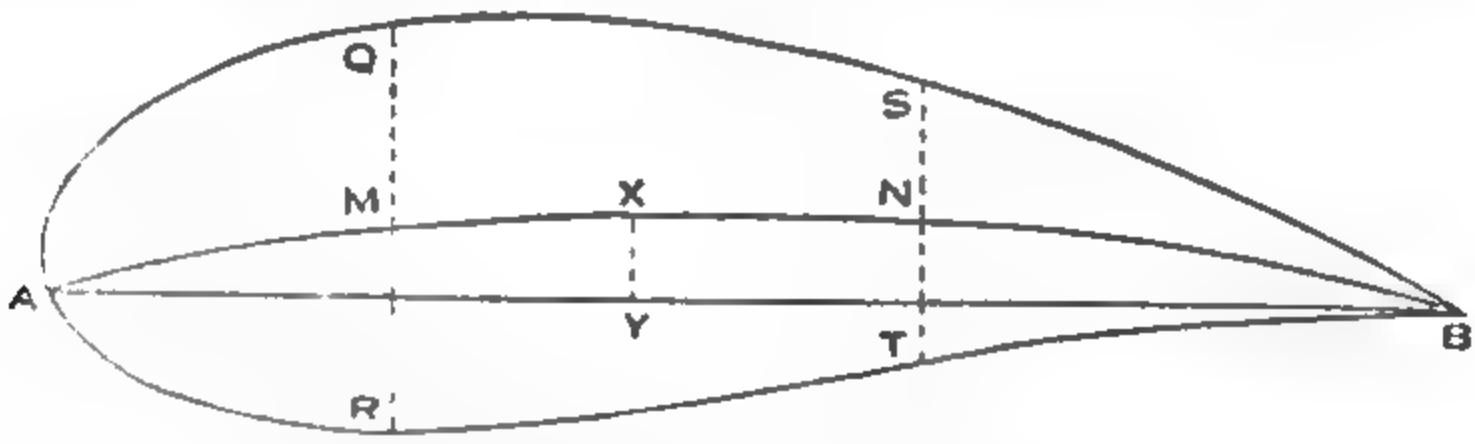


Fig. 252—Transverse section of an Aerofoil

known as the *profile*, is shown in Fig. 252. The front part of the aerofoil is called the trailing edge. The straight line AB joining the extremities of the leading and trailing edges is called the *chord*. The suitable curvatures of the top and the bottom surfaces are called *cambers* and the line running half-way between the upper and lower surfaces of the wing called the *mean camber* line. The curved line AMXB is the mean camber line. The thickness of the wing is measured at right angle to the mean camber line. Thus QR and ST are the thicknesses of the aerofoil at regions M and N respectively. The distance from

one wing tip to the other is called the *wing span*. The span ranges from 30 feet for a small plane to 212 feet in a large bomber.

The contour of the wing as perceived from above or below the aeroplane is called the *plan form*. There are various types of the plan forms. Fig. 253(*a*) shows the plan form of a *tapered wing*. The plan forms as represented in figs. 253(*b*) and 253(*c*) are called *positive* and *negative rakes*. Fig. 253(*d*) shows a *sweep back* wing. In case of

Fig. 253

a rectangular plan form the ratio of the span to the chord is called the *aspect ratio* of the wing. In case of non-rectangular plan forms the aspect ratio is the quotient of the square of the span and the area of the wing. The aspect ratio ranges from 5 in fast pursuit planes to 8 in heavy transport planes.

The angle between the chord and the relative wind is known as the *angle of attack*. The angle between the propeller axis or the direction of thrust and relative wind is known as the *angle of incidence*.

173. Pressure Distribution on Aerofoil in Motion.—When an aerofoil moves through air in a horizontal direction, the distribution

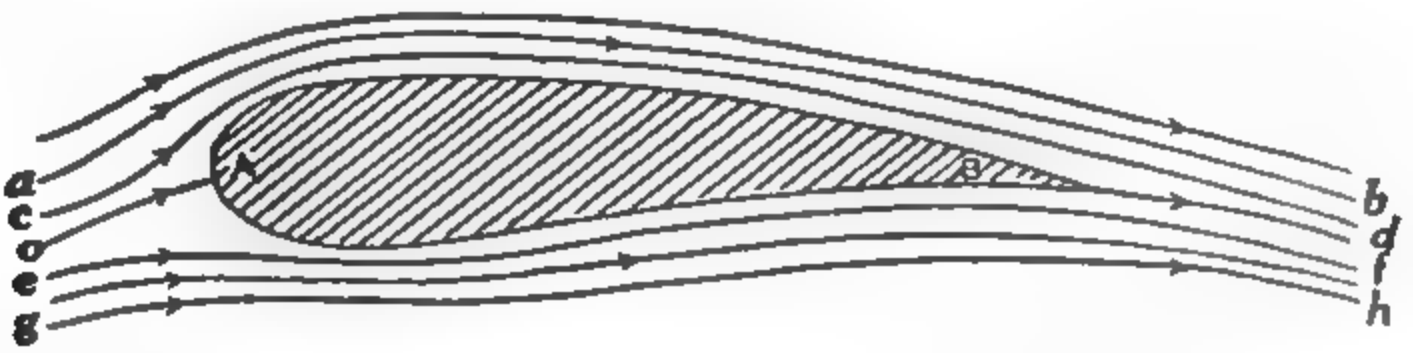


Fig. 254—Stream lines across an Aerofoil

of the stream lines around it is shown in fig. 254. A stream line OA on striking the leading edge of the foil at A divides in two directions,—one part passing over the upper cambered surface and

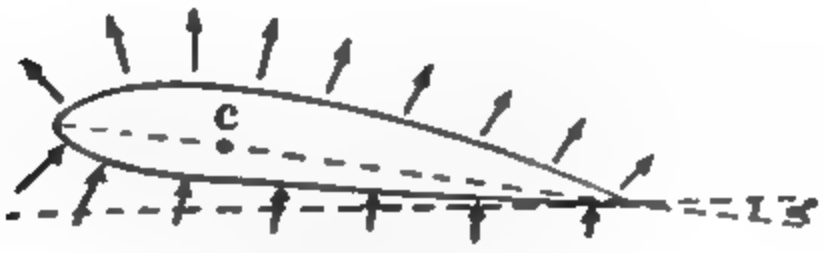


Fig. 255

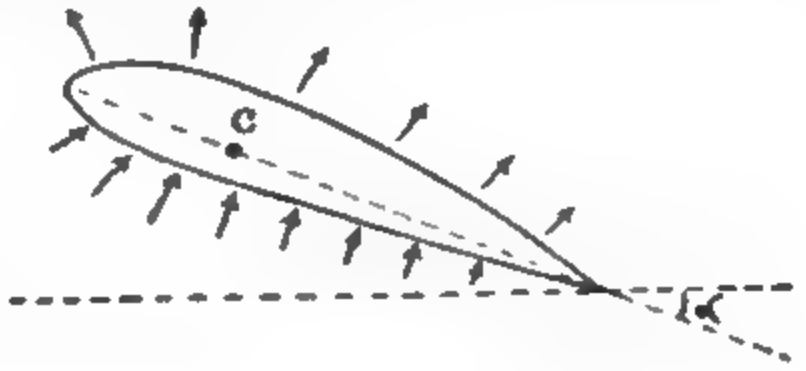


Fig. 256

the other just under the lower surface. These two parts meet at the trailing edge B and move in the direction B*d*. Now the length from A

to B along the upper surface is greater than that along the lower surface. Consequently the average speed of the relative wind within the upper stream tube *cabd* is higher than that of the lower one *efgh*. The region of highest speed within the upper stream tube although slightly varying with the angle of attack is near at the *frontal part* of the upper cambered surface. This lower part of the foil offers some sort of forward push to the air below it. Consequently the relative speed between the foil and air below it is less than that between the foil and air far from it.

Hence from the energy principles (Bernoulli's Theorem) we may say that the pressure above the foil is reduced than the normal pressure. Thus the upper surface of the foil experiences an *upward suction*. The amount of suction varies from point to point along the upper cambered surface. Conversely elementary areas of the lower surface of the foil experience pressures above the normal, varying from point to point. Both together contribute to the upward lift. Fig. 255 and 256 show pressure distribution on an aerofoil, when the angles of attack are 10° and 20° respectively.

Centre of Pressure on Aerofoil.—The total wind force on aerofoil is due to small forces which act upon the different parts of the foil. Hence the resultant of all these forces meets the chord at some point C, called the *centre of pressure*. Its position depends upon the angle between the chord and the relative wind or upon the angle of attack. At a small angle the centre of pressure is located near the rear end but when the angle is increased, the centre of pressure advances towards the leading edge (Fig. 255 & 256). For the most stable horizontal motion of an aeroplane, the centre of gravity of the plane should meet centre of pressure at the same point on the chord. The vertical component of the resultant air reaction should be equal to the weight of the aeroplane.

If the angle of attack is increased, the centre of pressure travels forward while centre of gravity remains fixed. Hence a couple acts upon the foil to rotate it in a direction tending to increase the angle of attack further. This is stabilised by the action of wind on the horizontal tail surface of the aeroplane to produce a torque in the opposite direction.

174. Lift and Drag on Aerofoil.—It has already been stated in Art. 170 that when a body moves in air, there is some reaction of air acting at every part of the body. The resultant of all the elementary reactions acts at some part of the body called the *centre of pressure*. This resultant reaction in case of an aerofoil acts at the centre of pressure C somewhere on the chord and at right angles to it (Fig. 257). The component CD of the reaction R parallel to the relative wind W is the drag and perpendicular component CL is the lift. The

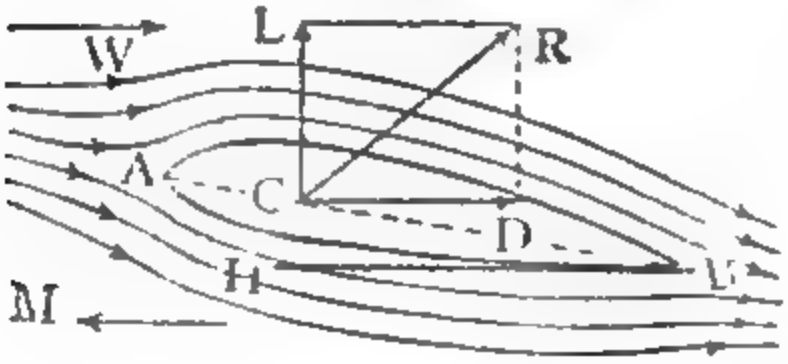


Fig. 257

weight of the foil with its associated components acts vertically downwards at the centre of gravity of the system.

When an air-craft flies in a horizontal direction, it is called a *level flight*. In fig. 258, the chord AB an aerofoil is supposed to move in a horizontal direction with a velocity V. Hence the relative wind is horizontal and so also the drag. The lift C_L is vertically upwards opposing the action of gravity W. When it moves in an oblique direction upwards, the flight is called a *climb*. In case of a climb, the relative wind makes a certain angle with the horizontal line and consequently the lift and the action of the gravity are not in one line (Fig. 259). The vertical component of the lift C_L has got to support the weight while the vertical component of the drag adds to the weight. Hence the speed is to be increased to support the plane in air. When the plane is moving downwards obliquely, the flight is called a *dive*. In case of a dive, the angle between the relative wind and the gravity is less than a right angle. The vertical components of the lift and drag oppose the action of gravity. Hence a lesser speed supports the weight of the plane.

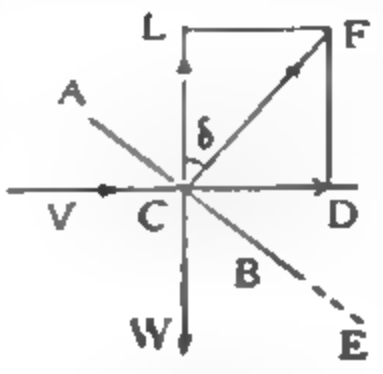


Fig. 258

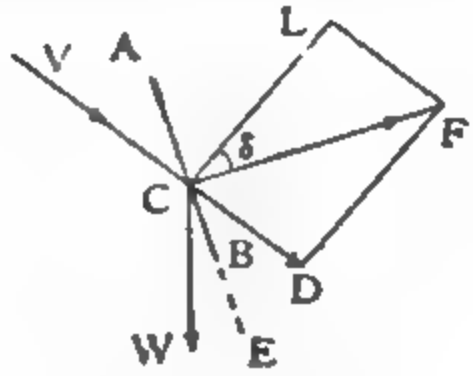


Fig. 259

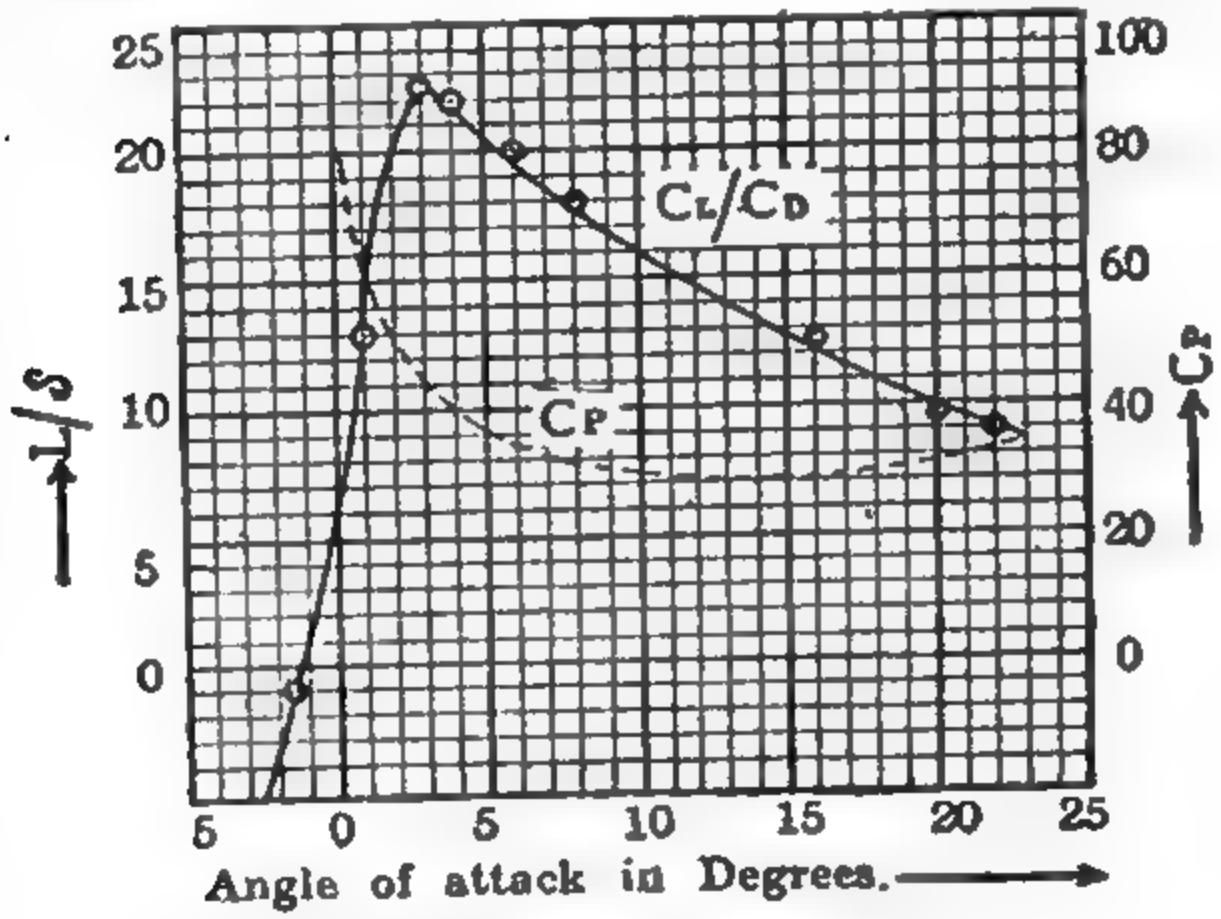


Fig. 260—Variation of Lift-Drag Ratio with Angles of Attack

Let the general value of the angle between the resultant reaction F and the lift L be δ. Then $L = F \cos \delta$ and $D = F \sin \delta$ where $F = \frac{1}{2} C \rho V^2 A$, where C is the force coefficient.

$$L = \tfrac{1}{2} C \rho V^2 A \cos \delta = \tfrac{1}{2} C_L \rho V^2 A \quad \text{where } C_L = C \cos \delta$$
$$D = \tfrac{1}{2} C \rho V^2 A \sin \delta = \tfrac{1}{2} C_D \rho V^2 A \quad \text{where } C_D = C \sin \delta \qquad (172,1)$$

The terms C_L and C_D are called *lift* and *drag co-efficients* respectively. The ratio of C_L and C_D are known as lift drag ratio.

It is evident that higher is the value of this ratio, the more is the load carrying capacity of the aircraft. Determination of the

two coefficients is made with a small model in an wind tunnel. It is found that the ratio of the lift and drag co-efficients depends upon the *angle of attack* and the *aspect ratio*. The variation of C_L and C_D with the various angle of attack are shown in Fig. 260, while the variation of the ratio C_L C_D and the centre of pressure are shown in fig. 261.

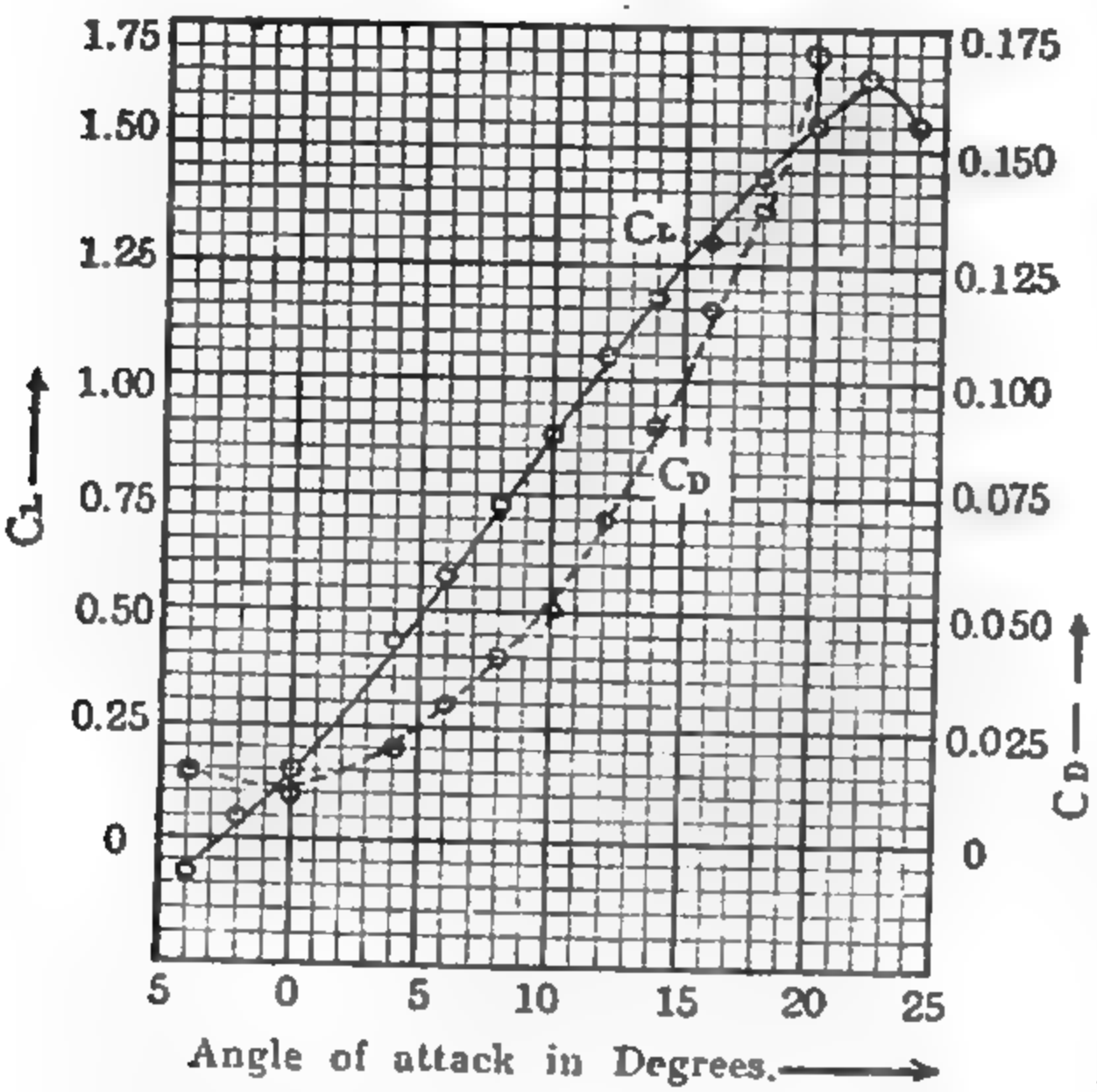


Fig. 261—Variation of C_L and C_D with Angle of Attack

From the expressions for the co-efficients of lift and drag, it follows therefore that

$V = \sqrt{L/\frac{1}{2}\rho A C_L}$ and

$V = \sqrt{D/\frac{1}{2}\rho A C_D}$.

In level flight the weight W of the plane and the cargo it carries must be equal to the force of lift L,

$$\therefore \quad V = \sqrt{\frac{W}{\frac{1}{2}\rho A C_L}} \qquad \ldots(174,2)$$

Study of Lift and Drag co-efficients.—With an increasing angle of attack the lift co-efficient steadily increases and reaches a maximum value at 22° after which there is a rapid fall. The angle of attack of 22° at which the lift co-efficient reaches a maximum value is called the *stalling angle* of the wing. The drag co-efficient is minimum when the angle of attack is zero and it increases when the angle of attack is either positive or negative (Fig. 261).

The maximum value of ratio C_L C_D occurs at an angle of attack of 3° (Fig. 260). There is a fall of the value of this ratio on either side of this angle, more steepy fall occuring on the lower side. The lift-drag ratio determines the load carrying capacity of an aerofoil for given speed. Hence for any fixed speed, the foil would be able to carry maximum load when the angle of attack is 3°. The lift-drag ratio of an aerofoil as seen from the graph reaches its maximum value of 22·5 while that of a flat plate scarcely exceeds 6.

The chain curve in fig. 260 represents the variation of the position of centre of pressure C_P with the angle of attack, the ordinate representing the percentage of the chord length behind the leading edge at which C_P occurs. The position of C_P is nearest the leading edge when the angle of attack is nearly 15°.

Example :—

1. An aeroplane while landing must go on a level flight to avoid jerks. What is the landing speed of a passenger aeroplane carrying a total load of 2 tons? The wing area of the plane is 200 sq. ft. and co-efficient of lift is 1·4 ($\rho = \cdot 08$).

A. Since it is a level flight,

$$V = \sqrt{\frac{W}{\frac{1}{2}\rho A C_L}} = \sqrt{\frac{2 \times 20 \times 4 \times 16}{\cdot 5 \times \cdot 08 \times 200 \times 1 \cdot 4}} = 15 \text{ ft./sec.}$$

175. Description of an Aeroplane.—The components of the aeroplane and the function of different parts are enumerated below (Fig. 262) :—

(1) The *wings* consist of slightly curved long plates rigidly attached to both sides of the body. The wings support the aeroplane in air when in motion.

(2) The *fuselage* Fu or body of the aeroplane carries the crew, cargo and passengers.

(3) The *propeller* or the air screw P consists of two, three or four blades connected to the main shaft. When the blades rotate air is pulled from the forward direction.

(4) The engine is mounted either at the fuselage or in front of the wing.

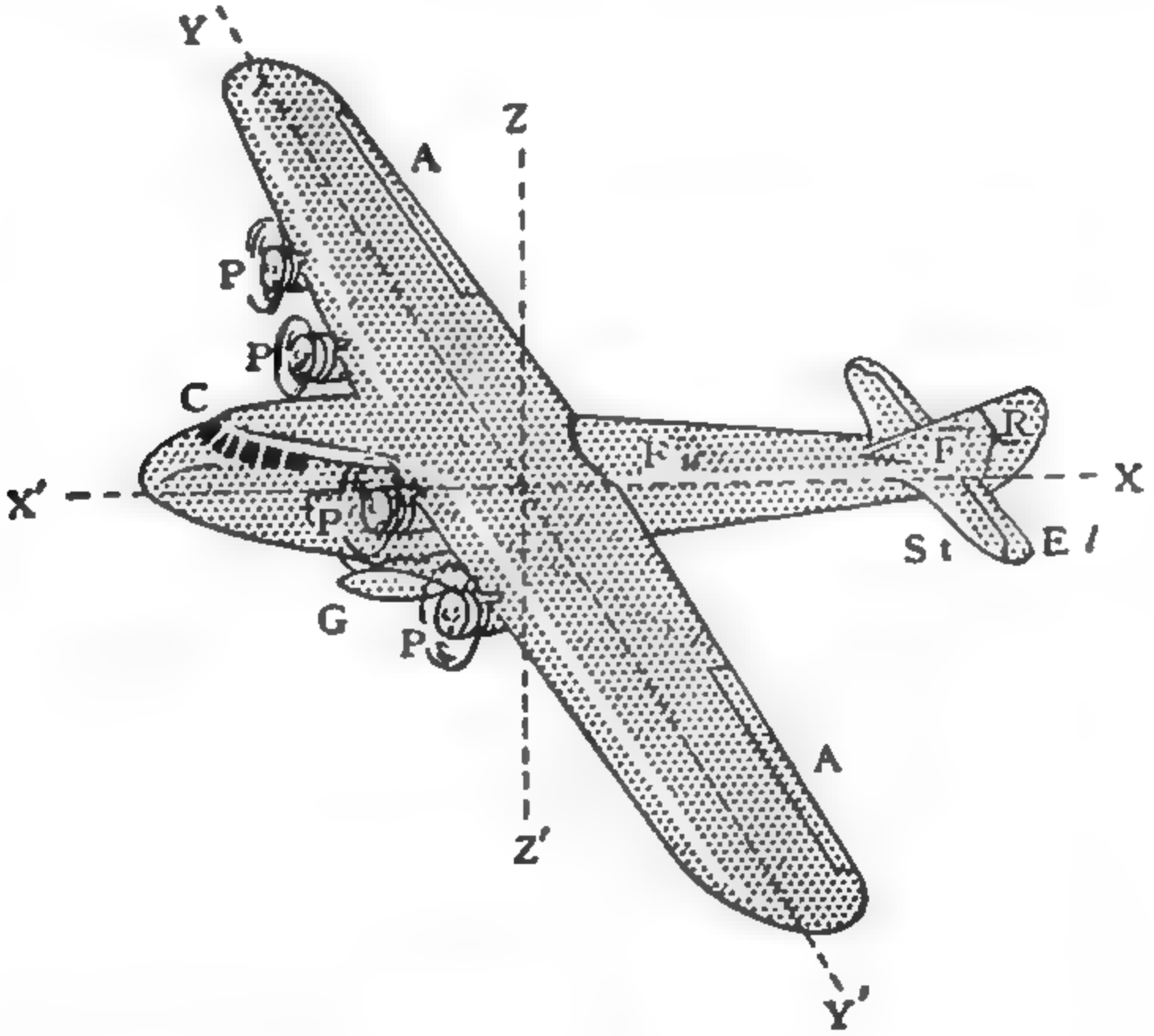


Fig. 262—An Aeroplane

(5) The *ailerons* A are flaps hinged to the rear of the wings and control the inclination of the plane while taking turns.

The rudder R is a part of the tail plate placed vertically and may be moved in a vertical plane. The reaction of this plate to the air current supplies the control on the flight in a horizontal plane.

The *elevator* El is a horizontal tail plate which may be inclined up or down. The reaction of the plate to the air current controls climbing and diving.

These three parts are popularly called the *control surfaces.*

(6) The landing gear G is an under-carriage attachment consisting of—

(*i*) rods and wheels supporting the aeroplane on the ground.

These can be withdrawn into the fuselage when flying in air to decrease air resistance.

(*ii*) pontoons for landing on water *e.g.*, sea-plane.

(*iii*) ski-rods for landing on snow.

176. The Propeller or Air Screw.—The function of the propeller is to impart to the aeroplane *a forward drive* through air. It looks

Fig. 263—An air screw

like an electric fan with a revolving shaft A and two blades attached to the shaft (Fig. 263). An electric fan sucks air from behind and throws it at the front while its body remains fixed in space. Its function is to generate a *spreading* air current, no matter whether vortices are produced in air. A propeller of the aeroplane on the other hand is designed so as to cut its way through the atmosphere imparting minimum air current and producing minimum vortices.

A propeller generally consists of two blades and connected to a shaft at opposite points. The length and average width of the blades vary according to the size and nature of the air-craft. The blades were originally made of wooden spars. For larger planes hollow steel blades capable of withstanding larger resistance were replaced. Nowadays blades are made of hard aluminium alloys. The propeller blade has got a continuous twist from the base to the tip and more over at any region the section of the blade presents the appearance of that of an

Fig. 264—Side view of an air screw

aerofoil so as to allow a stream-lined air flow round it. The twist introduced to provide for a constant angle of attack of the *relative wind* along the entire length of the blade (Fig. 264). This would be evident from the following considerations.

Suppose that the length of the blade is divided into small slices at right angles to its length. Let us take the section AB of any one slice (Fig 265) at a distance r from the axis of rotation. If the blade rotates n times per second, then this elementary part moves through a distance of $2\pi rn$ per second over a spiral curve. We can call it as the tangential velocity of the element. Let CG represent the tangential velocity of the blade element. If the air-craft moves linearly through a distance V in one second, then the wind would strike the plane with a velocity V from the front. But as the propeller rotates it sucks air in its front and consequently the head wind has an additional speed v, say. Thus the forward motion of the blade with respect to air is $V+v$, which is represented by the length CF.

Hence the moving blade element at any position has got two motions, a constant forward motion CF and a tangential velocity CG depending upon its distance from the axis of rotation. The element would, by the parallelogram law, move along CH and the relative wind

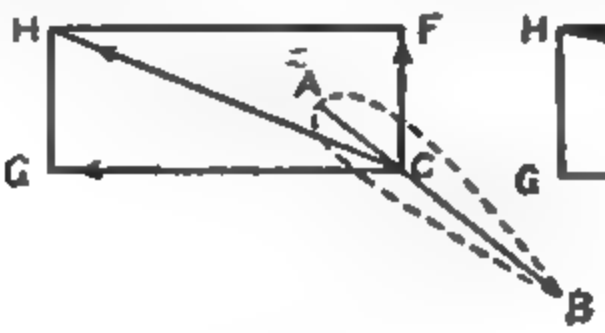


Fig. 265

Fig. 266

Fig. 267

would strike the blade in a direction HC. For the best possible ratio of the lift and drag, the angle between the direction of the relative wind and the chord of the blade element is about 3°. The twist of the blade should be such that at its any region the angle of attack is about 3°.

Near the wing tip $2\pi rn >> V+v$ since r is large (n is generally high being 1000 to 1800 revolutions per minute). Consequently CG is very large compared to CF and so the resultant CH would be inclined more in the direction of CG (Fig. 266). The direction of relative wind is almost at right angles to the direction of forward movement of the air-craft. For this reason the chord of the propeller is also placed nearly at the right angles to the propeller axis. Again near the shaft $2\pi rn << V+v$, since r is small. Consequently the direction of the resultant is almost parallel to the direction of aircraft in motion (Fig. 267). The chord of the blade element at this region is kept parallel to the shaft. As we recede from the shaft along the blade, the twist becomes proportional to its length. A given form of twist is *most suitable* for any particular speed and frequency of revolution of the propeller. This suitability is maintained by altering the twist of the blade of an aircraft by means of hydraulic transmission while in flight at various speeds.

Pitch of Air Screw.—The motion of the propeller through air is in some way similar to the motion of an ordinary screw through a nut. In case of a screw, the distance moved through for one revolution

is equal to the pitch of the screw. But in case of a propeller, the forward advance for one revolution is not a fixed quantity. A metal screw in a screw hole does not slip because of the rigid grip between the groove and the flange, while the air screw of the propeller undergoes certain amount of slip because air being a gaseous medium and serving as the screw hole of the propeller, moves backward as the propeller rotates and advances. The speed with which air moves behind the propeller is known as the *slip stream*. The *slip* of the propeller is the difference of the distances it travels with and without slip stream.

Propeller Efficiency.—The efficiency of a propoller is defined to be the ratio of the power consumed in driving the aeroplane forward to the power delivered by the engine in rotating the shaft. The efficiency ranges from 80% at high speeds to 60% at low speeds.

Propeller as a Gyroscope.—A gyroscope is any mass rotating uniformly about an axis passing through its centre of gravity ; for example the fly-wheel of an engine, the spinning top etc. Hence the propeller with the rotating shaft may be considered to be a type of a gyroscope. It is a property of a rotating body to tend to maintain its axis of revolution along a fixed direction in space. It is for this reason that the earth due to its diurnal revolution behaves as a huge gyroscope and maintains its axis of rotation always along a fixed direction.

To change the axial direction of a gyroscope an external torque is necessary, the direction of the torque to be applied depends upon the direction of the rotation of the gyroscope. A single-engined aeroplane moving in a particular direction in space tends to maintain this direction due to the gyroscopic effect. Suppose that the plane moves horizontally and the propeller rotates clockwise. In order to steer it for a climb in a vertical plane, if the elevators are only worked, the plane would be found to move continuously to the right as it ascends. Hence a counter torque is to be applied by the rudder to turn it to the left when it is found to nose up gradually. This follows from the principle of conservation of angular mementum. This is a serious defect of a single-engined air craft and is dangerous during swift manouvres. The latest types of air-craft are provided with two or four engines with shafts revolving in opposite direction. The gyroscopic effect of one engine being counteracted by the other. Hence there is no trouble with regard to steering of such air-crafts.

177. Harnessing Wind Power.—Energy of the moving wind has found application to small scale industries for a long time in Europe and Middle East. It is the earliest and cheapest natural power utilised for useful purposes. The principle of deriving power from air lies in setting a light propeller against an wind blast, which rotates due to reaction. Small toy propellers made of paper or a light sheet of tin and mounted on horizontal axis are often sold in fun fairs. The propeller consists of a number of blades fixed symmetrically around its axis of rotation and each blade is given a suitable curvature to be able to react with the wind with maximum efficiency (vide preceding Art).

The earliest utilisation of the air power is to be found in wind mills. An wind mill consists of a high structure, called the *air tower*, raised above ground. At the top of this structure is a horizontal shaft S which is mounted on roller bearing and with which is fixed the propeller P containing a number of blades. Usually there are four blades or sails but in some cases a large number of sails are fixed to a circular ring structure. The rotational motion of this shaft is conveyed to a vertical shaft V by toothed gear and from this vertical shaft power is derived.

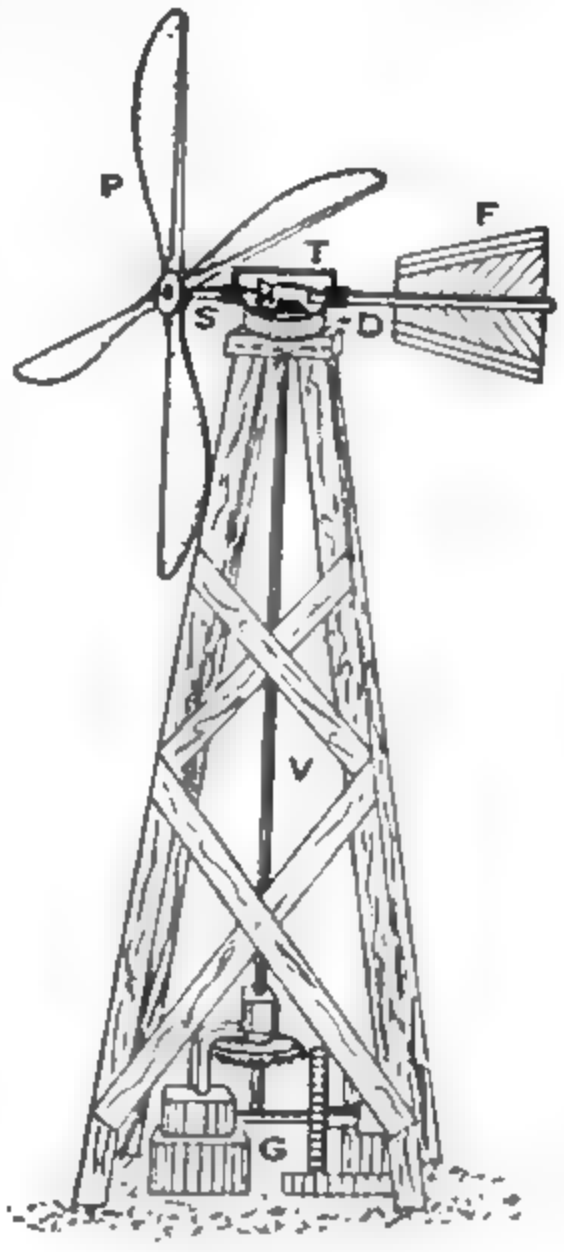


Fig. 268

Since direction of the wind changes from time to time, there is some device to change the direction of the propeller always against wind. This is done automatic by attaching a light vertical blade F behind the propeller. The propeller shaft is connected to the tail blade by an elbow joint T through a sleeve. The joint is mounted on a drum D which can freely revolve on a vertical axis. The blade always places itself with its axis parallel to the direction of the wind and so adjusts the position of the propeller. In some form of wind mills, the setting of the propellers automatically against the wind is done by means of a small fan fixed at right angles to the principal sails.

With a fixed form of the sails the speed of revolution of this propellers would depend upon the wind velocity and consequently the power supply would be different at different periods. Supply of a constant power is not possible with an wind mill but it can be partially controlled with an automatic 'reefing' arrangement introduced by William Cubitt. He constructed sails of wooden laths tied together with cords. In a high wind the laths are turned partly edgeways decreasing the wind reaction. Ordinary wind mills can run with a minimum wind velocity of about 10 miles an hour but the very best type requires only half this value. The latest type is designed in U. S. A. and from this machine power can be derived for 9 months a year on the average.

Although an wind mill is one of the earliest powers discovered by man, it has several disadvantages. The wind is irregular in motion and direction and sometimes it becomes so slow in motion that propellers do not work at all. Consequently regular supply of power cannot be achieved from an wind mill. Unlike water power, air cannot be stored in a reservoir for supply during stuffy weather. Due to all these difficulties, steam and other powers have gradually replaced air power.

But even when we part with the wind mill on its verge of retirement, we must say a few words of respect inasmuch as it saved

during 15th century and onwards the western part of Holland from the clutches of the sea. That part of the land is below the sea level and frequent floods would create havocs. The Dutch erected dams and utilised wind power in pumping out water. In England, things like milling, stamping, sawing etc. were formerly done by wind. Very satisfactory agricultural developments have been done by the wind mills in Holland, England, Australia and Argentine. In Nebraska, the wind power has been utilised in irrigating an apparently hopeless desert which is now transformed into a green and fertile expanse of wheat fields.

178. Equilibrium of Forces.—An aeroplane in flight is in equilibrium under the action of various forces. The wind force on the wing has its components L and D. The weight W of the plane with its components acts vertically downwards through the centre of gravity of the system. The propeller thrust T on the air is parallel to the longitudinal axis of the aeroplane. If the resultant of these forces is to be zero, they must meet at a point or be parallel. In this particular case they all meet at a point. Let the angle between the relative wind and the direction of propeller thrust be θ and the angle between the relative wind and horizontal plane be γ. For equilibrium, the force

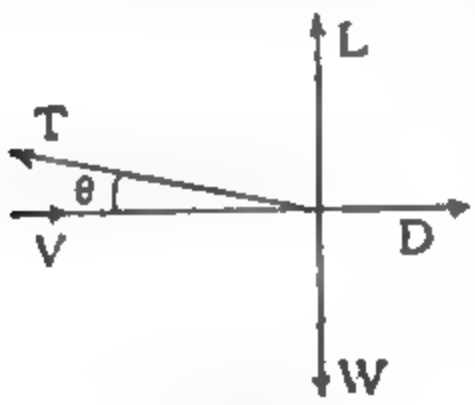


Fig. 269

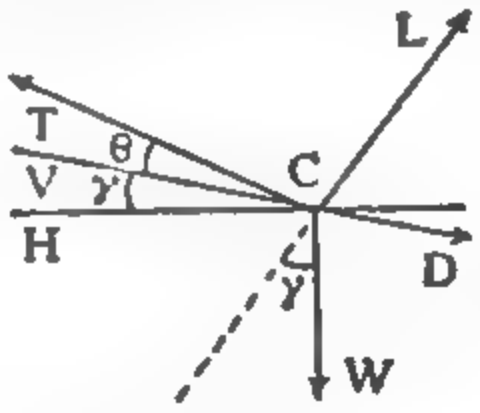


Fig. 270

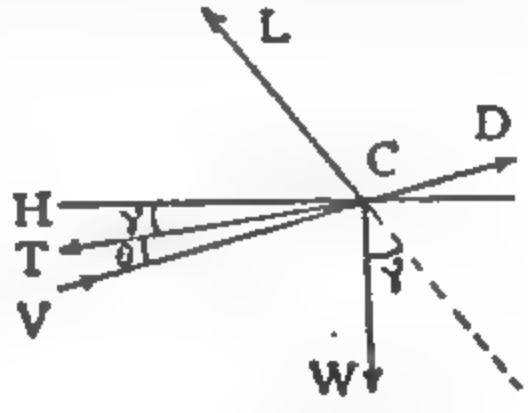


Fig. 271

compenents parallel to the wind must cancel each other and so also force components at right angles to the direction of relative wind.

Case I. *Level flight.*—In case of a level flight, $\gamma = 0$. (Fig. 269) since relative wind is horizontal. Then,

$$T \cos \theta = D \text{ and } T \sin \epsilon = L = W \qquad (178,1)$$

If in a level flight the propeller thrust is also horizontal $\theta = 0$, Then,

$$T = D \text{ and } L = W \qquad (178,2)$$

Case II. *Climb.*—In case of an aeroplane forging in an upward direction (Fig. 270) relative wind makes an angle γ with the horizontal. Then,

$$T \cos \theta = D + W \sin \gamma \text{ and } W \cos \gamma = L + T \sin \theta \qquad (178,3)$$

Case III. *Dive.*—In case of an aeroplane moving in a downward direction (Fig. 271).

$$W \cos \gamma = L + T \sin \theta \text{ and } T \cos \theta = D - W \sin \gamma \qquad (178,4)$$

Case IV. *Glide.*—In case of no thrust of the propeller ($T = 0$)

$$L = W \cos \gamma \text{ and } D = \pm W \sin \gamma \qquad (178,5)$$

179. Stability in Flight.—An air-craft in flight may take up various positions in air depending upon the load distribution, wind pressure and nature of the course of flight. When disturbed from such a position, the tendency of the air-craft to take up its original position is called stability.

An aeroplane may rotate about three perpendicular axes passing through its centre of gravity while in flight (Fig. 262).

Rolling.—Any rotatory or oscillatory motion of the aeroplane about an axis XX′ passing through the nose and tail tip is called *rolling*. In rolling one wing rises while the other drops down. The capacity of the air-craft to control rolling is called the *lateral stability*. The lateral stability is controlled by ailerons.

Pitching.—Any rotatory or oscillatory motion of the aeroplane about an axis YY′ passing through the wing tips is called *pitching*. In pitching the nose and the tail move up or down. The capacity of the aircraft to control pitching is called the *longitudinal stability*. The longitudinal stability is controlled by elevators.

Yawing.—Any rotatory or oscillatory motion of the aeroplane about an axis ZZ′ perpendicular to the wing surface is called *yawing*. In yawing the nose and the tail move in opposite directions with respect to the pilot along the plane of flight. This is controlled by the rudder.

180. Jet-propelled Craft.—The speed gaining capacity of a propeller-driven aeroplane is seriously hampered when the speed exceeds about 450 miles per hour, because of the excessive air drag upon the propeller blades. Hence for an efficient high speed flight some auxiliary device such as a jet-propulsion engine is to be installed with or without the air screw. Recently jet-propulsion device without the propeller is resorted to for high speeds. With jet planes a speed of 350 to 400 miles an hour near the earth's surface and nearly 600 miles an air at a high altitude are common nowadays. During August 1951, an American pilot Bridgeman attained a speed of 1238 miles an hour in a Skurocket jet plane having travelled almost twice as fast as the speed of sound. He broke the world's speed as well as the altitude record when flying at 79,494 ft. high up from the sea level.

In a jet-propelled machine, a gas is exploded in a chamber and the exhausted pipe faces the rear of the machine. As the exploded gas blasts through the exhaust in the rear direction, the machine by reaction receives a forward impulse. The rate of blasts is made as frequent as possible, so that the machine gets a large number of impulses over a period. The average of all these impulses per second is the thrust acting on the craft due to the jet reaction.

In one form of a machine, air forces its way through an opening of a tunnel at the nose of the jetplane. This air is compressed by an air pump and is mixed with the exhaust gas to raise its temperature. This compressed mixture is mixed with a highly inflamable fuel such

as gasolene and then exploded in a chamber. The exhaust of the combustion chamber leads to a nozzle having variable orifices.

The action of a jet-propelled craft depends upon the principle of conservation of linear momentum of two bodies *in contact*. If one of the two bodies be acted upon by an impulse in a certain direction, the other one by reaction receives an equal and opposite impulse. Consequently the second body would move away from the first gaining a certain speed in the opposite direction.

There are very many problems with regard to high speed flights. One of them is the heat developed due to friction with air. The meteorites, due to their extremely high velocities, are so much heated as to be ignited. The average speed of a meteroite has been calculated to be about 45,000 miles an hour. The rise of temperature in degrees centigrade is equal to the square of the speed of the body when expressed in units of hundreds of miles per hour. The German V_2 rocket launched against London had a speed of nearly 3000 miles per hour before falling to the earth. This means that the nose of the rocket had a rise of temperature of nearly 1000°C.

In a **rocket propelled** craft, the explosion of the fuel within the combustion chamber does not require atmospheric air ; the chemicals used within the machine supply all the energy of combustion. A rocket is therefore a self-contained or chemical-fuel motor and has a likelihood of travelling much better above the earth's atmosphere. Most of the V_2 rockets used alcohol and liquid oxygen as the explosive fuel. Although it is too early to make any definite statement at this stage, it may be possible that in near future we might see passenger rocket planes bound for Venus or Mars. A scheme is being drawn up to make a space station at a distance of 1200 miles from the earth's surface wherefrom to make a final journey to the moon by a convoy of three rocket planes The scientists are optimistic about the view that all necessary arrangements may be completed and the maiden journey undertaken by the year 1975.

181. High Altitude Flight.—The high altitude is characterised by a marked fall of atmospheric pressure, air density and temperature. With lowering density, the blade thrust on air decreases resulting in a consequent fall of the lift and drag. The speed and the load capacity of the air-craft diminish This is compensated by jet-propulsion.

As air pressure diminishes considerably, the sucking power of the fuel by the combusion chamber falls off. Consequently less and less fuel would be fed to the chamber with increasing height resulting in a decrease of horse-power of the engine. As an example a 1200 H. P. engine develops 500 H. P. at 20,000 ft. and 250 H. P. at 25000 ft. This loss of power may be compensated by maintaining the constancy of air pressure by a super-charger. The super-charger is in principle a compression pump geared to the engine.

At an altitude of 36,000 ft. where the temperature is $-55°C$, the effect of different contractabilities of metals becomes very perceptible.

Metal joints become leaky unless special precautions are taken ; rubber becomes brittle, pipe lines freeze and transmission cables slacken.

In addition to these effects there are also various physiological effects. Firstly due to lowering of temperature woolen and sheep skin suits are essential for the body protection. For very high altitudes the cabin should be electrically heated for further warming up The oxygen content of the higher atmosphere is so scanty that oxygen from cylinders is supplied to make up the defficiency. The cabin should be properly sealed to maintain the standard pressure inside.

Unless these precautions are adopted, the pilot is to suffer symptoms such as breathlessness, muscular weakness, giddiness and pain in ear, which may ultimately prove fatal. In spite of all the disadvantages there are some advantages of high altitude flight. The weather conditions fairly remain constant and consequently flight is very smooth. Due to rarefied air, the air resistance is smaller and a larger speed is possible for a given angle of attack and weight of the air-craft.

WILBUR AND ORVILLE WRIGHT

Wilbur Wright was born at Millville, Indiana, on April, 16, 1867. Orville was his younger brother. The father of Wrights, Milton Wright was a Bishop of the religious organisation known as United Brethren of Christ. The two boys received the school education in Richmond and Dayton. They began their careers at job printers and then became bicycle repairers.

Towards the end of the nineteenth century, they turned their attention to aerial flight which was in those days entirely confined in balloons. A few workers were busy in designing air-ships and such other lighter than air machines. But the Wright brothers aimed at a heavier than air machine to be driven by motor engines.

They first noticed the utility of wings of birds in the matter of flight and made a systematic study of the relation between the weights of birds and the corresponding wing spans. Then they formed some idea regarding the possible wing structure of the machine that would fly with required accessories. They first modelled gliders of various shapes without any engine. After studying successfully the lift of gliders, an engine was fitted to one glider to revolve propellers for a forward motion. The first flight in such a machine was made on December 17, 1903. The following year a more improved type of machine was constructed which could fly a distance of 20 miles.

In 1908, Wilbur Wright started for Europe with a newly-built machine for demonstration purpose. He made a successful flight in France. He died on May 30, 1912.

Summary

Wind Tunnel Experiment.—To study the wind stream and distribution of wind pressure on a model, it is suspended within a chamber known as an *wind tunnel* and wind is blown against it from a propeller.

Bernoulli's Theorem.—In case of flow of an ideal incompressible fluid free from viscous resistance, sum of the potential, kinetic and pressure energies at any point along a stream line is constant. In case of air, the statement may be simplified in the form. $\frac{1}{2}\rho\ v^2 + P = \text{const.}$

Venturi Tube Experiment.—It is a conical shaped pipe through which air can be blown. A number of manometers are attached at different regions. This apparatus is used to measure air speed. **Pitot tubes** are also appliances by which air speed may be found. They compare the static and dynamic pressures of the wind.

Drag. Lift and Skin friction are three resistances encountered by a body moving through air. An **Aerofoil** is a suitably shaped plate which can move through air with least possible resistance. The surfaces of the foil are properly cambered. When an aerofoil moves in air, there is a thrust of the air from below and a suction of the air from above. These two together constitute the resultant thrust.

Centre of Pessure of the aerofoil is the point on the chord where the resultant thrust of air acts. Its position depends upon the angle of attack of the foil.

An **Aeroplane** consists of wings, fuselage, propeller, engine, ailerons rudder, elevator and the landing gear. The **Propeller** may be considered to be a continuously twisted aerofoil providing the forword thrust of the air-craft. The **Stability** in flight depends upon rolling, pitching and yawing.

Jet-propelled Craft is used to secure greater efficiency of flight in higher altitude. **High Altitude Flight** requires arrangements for high speed of the craft maintaining pressure in the curburettor, hermetically sealed room for the maintenance of temperature and pressure for the passengers.

Exercises of Chapter XIII

Reference

Art. 158 1. State and explain Bernoulli's Theorem. How can it be applied in case of an air-craft in motion ?

Art. 159 2. A horizontal pipe of cross sectional area 6 sq. inches broadens out into a pipe of cross sectional area 10 sq. inches. The pressures recorded are 8 lb./sq. in. respectively. What volume of water is delivered per hour ?

A. 4,550 cu. ft./hour.

Arts. 120 & 163 3. Explain fully what you mean by surface tension and viscosity. Explain why rain drops are spherical. (P. U.—1951)

Arts. 161 & 168 4. Explain what is meant by stream line flow. Describe an experiment to demonstrate the deformation of stream lines by an obstacle.

Discuss the flow of air past a flat plate moving through air with a high velocity with its face inclined at a small angle to the direction of motion. Show how the lifting force is produced on the plate and explain how it varies with the angle of incidence of the plate.
(P. U.—1945)

Reference

5. Explan with illustrations giving neat sketches how can you use water power and wind to do your work. (C. U.—1953) — Arts. 162 & 177

6. How strong is the wind force against a wind shield 2 ft. × 3 ft. of a motor car moving at 40 miles per hour. — Art. 169

A. 32·5 lb.

7. What is a cambered wing? Draw a neat diagram of a section of it perpendicular to its span and indicate in it the lengths known as the chord, the upper camber and the lower camber. What are the essential points that are kept in view of its construction. (Pat. U.—1940) — Art. 172

8. How do you get the 'lift' that supports the aeroplane in the air, what is corresponding with drag? Define co-efficients of lift and drag; prove that in a horizontal flight. — Art. 174

$$V = \sqrt{\frac{W}{dSk}}$$

where V = velocity of the machine, S = area of wing, k = coeffi. of lift and d = density of air. (Pat. U.—1940)

9. Write short notes on variations of 'Lift' and 'Drag' with the angles of incidence. (Pat. U.—1953) — Art. 174

10. What is a stream lined body? Describe the structure of an aeroplane wing and discuss the factors upon which the lifting efficiency depends. (Utkal. U.—1949) — Arts. 174 & 175

11. Draw a neat sketch of an aeroplane showing its essential parts and explain fully the action of each part. (Pat. U.—1943, '51; *cf.* E. P. U.—1952) — Art. 175

12. What are the functions of (*a*) the propeller, (*b*) the wings, (*c*) the ailerons. Is there any limit to the height at which an aeroplane can ascent? (Pat. U.—1942) — Art. 175

13. Explain how an aeroplane manages to remain suspended during its flight and when it takes a turn. (Utkal. U.—1951, '53; *cf.* P. U.—1951) — Arts. 174 & 175

14. What in an air screw? Explain how it gives the forward motion to an aeroplane. (Pat. U.—1953; Utkal. U.—1947, '52) — Art. 176

15. Can an aeroplane fly without wings? Can an aeroplane fly in vacuum? Give reasons for your answer. (Utkal. U.—1952) — Art. 180

16. What are cambered wings in an aeroplane? Explain their action. Also explain with neat diagrams the action of the tail, elevator, fin and rudder. (Pat. U.—1949, '52) — Arts. 172 & 175

17. Explain clearly the part played by (*a*) the propeller, (*b*) the wings and (*c*) the ailerons in the flight of an aeroplane. (Utkal. U.—1948) — Art. 175

18. What is the stability of an aeroplane. Explain with the help of neat diagrams the action of parts of the plane which give stability to it. (Pat. U.—1952; Utkal. U.—1953) — Art. 179

19. Write a note on the flight of an aeroplane and indicate if there is a limit to the height to which it can ascend. (Utkal. U.—1948) — Art. 181

HEAT

CHAPTER I

THERMOMETRY

1. Heat.—From our earliest day we are familiar with the sensations of heat and cold. If we stand in the sun or before fire, we feel hot ; while, if we hold a piece of ice in our hand, we feel cold. The physical cause, due to which we have such feelings of hotness or coldness is termed **heat.**

Energy in mechanical or some other form may be transformed into heat. Thus, heat is produced when we rub our hands together or when a piece of iron is hammered on anvil. Again when coal burns in air, heat is produced. Here energy of combustion of coal appears in the form of heat. On the other hand, heat can be transformed into different forms of energy. Thus in a steam engine, heat produced by the combustion of coal converts water into steam within the boiler, which sets the engine in motion and so mechanical work is done at the expense of the heat. Heat is therefore a form of energy.

2. Temperature.—When heat is applied to a body which is not in a state of melting or vaporising, it grows hotter. Conversely when heat is taken away from it, it becomes colder. Generally speaking *temperature is the measure of hotness of a body.* If, on touching two bodies A and B, A appears to be *hotter* than B, A is said to be at a *higher* temperature than B. Thus temperature is the manifestation of heat in material bodies, which we can compare by our sense perception.

Accurately defined, temperature is *the thermal condition of a body which determines whether the body will communicate heat to or receive heat from another body, if put into thermal communication with each other.* If a hot metal ball be dropped into a mass of cold water, the ball after sometimes appears colder than before while the water appears warmer. Here, heat has left the hot ball and has passed into the cold water. So the passage of heat from one body to another depends on their difference of temperatures.

Distinction between Heat and Temperature.—Although the terms heat and temperature appear so much alike, they do not convey the same meaning :—

(1) Temperature indicates the thermal *condition* of a body, whereas the quantity of heat possessed by it is a form of *energy*.

(2) Heat generally increases the temperature of a body. But if a body melts or boils, it does so at definite temperatures, known as the melting or boiling point. The application of heat at these two states is utilised in melting or boiling out more substance without change of temperature. (Chap. VI).

(3) Two bodies may be at the same temperature although the quantity of heat possessed by each may be different. If, out of a large mass of boiling water, a small portion is taken off, the temperature of the two masses is the same, although the quantity of heat in the former is far greater than that in the latter.

(4) The flow of heat from one body to another depends only on their temperatures and not on the quantity of heat possessed by each. The amount of heat in a red-hot iron ball may be far less than that in a bucket of warm water ; but if the ball be put into the water the former will communicate some amount of heat to the latter.

(5) Temperature is analogous to surface level in the case of liquids. If the level of water in a vessel be higher than that in another, water will flow from the former to the latter when communication is made between the two vessels ; this takes place irrespective of the quantity of liquid in each. The flow ceases as soon as the level of water in both the vessels becomes the same. Similarly, heat will pass from a body at higher temperature to one at a lower temperature, irrespective of the quantity of heat possessed by each and there will be no further transference when the temperature of the two bodies becomes the same.

3. Effects of a Change of Temperature.—When a body is heated or cooled so that its temperature changes, the following among other effects may be observed :—

(*i*) *Change of Volume*—It is found that, in general, bodies increase in volume when heated and contract when cooled.

(*ii*) *Change of State*—On being heated, a solid at a certain temperature generally passes into the liquid state and similarly a liquid at another fixed temperature passes into the gaseous state. Thus, when ice is heated, it gradually melts into water and when the water is heated, it passes into steam. Conversely, steam being cooled changes into water and water being sufficiently cooled freezes into ice.

(*iii*) *Change of Physical Properties.*—Almost all the physical properties of matter are modified by a change of temperature. Thus the elasticity of solid, the surface tension and the solvent power of a liquid, the power of metals to conduct heat and electricity etc. are affected by changes of temperature. Zinc is hard and brittle at ordinary temperature but soft and flexible when heated ; iron is softened or tempered by alternate heating and cooling.

(*iv*) *Chemical and Electrical Effects.*—Many substances such as lime, are decomposed when sufficiently heated. Again some chemical union is brought about by application of heat. The electrical resistance of a conductor changes as its temperature is changed by heat. If a circuit is made by joining wires of two dissimilar metals and if one of the junctions be heated, an electric current flows through the circuit.

4. Measurement of Temperature.—All our ideas regarding difference of temperatures are primarily derived from the *sense of touch.* The temperature of the hand is therefore the criterion in these cases.

But this method of estimating the relative hotness or coldness of a body cannot be taken to be accurate for the following reasons :—

(i) Our sensations are not sufficiently delicate.

(ii) Judgement as to the temperature of a body is sometimes incorrect. For instance, if the right hand is placed in hot water and the left hand in cold water and then both be immersed into a mass of lukewarm water, the water appears *cold* to the right hand but *hot* to the left. Again different bodies at the same temperature may give different ideas of hotness or coldness, when touched by the hand. Thus, if a piece of wood and a piece of the polished iron lying side by side be touched, the latter will appear colder although in reality both are at the same temperature. The instrument that is devised for comparing and measuring different temperatures is known as a **thermometer.**

We take recourse to one or the other physical property of a substance which should change uniformly with temperature and which at the same time must be capable of easy and accurate measurement. It is an observed fact that when a substance is heated, its volume increases continuously with rise of the temperature. The change of the volume of a suitable substance may, therefore, be taken to indicate the corresponding change in temperature. Mercury is most suitable as a theremometric substance.

5. Mercury Thermometer and its Construction.—The most common form of thermometer is a mercury-in-glass thermometer. It consists of a small cylindrical bulb (Fig. 1) provided with stem of very narrow and uniform bore. The other end of the stem is closed and there is mercury inside. The volume of the enclosed mercury changes with change of temperature and for a definite temperature the volume occupied by mercury is also constant, as shown by a definite position of the mercury column along the stem. The stem is graduated in accordance with some standard scale, so that each division represents an equal change in temperature.

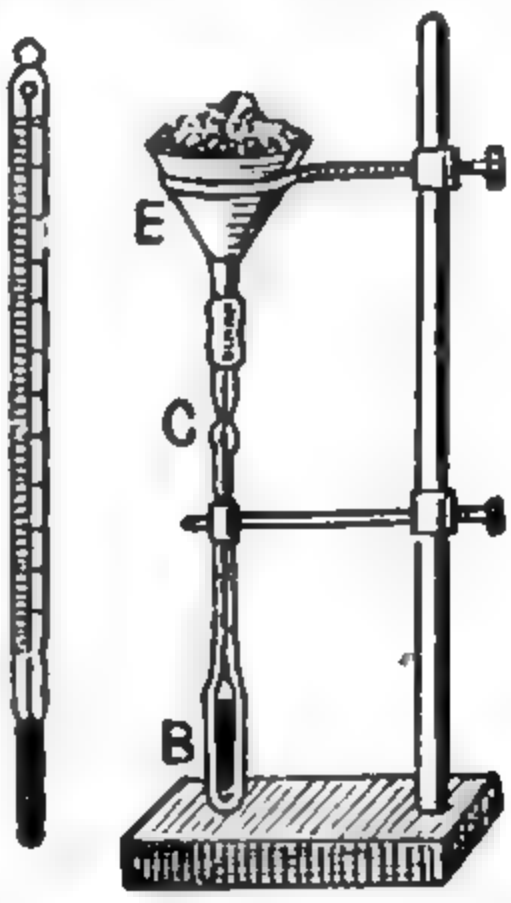


Fig. 1 Fig. 2

Construction of a Thermometer.—A piece of thick-walled *capillary glass* tube of *uniform* bore is taken. It is thoroughly cleaned and dried. A bulb B is blown at one end (Fig. 2). At C, a little below the open end, the tube is heated and drawn out so as to make a narrow neck. A small funnel E is then attached to the open end by a short piece of rubber tubing. The tube is held in a vertical position and a qnantity of pure and dry mercury is put into the funnel. As the bore is very narrow and as the tube is filled with air, the mercury does not run down the tube. To introduce the mercury into the bulb, the latter is gently heated ; the air in the bulb expands and a

portion of it bubbles out through mercury in the funnel. The bulb, being now removed from the flame, cools down ; the air inside contracts and sucks in some quantity of the mercury into the bulb. By repeating this process of alternate heating and cooling several times, the bulb is almost filled with mercury. The bulb is then strongly heated until the mercury boils and gives out mercury vapour ; as the mercury vapour continues to escape, it drives away all the air and moisture contained within. On cooling, mercury completely fills the bulb and the stem. While the bulb is still warm, the excess of the mercury in the funnel is taken out.

The bulb with the stem is now subjected to a temperature a little above the boiling point of water. Some of the mercury expands into the funnel and is removed. When the bulb slowly cools, the mercury contracts and as it just passes the constriction C, the tube is sealed at this point by means of a strong pointed flame. On cooling, the mercury contracts and fills up the bulb and a part of the stem.

The closed tube is then allowed to cool down for a considerable period before the instrument is *graduated*. In *selecting a scale* for the measurement of temperature with a mercury thermometer, it is necessary to mark on the stem the positions of the mercury thread corresponding to *two definite* temperatures. These are called the **fixed points** of thermometer. The lower fixed point is taken to correspond to the temperature at which pure ice melts under normal atmospheric pressure ; this is known as the *ice point*. The upper fixed point is taken to correspond to the temperature at which pure water boils under standard atmospheric pressure of 76 cm. of mercury. This is termed the *steam point*.

Determination of Fixed Points.—

(*a*) *The Lower fixed Point* or the **ice point.**—In order to determine the ice point, the bulb of the thermometer, constructed as above, is placed within small lumps of pure melting ice kept in a funnel (Fig. 3). The level of the mercury column gradually sinks and finally becomes stationary at a particular position. After sometime the fixed position is marked by a fine scratch made on the stem with a file. This gives the lower fixed point. It is also termed the *freezing point*. If the bulb of a centigrade thermometer is placed in melting ice in the above way and the mercury level stands at the zero mark, then its frezing point has been correctly marked.

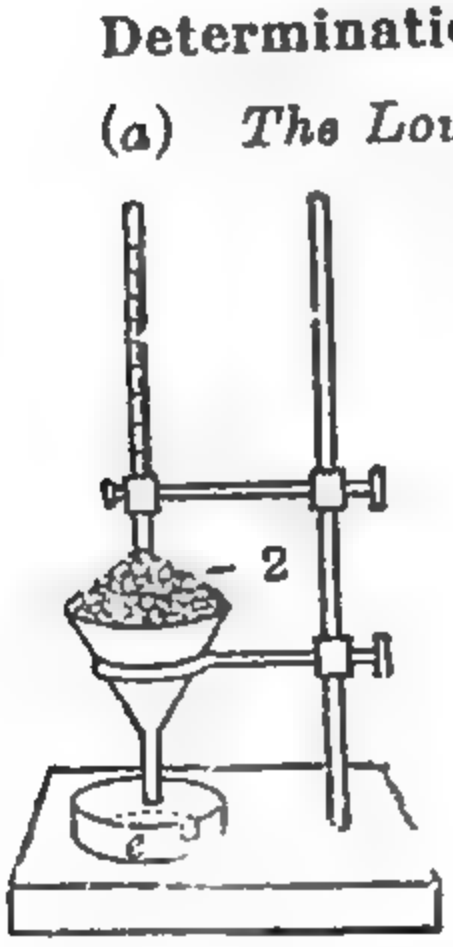


Fig. 3

(*b*) *The Upper Fixed Point* or the **steam point.**—To determine the upper fixed point, the thermometer is placed into steam of boiling water in a specially constructed vessel, called a *hypsometer*. This consists of a

copper vessel A (Fig. 4) with a vertical tube B which is surrounded by the jacket C. The jacket has two openings, one at the top and the other D at the side. A is partly filled with water and the upper opening is closed by cork. The thermometer, T is inserted through the cork into B so as to remain well above the water in A. As the water boils, the steam rises through A, passes into the jacket C and finally escapes through D. The steam in the jacket C protects that in B from being cooled by external air. An air manometer M attached to the side of the vertical tube, opposite to D indicates the pressure of the steam. The bulb and the stem being thus surrounded by steam, the level of mercury gradually rises and after some time attains a stationary position. The position of the tube should be so adjusted that this stationary position of the mercury level is just visible above the cork. After waiting for a few minutes when mercury has reached a steady temperature throughout, position of the mercury level is marked by a scratch. This mark indicates the temperature at which water boils under the atmospheric pressure at the time of the experiment, which is noted from the barometer.

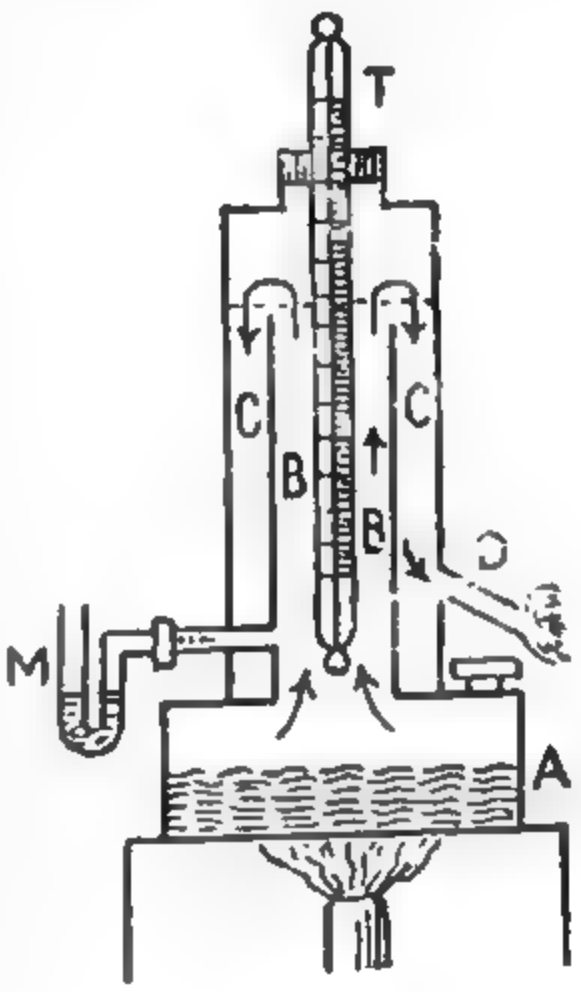


Fig. 4

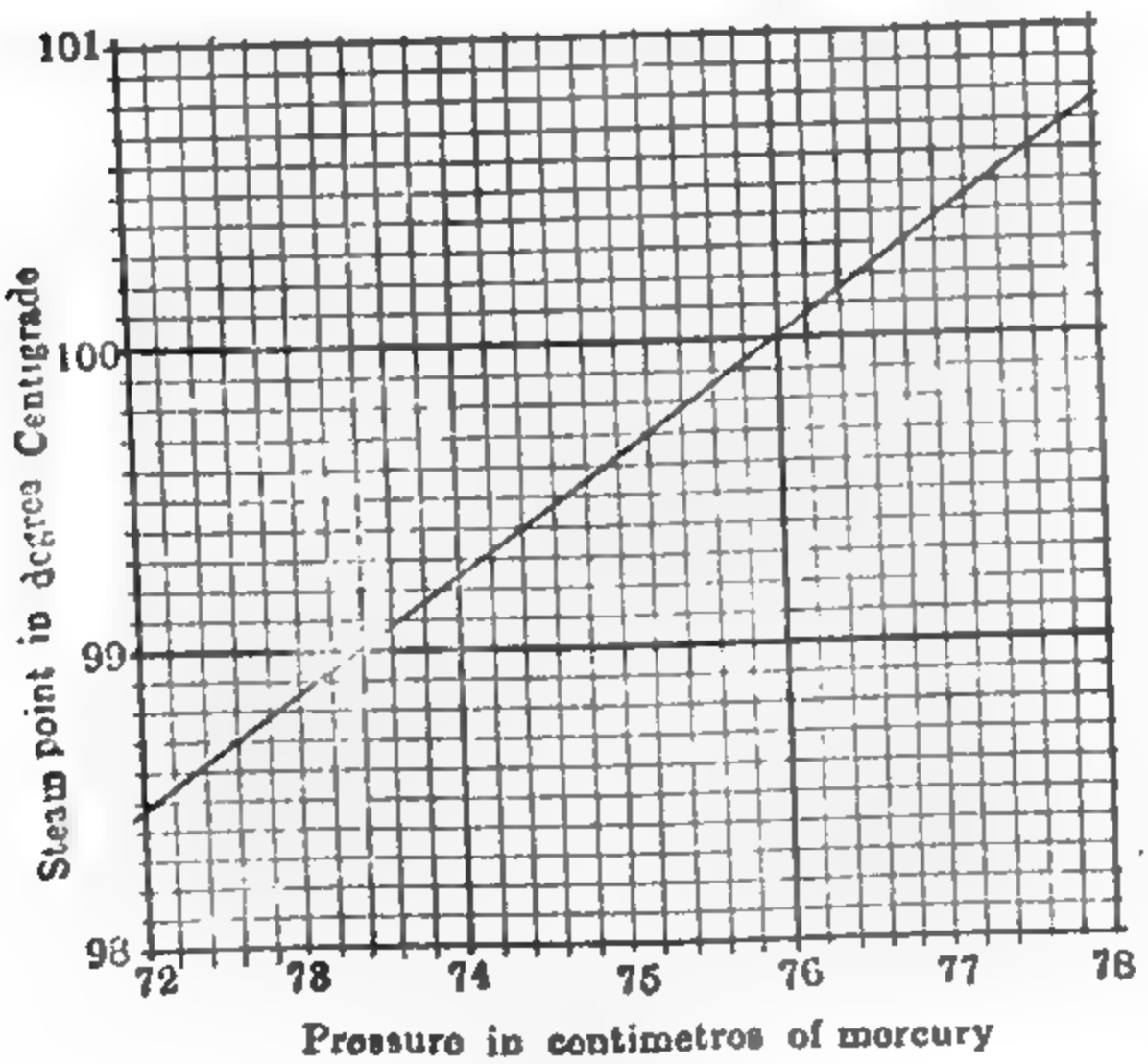


Fig. 5

As the observed atmospheric pressure is generally different from the standard pressure of 76 cm. and as pure water is found to boil at different temperatures under different pressures, the scratch mark so made does not tally with the defined steam point. The standard steam point according to a centigrade thermometer should be 100. The graph shows the variation of the observed steam point with a change of pressure (Fig. 5). It is found that near the normal atmospheric pressure, an alteration of pressure by about 28 mm. changes the boiling point of water by 1°C. The correction is to be added

or subtracted according as the observed pressure is below or above the normal atmospheric pressure.

Suppose that the scratch mark of the observed steam point is made when the barometric pressure is found to be 75 cm. The corresponding steam temperature from the graph is found to be 99·65. The length on the stem from the lower to the upper scratch marks is accurately read with a vernier microscope. Let it be l cm. If L cm. be the true length between the lower scratch mark and the standard steam point, then,

$$\frac{L}{l} = \frac{100}{99{\cdot}65} \text{ whence } L = l\,\frac{100}{99{\cdot}65} \text{ cm.}$$

Thus measuring a distance L cm. from the lower scratch mark, we put there the standard mark 100 for the upper fixed point.

After the two fixed points have been marked on the stem, the space between them is divided into a number of equal parts, according to a standard scale of temperature. Each of these divisions is called a **degree**. The graduations are continued some way both above and below the upper and the lower fixed points. To mark the graduations on the stem, the latter is coated with a thin layer of wax and equidistant lines are drawn with a fine pointer on the layer. By exposing the thermometer to the vapour of hydrofluoric acid, the marks are etched permanently on the stem.

6. **Scales of Temperature.**—There are three scales of temperature—(1) **Centigrade** (2) **Fahrenheit** and (3) **Reaumur.**

The *Centigrade scale* was designed by Elvius of Sweden in 1710 and later on by Christin in 1743. Some people associate also the name of Anders Celsius. Its zero is at the melting point of ice and the boiling point of water under normal atmospheric pressure is taken as 100°. The scale is used in scientific measurements by all nations.

The *Fahrenheit scale* was devised by Fahrenheit in 1717. Its zero was specified as the temperature of a given mixture of snow and common salt; the melting point of ice was taken as 32° of this scale and the boiling point of water under normal atmospheric pressure as 212°. This scale is generally used in Great Britain and the United States and also by doctors.

The *Reaumur scale* was designed by Reaumur near about 1730. Its zero is at the melting point of ice and the boiling point of water under normal atmospheric pressure corresponds to 80° on the scale. This scale is used for domestic purposes in Russia and in some parts of the European continent.

Comparison of Three Scales of Temperature.—The space between the two fixed points contains 100 divisions of the Centigrade, 180 of the Fahrenheit and 80 of the Reaumur Scale. This space is sometimes called the *fundamental interval* of a thermometer scale.

The positions of the two fixed points on the three scales are shown below :—

Scale	Ice-point	Steam-point	No. of equal divisions between fixed points
Centigrade (C)	0°	100°	100
Fahrenheit (F)	32°	212°	180
Reaumur (R)	0°	80°	80

$\therefore$ 100 Centigrade divisions = 180 Fahrenheit divisions = 80 Reaumur divisions

or $1°C = \frac{9}{5}$ of $1°F = \frac{4}{5}$ of $1°R$

Let us take a thermometer whose lower and upper fixed points are A and B respectively (Fig. 6) and let the three scales be placed by its side. Let us put the thermometer in a suitable bath and let P be the position of the top of the mercury column in AB. Let the reading on the three scales corresponding to the position P be denoted respectively by C, F and R. Then the distance AP represents C, F—32 and R degrees and AB represents 100, (212—32) and 80 degrees on the three scales. As AP is the same fraction of AB, whatever be the scale used.

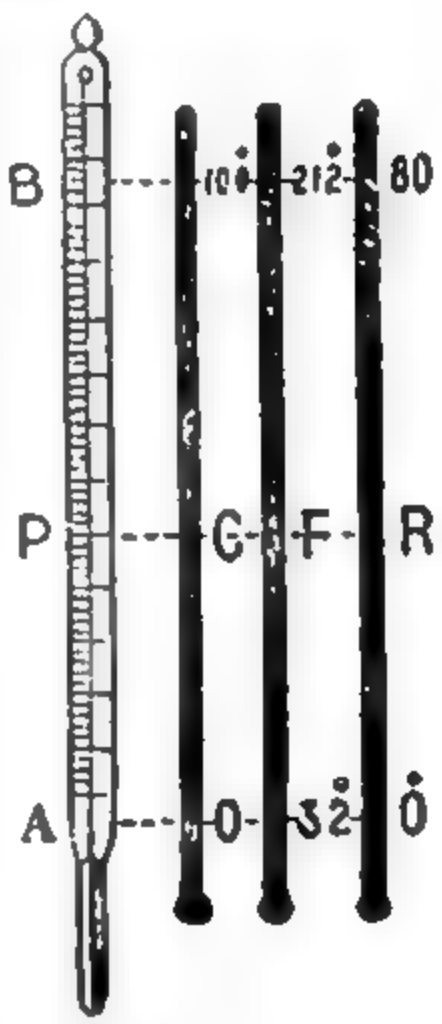


Fig. 6

$$\frac{AP}{AB} = \frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

$$\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4} \qquad \ldots(6.1)$$

Examples :—

1. The normal temperature of a healthy person is found to be 98·4°F. What would be corresponding readings on the Centigrade and the Reaumur scales ?

A. Here $\frac{C}{5} = \frac{98\cdot4-32}{9} = \frac{R}{4}$ whence $C = 36\cdot9°$ and $R = 29\cdot5°$

2. What is the temperature which when read on a Centigrade and a Reaumur thermometer differs by 2° ?

Let the reading on Centigrade Scale be x ; Then on Reaumur Scale $= x + 2$ or $x - 2$

Since $\frac{C}{5} = \frac{R}{4}$ we have $\frac{x}{5} = \frac{x+2}{4}$ or $\frac{x}{5} = \frac{x-2}{4}$

where $x = -10°C$. or $10°C$. and $R = -8°$ or $8°$ respectively.

3. If when the temperature is 0°C. a mercury thermometer reads 0·5°C. while at 100°C. it reads 100·8°C. Find the true temperature when the thermometer reads 20°C, assuming that the bore is cylindrical and the divisions are of uniform length.

A. Since at the true zero-point thermometer reads 0·5°C and at 100°C, it reads 100·8°C. there are (100·8 — 0·5) or 100·3 divisions between the two fixed points. Each division of the thermometer corresponds to 100/100·3 of a correct centigrade thermometer. The reading 20° of this thermometer is really (20 — 0·5) or 19·5 divisions above the freezing point.

$$\text{The corresponding true temperature} = \frac{19{\cdot}5 \times 100}{100{\cdot}3} = 19{\cdot}44°\text{C}$$

4. A faulty thermometer read 5°C in melting ice and 99°C in dry steam at normal pressure. Find the correct temperature in Fahrenheit when the thermometer reads 52°C. (Pat. U.—1951)

A. 99 —5 = 94° of the thermometer corresponds to 100° of a standard thermometer

$\therefore$ 1° of this thermometer $= \frac{100}{94}$ of the standard centigrade thermometer.

When the thermometer reads 52°C, it is in fact 52 — 5 = 47 scale divisions above the ice point.

This would be $\frac{47 \times 100}{94}$ = 50°C of a standard centigrade thermometer.

Again $\frac{C}{5} = \frac{F-32}{9}$ $\therefore$ $\frac{50}{5} = \frac{F-32}{9}$ whence F = 122° Fahrenheit.

7. Advantages of Mercury as a thermometric Substance.— (1) Mercury boils at 357°C, and freezes at − 39°C ; so it remains in the liquid state over a *long range of temperatures*. Ordinarily a mercury thermometer may be conveniently used for an approximate range of temperatures from − 30°C to about 300°C.

A mercury thermometer may also be used for measuring high temperature by filling the space above the mercury with nitrogen or carbon dioxide at a high pressure so as to prevent the boiling of mercury. A temperature upto 600°C can thus be measured by using glass with high melting point and the range can even be increased to 750°C by taking the tubes of silica.

(2) It is a *good conductor*, that is, it quickly transmits heat throughout its body and so all mercury in the thermometer bulb readily attains the temperature of the substance in which it is placed.

(3) Its expansibility is fairly large with a rise of temperature.

(4) Its expansion is *fairly* uniform so that its volume increases equally for equal rise of temperature at all parts of the scale.

(5) It has a *low thermal capacity* which means that, to increase its temperature through a range, a small quantity of heat is required.

(6) As the vapour pressure of mercury is very low at ordinary temperature, the space above mercury contains very little mercury vapour.

(7) It can be readily obtained pure. It is a shining opaque liquid and so its column within a glass tube can be easily recognised. It does not wet glass and does not, therefore, stick to the sides of the glass tube.

8. Alcohol Thermometer.— Alcohol is sometimes used as a thermometric substance. The liquid is coloured with some dye as otherwise the position of the top of the liquid column cannot be readily

observed. Its advantages and disadvantages as a thermometric substance in comparison to mercury are given below.

Advantages :—

(*i*) Alcohol freezes as – 130°C, while mercury freezes at – 39°C, so for measurement of *low temperatures*, an alcohol thermometer is preferable. Consequently, alcohol may be used in thermometers down to about – 128°C. (Liquid pentane may be used down to – 200°C).

(*ii*) For a given rise of temperature, alcohol expands much more than mercury ; so an alcohol thermometer is *more sensitive* than a mercury thermometer to temperature variations.

(*iii*) For a given volume, alcohol requires much less quantity of heat than mercury to be raised through the same range of temperature although the thermal capacity of alcohol is greater than that of mercury.

(*iv*) Alcohol is a light liquid and wets glass ; so when temperature rises, the thread of alcohol can move smoothly in a tube of very fine bore, while mercury tends to move in a jerky manner.

Disadvantages : —

(*i*) Alcohol boils at 78°C while mercury boils at 357°C, and so an alcohol thermometer cannot be used for accurately measuring temperatures higher than about 60°C.

(*ii*) Its conductivity is less than that of mercury.

(*iii*) The rate of expansion of alcohol is not uniform but it increases with rise of temperature. An alcohol thermometer is, therefore, graduated by comparison with a mercury thermometer, both being placed in the same bath.

(*iv*) It is highly volatile and so its vapour readily collects into the space above the liquid and exerts an appreciable pressure.

(*v*) An alcohol wets glass, a film of the liquid sticks to the walls of the tube when the temperature falls.

9. **Maximum and Minimum Thermometer.**—These thermometers are so constructed as to be able to record *automatically* the highest

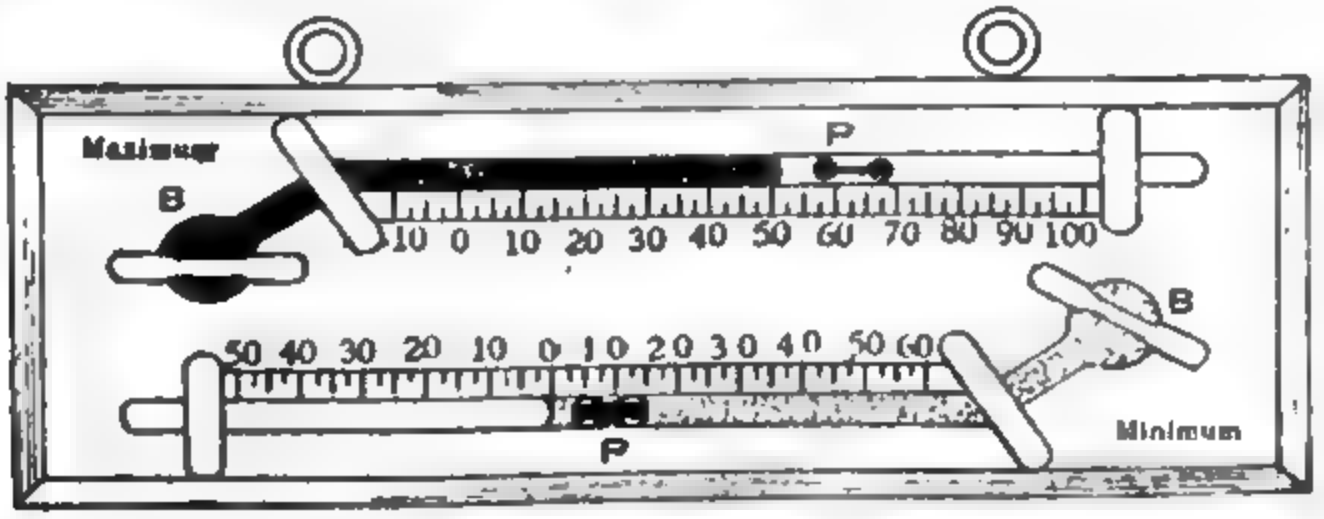


Fig. 7—Maximum & Minimum Thermometers

or the lowest temperature attained during a given period of time,

say, one day. They are used for meteorological and agricultural purposes.

Rutherford's Maximum Thermometer.—This is a mercury thermometer with the bulb B and a small steel index P, shaped like a double-headed pin resting within the stem (Fig. 7). The thermometer, is mounted horizontally in a wooden or metallic frame, so that the index can remain stationary at any position.

In setting the thermometer the steel index is moved from outside by a strong magnet till one end is brought just in contact with the surface of the mercury. As the temperature rises, the mercury expands and pushes the index forward. When the temperature falls, the mercury contracts, but the index is left at the position to which it had been pushed. The higher is the rise of temperature, the farther is the distance the index is pushed through. The end of the index towards the mercury surface records the maximum temperature attained during the period under observation.

In some form of a Maximum Thermometer, the stem is placed in a horizontal position and the bulb is kept at a slightly lower level. There is a constriction at the junction of the bulb and the stem. When temperature rises mercury is forced into the stem ; but when temperature falls mercury column within the stem is separated from the mercury within the bulb, automatically recording the maximum temperature. This sort of thermometer does not require the use of a separate index.

Rutherford's Minimum Thermometer.—In this thermometer alcohol is used in place of mercury. There is a small index P made of enamel or coloured glass and shaped like a double-headed pin, resting within the liquid in the stem (Fig. 7). The thermometer is fixed horizontally in a suitable frame. The instrument is set by tilting and the index is moved till one of its ends is just in contact with the liquid surface, the other end being within the liquid. As the temperature falls, the liquid contracts and the concave surface of alcohol drags the index along with it due to surface tension. But when the temperature rises, the alcohol expands, leaving the index at the positions attained. The end of the index towards the meniscus records the minimum temperature reached during the period under observation.

Six's Thermometer.—In this case the maximum and the minimum thermometers are combined into a single instrument. It essentially consists of a U-tube (Fig. 8) with bulbs at the two ends. The bulb B and a part of the stem are filled with alcohol ; the portion from P_1 to P_2 is filled with mercury. The tube above P_1 and a portion of the bulb *b* are filled with alcohol ; the remaining part of it contains alcohol vapour only, space being thus left for expansion.

To small steel indices P_1 and P_2 shaped like dumb-bells float on the ends of the mercury column. Each index is provided with a small side-spring which pressing against the wall of the tube keeps the index in position. The instrument is set by bringing the indices

just in contact with the respective mercury surface by means of a strong magnet working from outside. There are two graduated scales attached beside the limbs of the U-tube, the one of the 'maximum' side being graduated from below upwards and that one 'minimum' side, from above downwards.

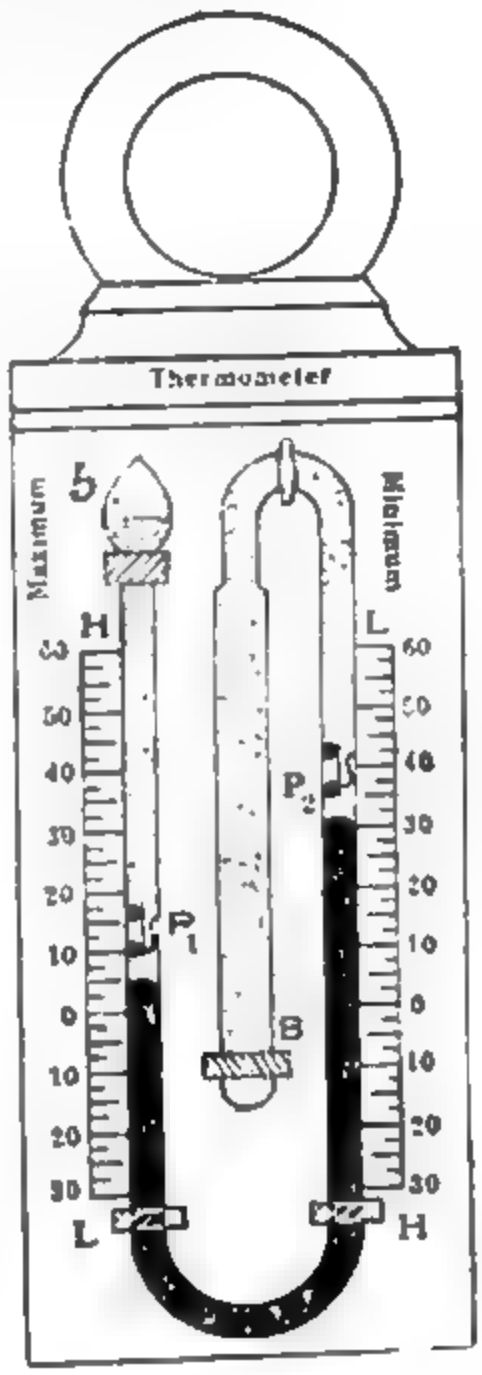


Fig 8

As the temperature rises, the alcohol in larger bulb B expands and pushes down the mercury column with which it is in contact. The other end of the mercury column rises, pushing forward the index P_1 and driving the alcohol in the other bulb *b*. When the temperature falls, the alcohol in the larger bulb contracts and the mercury level on this side rises, raising the index P_2, and leaving it at the highest position reached. The lower end of P_1 records the maximum temperature during the period under observation, the reading being obtained from the corresponding scale. The position of the lower end of P_2, from the corresponding scale, gives the *minimum* temperature. The instrument is re-adjusted after each observation.

Clinical Thermometer (*Doctor's thermometer*).—This is a form of sensitive maximum thermometer, used for recording the temperature of human body. It is a mercury thermometer with a constriction at C in the bore close above the bulb (Fig 9). It has a short stem which is graduated generally from 95°F, to 110°F, each degree being divided into fifths. The normal temperature of a healthy person varies between 96·6°F and 98·4°F. When the bulb is put into thermal communication

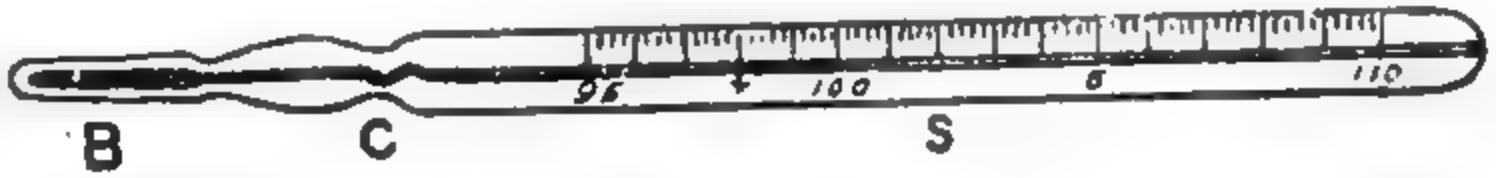


Fig. 9—Clinical Thermometer

with the human body, the temperature of the bulb rises and the mercury expands. The expansive force drives the mercury past the constriction into the stem. When the thermometer is removed, the temperature falls and the mercury below C contracts, but the thread above C cannot follows as it is obstructed by the constriction ; the farthest end of this portion indicates the maximum temperature of the body. When the thermometer is to be used again, it is shaken with the bulb downwards so that the mercury thread in the stem is made to pass through the constriction partly into the bulb.

***10. Sources of Errors in Mercury Thermometer.**—The following are some of the more important sources of error in a mercury thermometer and their corrections.

(1) *Change of Zero.*—By placing the bulb of an ordinary mercury thermometer in melting ice, it is sometimes found that the mercury stands at a slightly *higher* position than the freezing point marked on the stem. This occurs when a sufficient interval was not allowed between the scaling and the graduation of the thermometer tube. The tube contracts slowly and so the volume of the mercury appear to be in excess, showing a rise in the freezing point. The excess reading for ordinary glass is about '04°C.

(2) *Temperature of Steam.*—It has been mentioned in Art. 5 that in determining upper fixed point, a correction is to be applied if the atmospheric pressure at the time be not normal. But if both the fixed points possess certain errors, they may be eliminated by supplying for every such thermometer a correction table between 0° and 100°C.

(3) *Temperature of the Exposed Column.*—When a thermometer is used, a part of the stem always remains outside and is, therefore, at a a different temperature from the bulb and the rest of the stem. For this reason the readings which are recorded, would be slightly lower; hence it is better to immerse as much of the stem as possible. A correction for this may be applied.

(4) *Internal Pressure.*—A vertical column of mercury 20 cm. long exert a pressure on the thermometer bulb of nearly a quarter of atmospheric pressure. Hence when a comparatively higher temperature is recorded by a mercury thermometer, a pressure of this order is exerted on the bulb, distending the base. The reading is consequently lower. But if the thermometer is kept horizontal, the problem of internal pressure does not arise.

(5) *Non-uniformity of the Bore.*—In a thermometer, change of temperature is measured by the change of volume of a definite mass of the liquid in the bulb. The changes in volume produce corresponding movement of the liquid head along the stem. The distance on the stem between the two fixed points is divided into equal intervals or *degrees*, so that each degree represents equal change of temperature. This is true only when the bore is uniform; otherwise equal expansion of the liquid for equal rise of temperature, though regular, will not be indicated by equal length in different parts of the stem. In such a case, the thermometer may be calibrated all over the length of the bore by detaching a thread of mercury and measuring the number of degree divisons it occupies at various parts of the tube.

*11. Dial or **Index Thermometer.**—Sometimes the temperature of a source, remote from the observer, requires to be determined or the thermometer has to be inserted in a position where it is difficult to read. For this reason a dial thermometer is used in preference to ordinary mercury-in-glass pattern already described. The apparatus consists of a nickel plated copper bulb B connected to a metal tubing T by a gas tight joint (Fig. 10) The other end of the tubing is served to the body of the instrument carrying a pressure gauge with a pointer moving over a circular dial graduated generally in Fahrenheit degrees.

The bulb and the tube are filled with a liquid. When the bulb is introduced into a hot source, the liquid inside it expands and partly vaporises and the force of expansion communicated through the gauge causes the pointer to move over the dial and record the temperature.

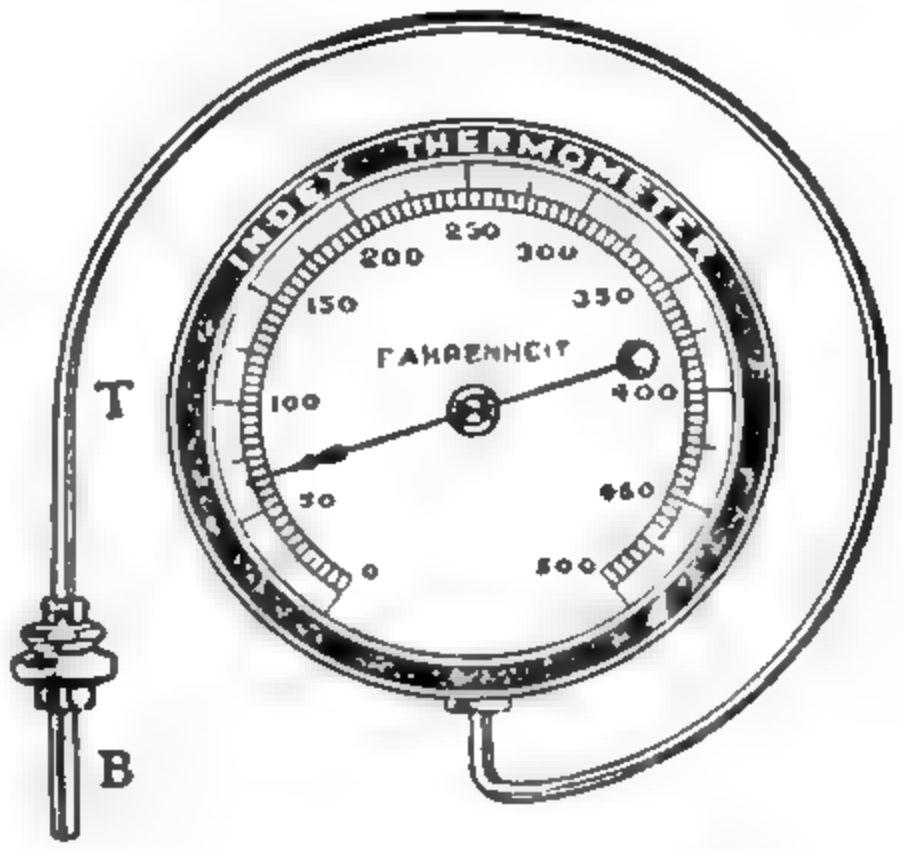


Fig. 10 – Dial Thermometer

There are various patterns and sizes of index thermometers for different purposes. Sometimes an index thermometer is utilised to ring a bell or light a lamp or operate a switch automatically when a certain temperature is reached. It is very widely used as a fire alarm, for measuring temperatures of oil engines, as a radiator thermometer on motor cars and aeroplanes, for taking temperature of liquid in tanks, for ovens and stoves and transformers. The dial has different ranges of calibration from – 40° to 1000° F.

12. Principle of Temperature Measurements.—It has already been stated that any property of a substance, such as expansion in volume, electrical resistance, thermo-electromotive force at the junction of two metals or the vapour pressure of a liquid, which changes with temperature, may be used to reckon a temperature. Suppose that at the freezing point of water, the magnitude of any property of a substance is X_0. The same property when measured at the boiling point of water would have a different magnitude, say, X_{100}. On the centigrade scale this change in magnitude of $X_{100} - X_0$ corresponds to a difference of temperatures of 100°C. Therefore 1° on the centigrade scale is defined as that change of temperature which causes a change of value of $(X_{100} - X_0)/100$.

Let X_t be the magnitude of the property at some temperature. Then between the freezing point and this temperature, the change in value of the property is $X_t - X_0$. We define this temperature as t°C, such that the change of property for 1°C remains the same as before.

$$\text{Thus, } \frac{X_t - X_0}{t} = \frac{X_{100} - X_0}{100} \quad \text{or} \quad t^\circ\text{C} = \frac{X_t - X_0}{X_{100} - X_0} \times 100 \qquad \ldots(12,1)$$

In selecting the scale, therefore, we assume that the property of a substance changes by equal amount for each degree rise of temperature. The condition is shown by the straight line ABC drawn on a graph paper (Fig. 11) with abscissa as temperature and ordinate as the property of the substance ; the points A, B and C correspond to $(X_0, 0)$, (X_t, t) and $(X_{100}, 100)$. Then from this graph, the relation in eqn. (12, 1) can be easily found. But in fact no physical property of a substance varies *exactly uniformly* with a rise of temperature. Suppose

that the expansion in volume of a particular liquid with temperature is shown by the dotted curve ADC which is not straight. Now if this liquid is used in a thermometer, whose fixed points are standardised at 0° and 100°C, the thermometer will not indicate exactly any other temperature as defined by eqn. (12,1). For the value X_t, the liquid thermometer would read $\theta°$ and not the temperature $t°$. For this reason there is a slight variation in the readings of particular thermometers when any temperature other than the two fixed points is read. Of all substances, the expansibility of hydrogen with temperature is very nearly uniform. Hence a hydrogen thermometer (Chap. IV) is the standard thermometer accepted by the International Committee of Weights and Measures. The linear scale ABC, which is an ideal one, is called the *Absolute Thermodynamic Scale.*

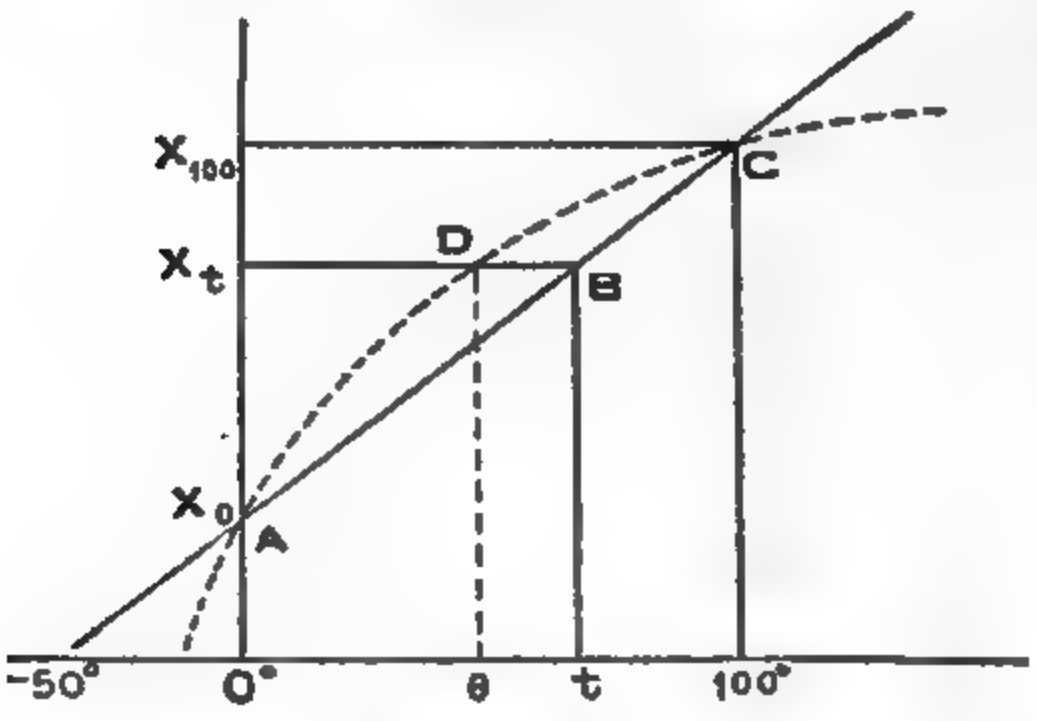


Fig. 11

ANDERS CELSIUS (1701—1744)

Celsius was born at Upsala, Sweden, on November 27,1701. His grandfather was a famous Swedish astronomer. After graduating from the University of Upsala, he was appointed a Professor of Astronomy. In order to build an Observatory at the University, he made a tour of inspection to various Observatories of Europe and in 1732 an upto-date Observatory was erected at Upsala. Meanwhile he published some papers regarding the phenomena of aurora borealis.

It was while he was engaged in astronomical studies that he constructed the centigrade thermometer in 1741. He divided the range from the melting point of ice to the boiling point of water into 100 equal parts and called each part a degree centigrade. Such a scale is widely used for scientific purposes. He went with his newly constructed thermometer to Lapland to measure the temperature of arctic region. He died a premature death on April 25, 1744.

Summary

Heat and Temperature.—Heat is a form of energy while temperature denotes a particular thermal condition of body. Flow of heat depends upon difference of temperatures.

Thermometers are instruments for accurate measurements and comparison of temperatures; the most common one is the mercury-in-glass thermometer.

The fixed points of a thermometer—called the Ice point and the Steam point—are respectively the melting point of ice and the boiling point of water under normal atmospheric pressure.

Scales of Temperature.—(*i*) *Centigrade*, (*ii*) *Fahrenheit* and (*iii*) *Reaumur*.

Comparison of the scales.— $\frac{C}{5} = \frac{F-32}{9} = \frac{R}{4}$

Maximum and Minimum Thermometers.—(*i*) *Rutherford's* (*ii*) *Six's*.

Clinical Thermometer is used for measuring the temperature of the human body. **A Dial Thermometer** is used to measure temperature of a source romote from the observer.

Exercises on Chapter I

Reference

1. What do you mean by the temperature of a substance? Describe with a neat diagram the Six's thermometer and discuss the principle on which it works. State some of its uses. — Arts. 2 & 9

Determine the temperature which is indicated by the same number on Centigrade and Fahrenheit scale? (C. U.—1942; '48)

A. $-40°C$, or $-40°F$.

2. How would you check the freezing and boiling points of a thermometer? — Art. 5

The freezing point on a thermometer is marked 20 and the boiling point 150. What reading would this thermometer give for a temperature of 45°C.? (C. U.—1937)

A. 78·5°.

3. How will you construct a mercury thermometer and graduate it so that your reading will agree with that of another thermometer. All precautions and corrections should be mentioned. — Art. 5
(Pat. U.—1951; Del. H. S.—1950; U. P. B.—1939)

4. (*a*) What is meant by Fundamental Interval of a thermometer. Describe an experiment to determine it accurately. (Pat. U.—1940) — Art. 5

(*b*) Describe fully the method followed to mark the scale on a mercury-in-glass thermometer. (C. U.—1952)

5. Describe the construction of a mercurial thermometer. — Art. 5
(C. U.—1926, '32, '41; Pat. U.—1922, '32)

6. Describe how the fixed points of a mercury thermometer are determined.

A faulty thermometer reads 1° when placed in melting ice and 96° in steam at normal barometric pressure. Find the correct temperature when the thermometer reads 39°, the bore of the thread and graduations being supposed uniform. (Dac. U.—1943) — Arts. 5 & 6

A. 40°C.

Reference

Art. 5 7. Describe how you would construct and graduate a liquid-in-glass thermometer to read temperatures between 2°C and 10°C. Another sensitive thermometer for this range will be supplied.

Give reasons for choosing a particular liquid as the thermometric substance. Also discuss whether water can be used for this range, and explain how you would read the temperature without any ambiguity with water-in-glass thermometer in the above range. (Pat. U.—1952)

Art. 9 8. Describe any two forms of maximum or minimum thermometer. (Pat. U.—1921; All. U.—1926)

Art. 8 9. Give an account of the various methods for measurement of temperature and classify them according to the range of temperature for which they are suitable. Give their relative accuracy of measurement. (R. P. B.—1948)

Art. 9 10. Describe a clinical thermometer. What makes it possible to read the temperature after removing the thermometer from the body? (C. U.—1937)

CHAPTER II

EXPANSION OF SOLIDS

13. Expansion of Solids.—Solids in most cases expand when heated. This can be shown by an apparatus known as ***Gravesand's Ring.*** It consists of a metal ring (Fig 12) clamped to a stand. A ball of the same metal suspended by a chain can just pass through the ring when both are at the same temperature. When heated in a flame, the ball will no longer pass through the ring, showing that it has expanded. On cooling, the ball contracts and slips through the ring.

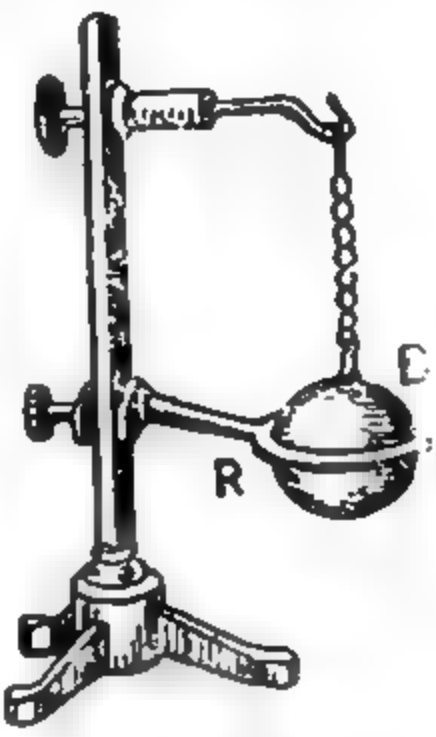


Fig. 12

A short metal rod with flat ends and provided with a wooden handle fits exactly into a metal gauge at ordinary temperature. When heated, the rod no longer fits into the gauge, as it has expanded.

Two similar strips, one of iron and the other of copper, rivetted together at several points, are taken (Fig. 13). When heated the composite bar slowly gets curved, the copper remaining on the outside. If on the other hand, the bar is placed for sometime in ice, it bends in the opposite direction with copper strip on the inside. This shows that copper expands or contracts more than iron. It can be shown in a similar way that different solids expand differently.

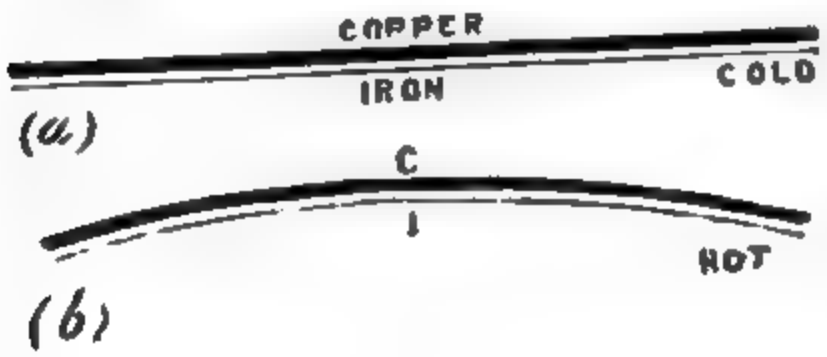


Fig. 13

When a solid is heated, it generally expands in all directions, although the expansion is usually very small. The increase in length of the body is called *linear expansion*, that in area is called *superficial expansion* and the increase in volume is called *cubical expansion*.

14. Co-efficient of Linear Expansion.—When a solid is heated, it is found that the increase in length (*i*) *is proportional to the original length of the body*, (*ii*) *is proportional to the increase of temperature to which the body is subjected, and* (*iii*) *depends on the nature of the material of the solid.*

The co-efficient of linear expansion of a solid is defined to be *the increase in length per unit length of the material for a rise of temperature of 1°C.*

The *coefficient of linear expansion of iron is* 0·000012 ; this statement means that a rod of iron 1 cm. in length increases by 0·000012 cm. for a rise of temperature of 1°C.

Let the original length of the rod at 0°C. $= l_o$
length of the rod when heated through t°C $= l_t$
coefficient of linear expansion of the solid $= \alpha$
Then the increase in length for a rise of t°C $= l_t - l_o$
∴ the increase in length for a rise of 1°C $= (l_t - l_o)/t$

Hence by definition, the mean coefficient of linear expansion between 0°C and t°C is given by

$$\alpha = \frac{l_t - l_o}{l_o t} \text{ per °C.} \qquad \ldots(14,1)$$

In other words, the coefficient of linear expansion

$$= \frac{\text{increase in length}}{\text{original length} \times \text{rise of temperature}}$$

From eqn. (14,1), it is thus apparent that the co-efficient of linear expansion of a material *would have the same value, whether its length is measured in centimetres or in feet.* Therefore the coefficient of expansion is independent of the unit of length. But it depends upon the scale of temperature in which the value is found. For example, if the Fahrenheit scale is used, the coefficient of linear expansion becomes different from that measured in centigrade scale. As 1°F. $= \frac{5}{9}$ of 1°C. the coefficient of linear expansion of iron,

$$= \tfrac{5}{9} \times 0{\cdot}000012 = 0{\cdot}0000066 \text{ per °F.}$$

Again from eqn. (14,1) we have $l_t - l_o = l_o \alpha t$

Hence $l_t = l_o + l_o \alpha t = l_o(1 + \alpha t)$ $\qquad \ldots(14,2)$

For a wide range of temperature, the coefficient of linear expansion is nearly constant.

15. Expansions at different Temperatures.—A body may be initially not always at 0°C but at some other temperature say t_1°C. Then its length at a different temperature t_2°C, may be obtained in the following way ;—

Let l_o, l_1 and l_2 be the length at 0°C, t°C, and t_2°C, respectively and let t_2 be higher than t_1.

Then $l_1 = l_o(1 + \alpha t_1)$ and $l_2 = l_o(1 + \alpha t_2)$

$$\therefore \quad \frac{l_o}{l_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1} = (1 + \alpha t_2)(1 + \alpha t_1)^{-1}$$

Since α is a very small quantity, square or higher powers of α may be neglected. Expanding $(1 + \alpha t_1)^{-1}$ binomially upto two terms, we have

$$(1 + \alpha t_1)^{-1} = 1 - \alpha t_1$$

$$\frac{l_2}{l_1} = (1 + \alpha t_2)(1 - \alpha t_1) = 1 + (t_2 - t_1)\alpha - \alpha^2 t_2 t_1$$

neglecting terms containing higher powers of α

$$\therefore \quad l_2 = l_1\{1 + (t_2 - t_1)\alpha\} \qquad \ldots(15,1)$$

$$\text{Or,} \quad \alpha = \frac{l_2 - l_1}{l_1(t_2 - t_1)} \text{ per °C} \qquad \ldots(15,2)$$

This expression is similar to the eqn. (14,1) except that the original

length is taken not at 0°C. Thus assuming the co-efficient of linear expansion of a solid to be very small, it is immaterial whether the original length is measured at 0°C. or at the room temperature

16. Determination of the Co-efficient of Linear Expansion of Solid by Pullinger's apparatus. —The apparatus consists of a steam jacket J (Fig. 14) held vertically in suitable frame F. The ends of the jacket are closed by rubber corks. It is provided with inlet and outlet tubes for steam and with two other side tubes for inserting thermometers TT. A rod R, about a metre long and of the material of which the co-efficient of linear expansion is to be determined, passes into the jacket through the cork and stands vertically inside the jacket. The lower end of the rod rests firmly on a slab of marble or glass or on a vertical pivot, fixed to the base board of the apparatus and is thus prevented from expanding in the downward direction. The upper end of the rod which is free to expand passes just outside the jacket and projects through a hole in a glass plate G placed horizontally at the top of the frame. A spherometer S is placed with the three outer legs on the plate G while the central leg passes through the hole and can be screwed down to touch the upper end of R.

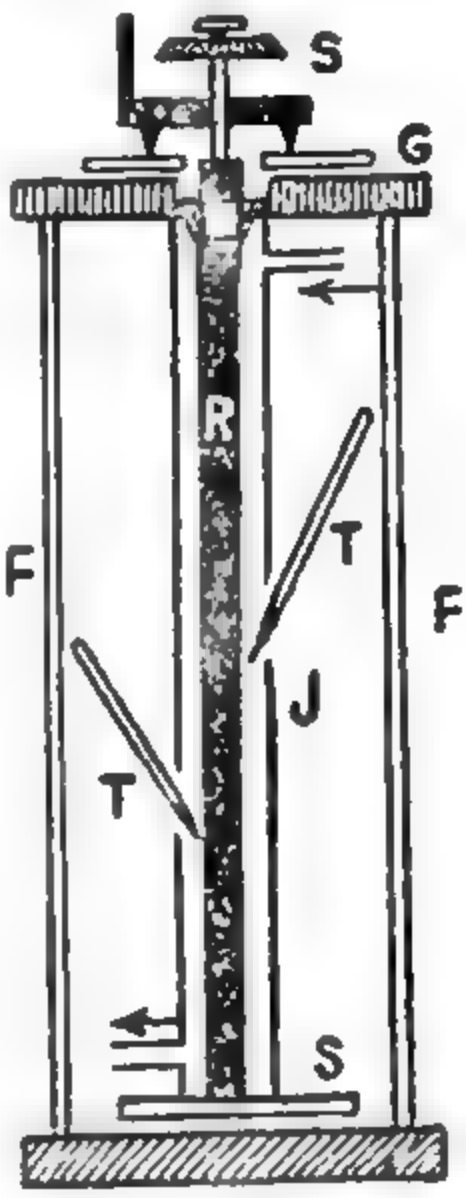


Fig 14

The rod is taken out and its length is measured at the room temperature with a metre scale. It is then placed in position in the jacket tube and after sometime its temperature is noted from the thermometers ; if the two thermometers give different readings, the mean of the readings is to be taken. The spherometer is placed on the plate G and the central leg is screwed down just to touch the upper end of R. The reading of the spherometer is noted and the central leg is screwed up to make room for the rod to expand. Steam from a boiler is now passed through the jacket J. The temperature inside gradually rises and finally becomes constant, as would be shown by the thermometers. If the steady temperatures, as recorded by the thermometers, show slightly different readings ; the mean of the two temperatures is to be taken as the final temperature. After waiting for some time to be sure that the rod has attained a constant temperature throughout, the central leg of the spherometer is again screwed down just to touch the upper end of R and the reading is noted. The difference between the two readings of the spherometer gives the increase in the length of the rod for the rise of temperature during the experiment.

Let the initial length of the rod $= l$
increase in its length, as given by the spherometer $= e$
initial and final temperatures of the rod respectively $= t_1°$ and $t_2°$C

Then from (15,2) $$\alpha = \frac{e}{l \times (t_2 - t_1)}$$

For further details of the experiment *vide* J. Chatterjee's Intermediate Practical Physics. Sometimes a hollow metal pipe with closed ends and having arrangements to circulate water or steam within it and to insert thermometers, is used instead of a solid rod. The pipe is covered on its outsides with nonconducting substance such as felt or flannel to prevent much heat being lost. The expansion of a solid rod or a hollow pipe of the same length and of the same material is equal. To know the instant of contact between the rod and the spherometer, one terminal of an electrical battery is connected to the rod while the other terminal is connected through a galvanometer to the spherometer. So long as there is an air gap between the central leg and the rod, the electrical circuit is not complete and the galvanometer shows no deflection. But the moment the two are just in contact, there is a deflection in the galvanometer. The spherometer is then read. The rest is similar to the experiment, as mentioned above.

Co-efficients of Linear Expansion of Solids

(Mean Co-efficients between 0°C and 100°C.)

Invar	0·0000009	Nickel	0·0000130
Glass	0·0000089	Copper	0·0000167
Platinum	0·0000089	Brass	0·0000189
Steel	0·0000110	Aluminium	0·0000238
Iron	0·0000116	Zinc	0·0000298

17. Travelling Microscope Method.—This is a very accurate method of measuring a small change in length or comparing two lengths which are nearly equal to each other. Hence this is sometimes called the *comparator method*. The specimen, whose linear expansion is to be

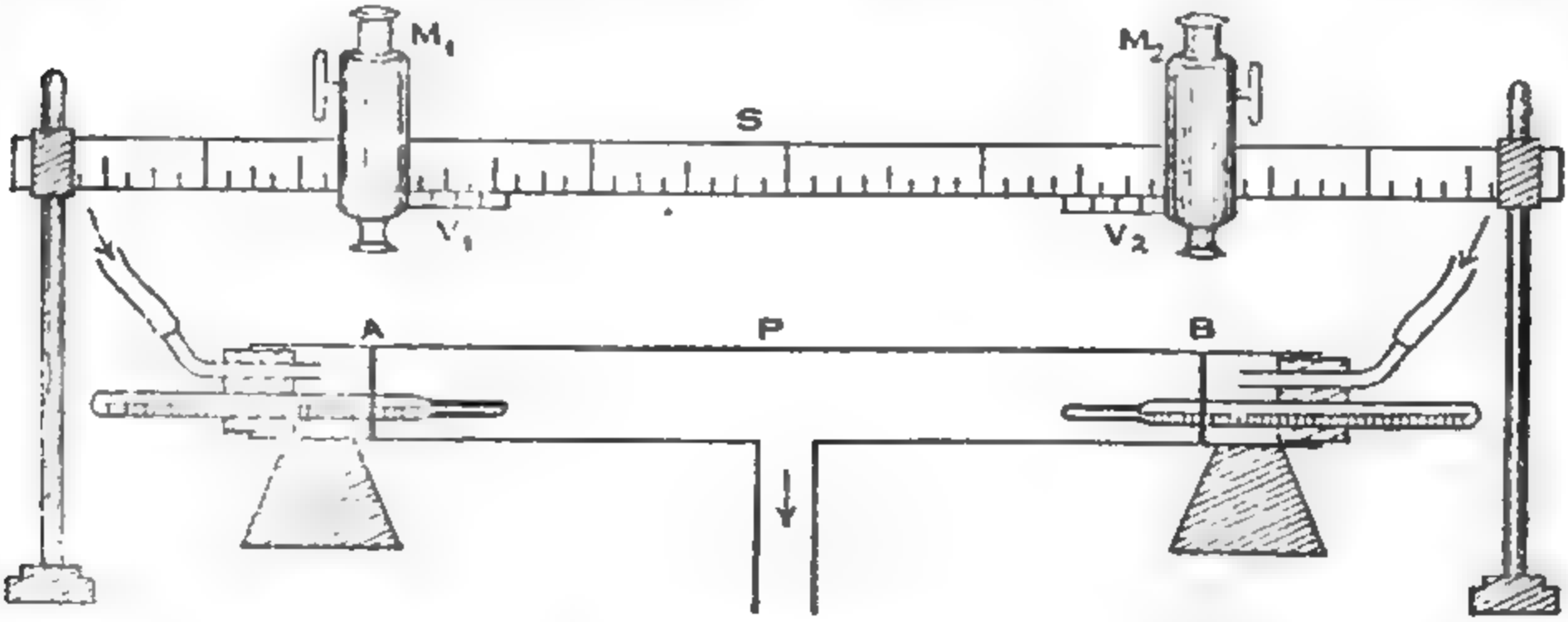


Fig. 15—Comparator Method

measured, is taken in the form of a rod or a pipe. In fig. 15, P represents a hollow pipe of nearly one metre in length closed at two ends by cork or rubber stoppers. Each stopper is provided with two holes, through

which a thermometer and an inlet glass pipe may be introduced. There is an exit pipe fitted at the central part of the pipe. The pipe with its equipments may be placed in a horizontal position on two blocks of wood.

Two microscopes M_1 and M_2, which may be slided over a horizontal metal scale S, carry two verniers V_1 and V_2 or micrometer screws. Slight lateral movement of any microscope may be accurately measured with such attachments. Two marks A and B are made at the ends of the pipe.

Thermometers and pipe fittings are made as shown and the inlet pipes are connected through rubber tubings to a boiler containing water at the room temperature. The length between the two marks A and B is measured with the scale S or any other metre scale to the nearest millimetre. Let the length so read at the room temperature t_1°C be l cm. The microscopes are then focussed on the marks and their positions are read accurately with the vernier attachments. Let the initial readings of the microscopes be C_1 and C_2 cm. respectively.

Then the boiler is heated and steam being formed circulates through the pipe and passes out through the exit. Thermometers record a rise in temperature. When they record a steady high temperature, t_2°C say, the microscopes are slided properly and the marks are again focussed. Let the new readings of the microscopes be C_3 and C_4 respectively. Then the elongation on one side is $(C_3 - C_1)$ cm. and on the other side $(C_4 - C_2)$ cm. Thus the total elongation is the sum of the two and call it x cm.

Then, coefficient of Linear Expansion $\alpha = \dfrac{x}{l(t_2 - t_1)}$

18. Co-efficient of Superficial Expansion.—It is defined to be *the increase in area per unit area of a surface for a rise of temperature of* 1°C.

Let original area of the surface at 0°C $= S_0$

area of the surface at t°C $= S_t$

coeff. of superficial expansion of the solid $= \beta$

Then by definition, $\beta = \dfrac{S_t - S_0}{S_0 t}$ ... (18,1)

$\therefore \quad S_t - S_0 = S_0 \beta t \quad$ or $\quad S_t = S_0(1 + \beta t)$ ... (18,2)

Relation between α and β.—Consider a square lamina ABCD of a homogeneous solid of side l_0 cm. at 0°C. (Fig. 16) ; then its area at 0°C is given by $S_0 = l_0^2$ sq. cm. At t°C, let the square takes a larger shape as given by AEFG. Then the increase of area is given by the shaded part BCDGFE.

At t°C., the length of each side becomes $l_t = l_0(1 + \alpha t)$

$\therefore$ the area at t°C. is given by $S_t = \{l_0(1 + \alpha t)\}^2$

Again considering the area as a whole $S_t = S_0(1 + \beta t)$

Hence $S_0(1+\beta t) = \{l_0(1+\alpha t)\}^2 = l_0 2(1+2\alpha t)$

neglecting $\alpha^2 t^2$, as α is small.

$\therefore \quad 1+\beta t = 1+2\alpha t \quad$ or $\quad \beta = 2\alpha \qquad (18,3)$

Thus *the coefficient of superficial expansion* = *2 × coefficient of linear expansion.*

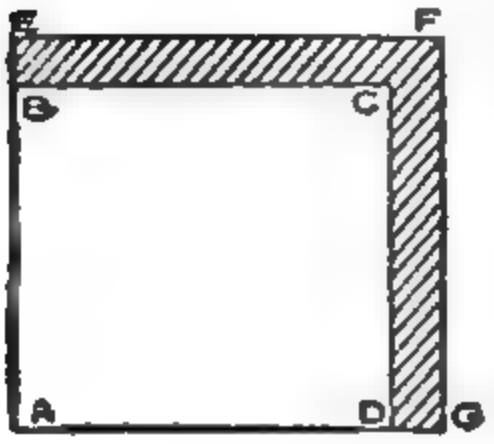


Fig. 16

It is evident from eqn. (18,3) that if the co-efficient of linear expansion of a solid be known, its coefficient of superficial expansion may be readily found. For example, the coefficient of linear expansion α for a brass is $1{\cdot}89 \times 10^{-5}$ and if the rise of temperature of a certain unit brass surface be 100°C, then its area $(1+\alpha t)^2$ becomes equal to 1·0037835. The quantity $1+2\alpha t$ is equal to 1·00378. These two values are equal to the required degree of accuracy upto the fifth place of decimals. Thus $2\alpha = \beta$.

19. **Co-efficient of Cubical Expansion of Solid.**—It is defined to be *the increase of volume per unit volume of a body for a rise of temperature of* 1°C.

Let the original volume of the body at 0°C be V_0 ; volume of the body at t°C be V_t and coefficient of cubical expansion of the solid be γ

Hence by definition, $\qquad \gamma = \dfrac{V_t - V_0}{V_0 t} \qquad (19,1)$

$\therefore \quad V_t = V_0 + V_0\gamma t$ or $V_t = V_0(1+\gamma t) \quad \ldots \quad (19,2)$

Relation between α and γ.—Let us consider a cube of the substance, the length of each side being l_0 cm. at 0°C. ; if V_0 be its volume at 0°C. $V_0 = l_0^3$.

At t°C , the length of each side becomes $l_t = l_0(1+\alpha t)$

$\therefore$ the volume at t°C is given by $\quad V_t = \{l_0(1+\alpha t)\}^3$

Again taking the volume as a whole, $\quad V_t = V_0(1+\gamma t)$

Hence $\quad V_0(1+\gamma t) = \{l_0(1+\alpha t)\}^3 \quad = l_0^3(1+3\alpha t)$

neglecting terms containing higher powers of α,

$1+\gamma t = 1+3\alpha t \quad$ or $\quad \gamma = 3\alpha \quad \ldots \quad (19,3)$

i e., coeff. of cubical expansion = 3 × coeff. of linear expansion.

It is evident from eqn. (19,3) that the coefficient of cubical expansion of a solid can be *indirectly* determined, if the coefficient of linear expansion is known. It is to be remembered that for a particular rise of temperature a hollow body and a solid one of the same material and of *same external* volume increase by the same amount.

If the initial temperature of the solid be t_1°C. instead of 0°C. and

if V_1 be the corresponding volume, it can be established exactly as in eqn. (15,1) that the volume V_2 at t_2 is given by,

$$V_2 = V_1\{1 + \gamma(t_2 - t_1)\} \quad \ldots (19,4)$$

$$\text{whence } \gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \quad \ldots (19,5)$$

Examples :—

1. The length of a copper rod at 50°C, is 200·166 cm. and at 200°C., it is 200·664 cm. Find the length at 0°C. and the co-efficient of linear expansion of copper.

A. Let l_0 cm. be the length of the rod at 0°C. and let α be the co-efficient of linear expansion of copper.

Then $200\cdot166 = l_0(1 + 50\alpha)$ and $200\cdot664 = l_0(1 + 200\alpha)$

$$\therefore \quad \frac{200\cdot664}{200\cdot166} = \frac{1 + 200\alpha}{1 + 50\alpha} \qquad \text{whence } \alpha = 0\cdot0000166$$

Substituting the value of α, $l_0(1 + 50 \times 0\cdot0000166) = 200\cdot166$

whence $l_0 = 200$ cm.

2. Railway lines are laid with gaps to allow for expansion. If the gap between steel lines 66 ft. long is 0·5 in. at 10°C, at what temperature will the lines just touch ? Co-efficient of linear expansion of steel is 11×10^{-6}. (C. U.—1953 ; Gau. U.—1951)

A. Each line can expand ·25 inch at each end making a total elongation of ·5 inch in order that consecutive lines may just touch each other.

Now, elongation = ·5 in. = ·0416 ft.

Let the temperature be t°C.

Then $\cdot0416 = 66 \times 11 \times 10^{-6} \times (t - 10)$ whence $t = 67\cdot3$°C.

3. An iron ring which is 1 ft. in diameter is to be shrunk on a pulley which is 1·005 ft. in diameter. If the temperature of the ring is 10°C, find the temperature to which it must be raised so that it will slip on the circumference of the pulley. Co-efficient of linear expansion of iron = 0·000012.

(E. P. U.—1953)

A. The iron ring of diameter 1 ft. has a circumferencial length of $\pi \times 1$ ft. In order that it can just fit on a pulley whose circumferencial length is $\pi \times 1\cdot005$ ft. the circumferencial elongation of the ring would be $\pi(1\cdot005 - 1) = \pi \times \cdot005$ ft. Now this elongation must be brought about by a change of temperature. Let the required temperature be t°C.

$$\therefore \quad \pi \times \cdot005 = \pi \times 1 \times \cdot000012 \times (t - 10)$$

$$\text{whence} \quad t = \frac{5000}{12} + 10 = 426\cdot6\text{°C.}$$

4. A zinc rod is measured by means of a brass scale, which is correct at 0°C and found to be 1·0001 metre long at 10°C. What is the real length of the rod at 10°C. Mean co-efficient of linear expansion of zinc is ·000029 and of brass is ·000019 per deg. C. (Nag. U.—1951 ; Utkal. U.—1951)

A. As the brass scale is correct at 0°C, 1 metre of the scale at 0°C would be $1 + \cdot000019 \times 10 = 1\cdot00019$ metres at 10°C. But 1·00019 metres are apparently read by the scale at 10°C as 1 metre. Hence the apparent reading of 1·0001 metre at 10°C would correspond to $1\cdot0001 \times 1\cdot00019 = 1\cdot00029$ metres in the correct scale at 0°C. Thus the length of the zinc rod at 10°C is 1·00029 metres.

If l_0 be the length of the rod at 0°C, then according to the condition,

$1 \cdot 00029 = l_0 (1 + \cdot 000029 \times 10) = l_0 \times 1 \cdot 00029$

whence $l_0 = 1$ metre.

5. An iron scale is correct at 0°C and the length of a zinc rod as measured by the scale is 100 cm. when both the scale and the rod are at 0°C. What will be the length of the rod as measured by the iron scale when both of them are at 100°C, the co-efficient of linear expansion of iron and of zinc being ·000012 and ·000026 respectively per degree centigrade. (Pat. U.—1952)

A. At 0°C the length of the zinc rod = 100 cm.

At 100°C the true length of the zinc rod $= 100\ (1 + \cdot 000026 \times 100)$ cm. When the zinc rod is measured with an iron scale at 100°C, then each centimetre of the scale at 0°C becomes $(1 + \cdot 000012 \times 10)$ cm. at 100°C. Thus each true centimeter length at 100°C would apparent read $\frac{1}{1 \cdot 0012}$ cm. = ·9988 cm.

∴ True length of the zinc rod at 100°C which is 100·26 cm. would be apparently read by the iron scale at 100°C to be $100 \cdot 26 \times \cdot 9988 = 100 \cdot 14$ cm.

6. A lump of iron has a value of 10 cu. ft at 100°C. Find its volume at 25°C. (α for iron = 0·000012).

A. We have γ for iron $= 3\alpha = 0 \cdot 000036$.

Let the required volume at 25°C. be V_1 cu. ft.

From the relation $V_2 = V_1 \{1 + \alpha\ (t_2 - t_1)\}$

$10 = V_1 \{1 + 0 \cdot 000036\ (100 - 25)\}$

whence $V_1 = 9\ 97$ cu. ft.

20. Practical Cases of Expansion of Solids.—The expansion or contraction of solids due to temperature variation is practically utilised in very cases. In lying down rails on a railway track, small spaces are left between consecutive rails in order to allow room for expansion when they get heated, The slots in the fishplates serve the same purpose. Telegraphic wires are given enough sag to allow for the contraction in winter. The metallic water pipes and gas mains have telescopic joints to allow for change in length due to variations in temperature.

In the construction of an iron bridge, each part is allowed freedom of expansion. The ends of the girders, for example, are not rigidly fixed on the supports but are placed upon rollers, so that they can expand or contract without causing any strain on the supports. The ends of the iron bars in a furnace are kept free ; otherwise they would bend due to expansion on heating. When iron pipes are used for conveying hot water or steam, they are fitted with some device allowing for expansion and contraction.

On account of unequal contraction of different parts, metal castings may sometimes crack on cooling. Thus in casting a wheel with thin rim and massive arms, the rim cools and solidifies much more rapidly than the arms and then opposes the contraction of the latter. A great stress is thus set up which may break the rim or some of the arms. This is avoided by making the arms bent.

If very hot water is poured into a thick bottomed drinking vessel or glass, it would often crack. As glass is a bad conductor of heat, the inner and the outer layers of the thick portion are not readily raised tc the same temperature and so expand unequally, causing the glass to crack. Similar effect may be observed when iced water is put into a glass tumbler. A vessel of fused quartz would stand quick variation in temperature, as its expansion is extremely small.

Sometimes metallic wires are sealed into glass vessels for making air-tight joints *e g.*, in electric lamps. If a copper wire is fused into glass, the glass will crack during cooling, as copper contracts more than glass. But a platinum wire can be safely sealed into glass, as the coefficient of linear expansion of the latter is practically the same as that of the former. Now-a-days for the sake of economy, a nickel-iron alloy called platinite is used for the construction of electric lamps.

When metal rods or chains are used as measuring instruments temperature should always be noted and a correction applied for any variation. The ***Standard Yard*** is taken to be the distance, at 62°F., between two fine marks on gold or platinum plugs inserted into a bronze bar kept at the Standard Office. The standard metre is correct at 0°C.

The iron tyres of cart wheels are made of slightly smaller diameter than the wheels. The tyre is very strongly heated so as to be easily put round the wheel. As the tyre cools, it contracts and fits tightly upon the wheel. Boiler plates are rivetted with red-hot rivets, on cooling, the rivets contract and grip the plates so tightly as to make the joints steam-proof. Big guns are often made by sliding several cylinders one above the other upon an inner tube. Each cylinder in red-hot condition is put round the inner one and on cooling fits tightly upon the latter. By these means, the enormous pressure developed due to the combustion of the powder is uniformly distributed throughout the body of the gun. Sometimes a stopper sticks tight into the mouth of a glass bottle. If the bottle is slightly warmed at the neck, the neck gets heated earlier and expands making the stopper loose.

Thermostat is an instrument in which the differential expansion of a compound bar of two dissimilar metals is utilised. It is used in fire alarms and in automatically controlling temperature in incubators. In constructing chronometers, watches and clocks, arrangements are to be made to avoid the effects of change of temperature on the time-keeping parts. For this purposes clocks are provided with compensated pendulums and watches and chronometers, with compensated balance wheels.

21. Compensated Pendulum.—The time for one complete oscillation of a simple pendulum is given by the relation,

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l is the distance from the point of suspension to the centre of gravity of the bob and g is the acceleration due to the gravity.

The pendulum in a clock should be so designed that the time period may remain constant. A change of temperature affects the length l and consequently the time period T. When the temperature rises, l increases, therefore T also increases and so the clock goes slow. When the temperature falls, l decreases, consequently T also decreases, and so the clock goes fast. Hence with an ordinary pendulum a clock has a tendency to lose in hot seasons and gain in cold seasons. In order that the clock may keep a current time at all seasons, the pendulum of a clock must be provided with special devices, so that the *effective* length of the pendulum remains unaltered in spite of the variations in temperature. Such a pendulum is termed a **Compensated Pendulum.**

In an ordinary clock pendulum, there is a heavy metal disc, which can be slided up and down a rod. There is a small nut screwed at the end of the rod just under the disc. As temperature rises, the rod increases in length, the disc or the bob moves down and so that effective length increases. To effect compensation in this case, the nut is screwed up a bit so as to raise the centre of gravity of the pendulum as a whole to its former position and to make the effective length attain the former value. When the rod contracts due to fall in temperature, the nut is screwed down to effect the adjustment. Compensation by this method is not automatic but is to be done by trial.

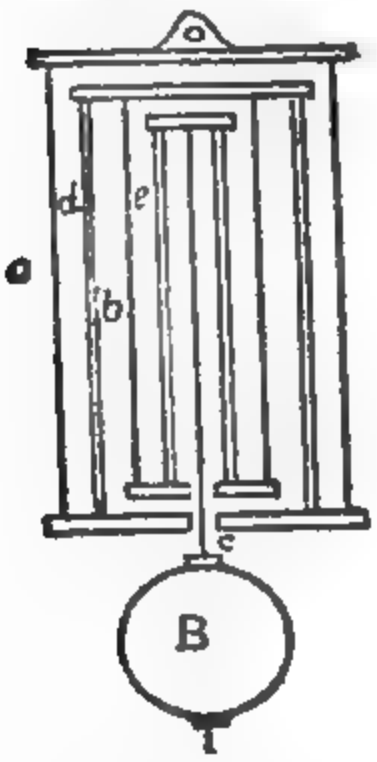


Fig. 17

Fig. 18

In one form of clocks, the *automatic* compensation is accomplished by the following arrangement :—

Harrison's Gridiron Pendulum.—The principal of this pendulum may be understood from the following explanation. Let AB and CD (Fig. 18) are two rods of different materials and of unequal lengths joined by a cross-bar BC. The end A being fixed, the rod AB is allowed to expand downwards while the rod CD being rivetted at C can expand upwards. If the lengths and the material of the rods be so chosen that for an equal rise of temperature each increases by the same amount, than the downward expansion of AB is equal to the upward expansion of CD. Hence the lowering of the end B is the same as the rise of the end D, so that the distance between A and D remains constant whatever be the change of temperatures.

If α and α' be the coefficients of linear expansion, l and l' the lengths of AB and CD respectively, then for a rise or fall of temperature t°C.

$$l\alpha t = l'\alpha' t \text{ or } l\alpha = l'\alpha' \text{ or } \frac{l}{l'} = \frac{\alpha'}{\alpha}$$

that is, the lengths of the rods must be inversely proportional to the respective co-efficients of linear expansion. Evidently the shorter rod CD should be of more expansible material than AB.

In an *actual gridiron pendulum*, the bob is carried by a frame work made of 9 parallel rods (Fig. 17), alternately of brass and steel ; there are 4 brass rods (shown in thick lines) and 5 steel rods (shown in thin lines) including the central one which carries the bob B. The ends of the rods are attached to cross-pieces in such a way that the steel rods are fixed at the upper ends and can expand downwards, tending to lower the position of B, while the brass rods being fixed at the lower ends can expand upwards and tend to raise the position of B. For the sake of compensation, the total downward expansion of the steel rods must be equal to the total upward expansion of the brass rods. Again all the bars excepting the central one occur in pairs ; as each pair of similar rods, of the same metal and attached to the same cross-piece, expand in the same direction by the same amount, the expansion of one rod of each pair is to be considered in the determination of the displacement of the bob.

Let l be the total length of the three steel rods (a, b, c) is one half of the frame and let l' be the total length of the brass rods (d, e) in the same half. If α and α' be the co-efficients of linear expansion of steel and brass, then $l\alpha = l'\alpha'$.

$$\frac{l}{l'} = \frac{\alpha'}{\alpha} = \frac{19 \times 10^{-6}}{12 \times 10^{-6}} = \frac{19}{12}$$

so that for the sake of compensation, the effective lengths of the steel and brass rods are approximately in the ratio 3 : 2.

Now-a-days the process of compensation is effected more easily by having the pendulum rods made of an alloy of steel and nickel prepared by M. Guillaume and named **invar** (invariable steel). As its coefficient of linear expansion is only 0·0000009, the change in length for ordinary changes of temperature is practically inappreciable. Standards of length are also made of invar. Substances like fused quartz and silica have also very negligible expansion due to temperature changes. Consequently vessels made of these substances have little chances of crack due to sudden temperature variations.

Compensated Balance Wheel of a Chronometer or a Watch.— The motion of the hands of a watch or a chronometer is mainly controlled by the vibrating balance wheel, the period of oscillation of the wheel depending upon its diameter. An ordinary wheel would increase in diameter when temperature rises, and as a result its period of oscillation becomes larger. Hence the watch in hot seasons would go slower.

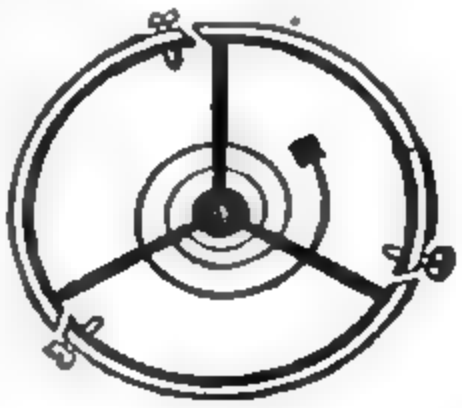


Fig. 19

The *compensation* is secured in the following way. The rim of the wheel is divided into three sections, each of which is supported at one end by a spoke of wheel and carries a compensating mass screwed at the other

end (Fig. 19). The segments are made of strips of two dissimilar metals, the more expansible one remaining outside.

As temperature rises, the spokes increase in length and carry the ends of the respective segments of the rim outwards ; but owing to the greater expansion of the outer metal of each strip, the other end of the segment moves inwards thus bringing the compensating masses nearer the centre. If the wheel is so constructed that outward displacement of the masses due to expansion of the spokes is equal to the inward displacement due to the curling of the segments, the average diameter of the wheel does not change and consequently the period of oscillation of the balance wheel is unaffected by changes of temperature and the watch keeps right time.

Examples :—

1. A gridiron pendulum is made of 4 brass rods, of length each 50 cm. and 5 iron rods. Find the length of each iron rod. (for iron $\alpha = 0{\cdot}0000117$ and for brass $\alpha = 0{\cdot}0000187$).

A. On each side of the central iron rod carrying the bob, there are one pair of iron and one pair of brass rods ; but as the expansion of the rods of the same material on either side is the same, 3 iron rods and 2 brass rods are to be taken as doing compensation.

Let l be the length of each iron rod and t the rise of temperature.

Here expansion of 3 iron rods = expansion of 2 brass rods.

$\therefore \quad 3 \times l \times 0{\cdot}0000117 \times t = 2 \times 50 \times 0{\cdot}0000187 \times t$

whence $\qquad l = 53{\cdot}3\text{cm}.$

2. A clock which keeps correct time at 25°C. has a pendulum made of brass. How many seconds will it gain per day when the temperature falls to the freezing point ? (Co-efficient of linear expansion of brass is 0·0000187).

A. Let l_0 be the length of the pendulum at 0°C. and l at 25°C. let t_0 and t be the corresponding time periods.

$$\text{Then } \frac{t}{t_0} = \sqrt{\frac{l}{l_0}} = \sqrt{\frac{l_0(1 + 25 \times 0{\cdot}0000187)}{l_0}} = (1 + 0{\cdot}0004675)^{\frac{1}{2}}$$

$$= 1 + \tfrac{1}{2} \times 0{\cdot}0004675 = 1{\cdot}0002337$$

But as the pendulum keeps correct time at 25°C. $t = 1$ sec. and so at this temperature the pendulum makes $(60 \times 60 \times 24)$ or 86400 complete oscillations in a day of 86400 seconds.

At 0°C. the time period is given by $t_0 = \dfrac{1}{1{\cdot}0002337}$ sec.

$\therefore$ Number of oscillations made per day

$$= 86400 \div \frac{1}{1{\cdot}0002337} = 86420{\cdot}2$$

$\therefore$ The clock gains by $(86420{\cdot}2 - 86400) = 20{\cdot}2$ seconds.

3. An iron clock pendulum makes 86405 oscillations in one day ; at the end of the next day the clock has lost 10 seconds. Find the change in temperature. (The co-efficient of linear expansion of iron is 0·0000117).

(U. P. B.—1937 ; Pat. U.—1924)

A. Let the change in temperature be θ °C.

In a pendulum $t = 2\pi\sqrt{l/g}$

Let l be the length and t the time period of a pendulum on the first day and l' the length t' the time period on the next day.

A correct pendulum makes $24 \times 60 \times 60$ or 86400 oscillations a day. As the pendulum makes 86405 oscillations on the 1st day, so $t = (86400 \div 86405)$ sec.

As the pendulum loses 10 seconds on the next day it make 86395 oscillations on the day; $t' = (86400 \div 86935)$ sec.

$$\therefore \quad \frac{t'}{t} = \frac{86405}{86395} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l(1+\alpha\theta)}{l}} = \sqrt{1+\alpha\theta} = (1+0{\cdot}0000172\theta)^{\frac{1}{2}}$$

whence $\theta = 19{\cdot}8°C.$

GABRIEL DANIEL FAHRENHEIT (1686—1736)

Fahrenheit was born at Danzig, Germany, on May 14, 1686. His parents wanted him to become a business man, but from his boyhood he showed a tendency towards sciences. He studied in Germany, England and Holland. The liquid used in the thermometers in those days was alcohol. He is the pioneer of substituting mercury as the thermometric substance.

He standardised the scale of his thermometer by observing the lowest point of thermometer as attained during the winter of 1709 at Danzig. This he called the zero of his scale.

The head of the mercury column at the boiling point of water was marked 212. In this way he obtained the idea of degree Fahrenheit. Although a slight correction had to be made for the International standard of the Fahrenheit scale, his original scale has been very nearly retained. He also devised an improved form of hydrometer. He was elected a Fellow of the Royal Society. He died in Holland on September 16,1736.

Summary

Expansion of Solids may be (i) linear, (ii) superficial and (iii) cubical.

Co-efficient of Linear Expansion of solid is *the increase in length per unit length of a rod of the material for a rise of temperature of 1°C. If l_0 be the length of the rod at 0°C and l_t, that at t°C.*

$$\alpha = \frac{l_t - l_0}{l_0 t} \text{ or } l_t = l_0(1+\alpha t)$$

Determination of α can be made by *Pullinger's apparatus and by Comparator method.*

Co-efficient of Superficial Expansion β is given by

$$\beta = \frac{S_t - S_0}{S_0 t} \text{ or } S_t = S_0(1+\beta t) \text{ ; also } \beta = 2\alpha$$

Co-efficient of Cubical Expansion γ is given by

$$\gamma = \frac{V_t - V_0}{V_0 t} \text{ or } V_t = V_0(1+\gamma t) \text{ ; also } \gamma = 3\alpha$$

Effects of Expansions of solids are to be taken into account in many practical cases of which the effect on the time-keeping parts of clocks and watches is a very important one. This is avoided by compensated pendulums and compensated balance wheels.

Exercises on Chapter II

Reference

Arts. 14 & 19 1. Define co-efficients of linear and cubical expansions. Do they differ when the lengths are measured in centimeters or feet?
(Dac. U.—1943; C. U.—1934, '37)

Arts. 14 & 19 2. Define the co-efficients of linear and cubical expansions of a solid. Show that the latter is three times the former. (E. P. U.—1951; P. U.—1952; *Cf.* Pat. U.—1952; Gau. U.—1951)

Art. 16 3. Describe an experiment to determine the co-efficient of expansion of a metal in the form of a rod.

The length of a tabular railway bridge is 461 metres. Find the total linear expansion of the tube between $-5°C.$ and $+35°C.$ The tube is made of steel, the co-efficient of linear expansion of which is 12×10^{-6}. (Dac. U.—1941)

A. 22·13 cm.

Art. 14 4. The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature from 36°F. in winter to 117°F. in summer. Given expansibility of iron $= 12 \times 10^{-6}$ per degree centigrade.

A. 0·21 mile nearly.

Art. 14 5. A glass rod when measured with a zinc scale, both being at 20°C appears to be one metre long. If the scale is correct at 0°C what is the true length of the glass rod at 0°C? The co-efficient of linear expansion of the glass is 8×10^{-6} and that of zinc is 26×10^{-6}.

A. 100·036 cm.

Art. 14 6. The height of a barometer appears to be 76·4 cm. according to a brass scale which is correct at 0°C. If the temperature at the time of reading is 30°C. what is the actual height of that mercury column? The co-efficient of linear expansion of brass is 0·0000189. How is this quantity determined experimentally?

A. 76·44 cm.

Art. 14 7. A steel tyre 4·0 feet in diameter is to be shrunk on to a cart wheel of which the average diameter is $\frac{1}{8}$ in. greater than the inside diameter of the tyre. Assuming the co-efficient of linear expansion of the material of the tyre to be 0·0000112, calculate the necessary rise of temperature of the tyre in order that it may easily slip on the wheel.

A. 232·6°C.

Art. 14 8. A man wishes to fit an aluminium ring on a steel rod of 1 inch diameter but it is ·001 too small in diameter. How much should its temperature be raised before it will just slip on? Subsequently he wishes to remove it again, but now he has to heat both metals together. Through how many degrees must this be done? (Coeff. of expansion

Reference

of aluminium = ·000025; coeff. of expansion of steel = ·000010)
(Gau. U.—1953)

A. 40°C; 66°7C.

Art. 14

9. One end of steel rod of length 61 cm. is fixed and the other presses against an end of a lever 10·5 cm. from the fulcrum. The rod on being heated through 500°C. turns the lever through 2°. Find the co-efficient of linear expansion of rod (90° = $\pi/2$ radians).

Hints:— $\frac{\text{Increase}}{10{\cdot}6} = \theta$ radians $= \frac{2 \times \pi}{180}$ radians

$\therefore$ Increase in length $= 10{\cdot}5 \times 2 \times \frac{22}{7} \times \frac{1}{180} = 0{\cdot}366$ cm.

A. 0·000012 per °C.

Art. 16

10. Describe any method of finding the co-efficient of linear expansion of a metal rod. (Gau. U.—1953; C. U.—12942; Nag. U.—1951; E. P. U.—1951)

Art. 19

11. Compare the density of lead at 100°C. with that at — 100°C assuming that its co-efficient of expansion remains constant within these limits of temperature (co-efficient of linear expansion = ·000028)
(C. U.—1942)

A. ·983 : 1.

Arts. 14 & 19

12. Explain what is meant by the co-efficient of the linear expansion. State the relation it bears with the co-efficient of cubical expansion.
(C. U.—1943, '53)

Art. 17

13. Describe any method of determining the co-efficient of linear expansion of solid. (C. U.—1953; P. U.—1929; Dac. U.—1934; E. P. U.—1953; Pat. U.—1952; Gau. U.—1951)

Art. 19

14. A sheet of brass is 50 cm. long and 10 cm. broad at 0°C.; the area of the surface increases by 1·9 sq. cm. at 100°C. Find the co-efficient of linear expansion of brass.

A. 19×10^{-6} per °C.

Art. 19

15. The volume of a lead bullet at 0°C. is 25 c.c. The volume increases at 98°C. by 0·021 c.c. Find the co-efficient of linear expansion of lead.

A. $28{\cdot}6 \times 10^{-6}$ per °C.

Art. 1

16. A bar of steel is heated through 200°F. and securely fixed at each end while at this increased temperature. What force per sq. inch of section is the bar capable of exerting when it has coolel down to the original temperature, provided that the fixing at both the ends remain rigid? Mean co-efficient of expansion between 32°F. to 212°F = $6{\cdot}7 \times 10^{-6}$ and the modulus of elasticity = 14000 ton per sq. inch.

A. 18·76 tons wt.

Art. 21

17. Two flat circular discs, one of platinum and the other of silver, of equal radius and each 2 mm. thick are securely fastened back to back when at 0°C. They are then suspended in water boiling at normal pressure. Show that they will be bent so that they form a section of a hollow sphere and determine their radius of curvature. Given the co-efficient of linear expansion of silver = $18{\cdot}8 \times 10^{-6}$ per degree centigrade.

A. Radius of curvature = 404 cm.

Reference

Art. 21 18. Describe the effect of varying temperature on the rate of a clock or a watch. Explain how chronometers are constructed so as to keep accurate time in spite of changes in temperature.
(C. U.—1935; All. U.—1932)

Art. 21 19. Why should the time of oscillation of a clock-pendulum change with rise of temperature? What arrangement is made to make the clock correct time both in warm and cold weather? Given that the co-efficient of linear expansion of brass is 19×10^{-6} and that of steel 11×10^{-6}, what must be the relative lengths of the bars of the metal used in gridiron pendulum. (P. U.—1930)

A. 11 : 19.

Art. 21 20. What is a compensated pendulum? Explain the principle of construction of Harrison's gridiron pendulum.

A gridiron pendulum is made of 5 iron rods and 4 brass rods. Each of the brass rods is 50 cm. in length. Find the length of each iron rod. The linear co-efficient of expansion of iron is 12×10^{-6} and of brass is 18×10^{-6}. (C. U.—1948)

A. 50 cm.

Art. 21 21. The pendulum rod of a clock is made of wrought iron and the pendulum swings once per second. If the change of temperature is 25°C. find the alteration in the length of the pendulum. Co-efficient of linear expansion of the wrought iron $= 11 \cdot 9 \times 10^{-6}$, $g = 980$ cm./sec.^{2}.

A. 0·029 cm.

CHAPTER III

EXPANSION OF LIQUIDS

22. Expansion of Liquid.—Like solids, liquids also expand when heated. But as liquids have no definite shape of their own, but have fixed volumes, they possess volume or *cubical expansion* only. Therefore expansion of a liquid always means cubical expansion. Different liquids expand by different extents for a given rise in temperature. Fig. 20 represents a number of glass bulbs of equal capacity A, B, C etc., each provided with a thin stem and fixed vertically in a suitable frame V. All the bulbs are enclosed in a trough at the base of the apparatus. Different liquids are poured to the same height into the bulbs at the room temperature so that they occupy equal volume at this temperature. Now hot water is placed into the trough, so that each liquid is heated equally. But the rise of level in different vessels is different. This shows that the expansibility differs with liquids. The volume expansion of a liquid is more than that of a solid for a given rise of temperature.

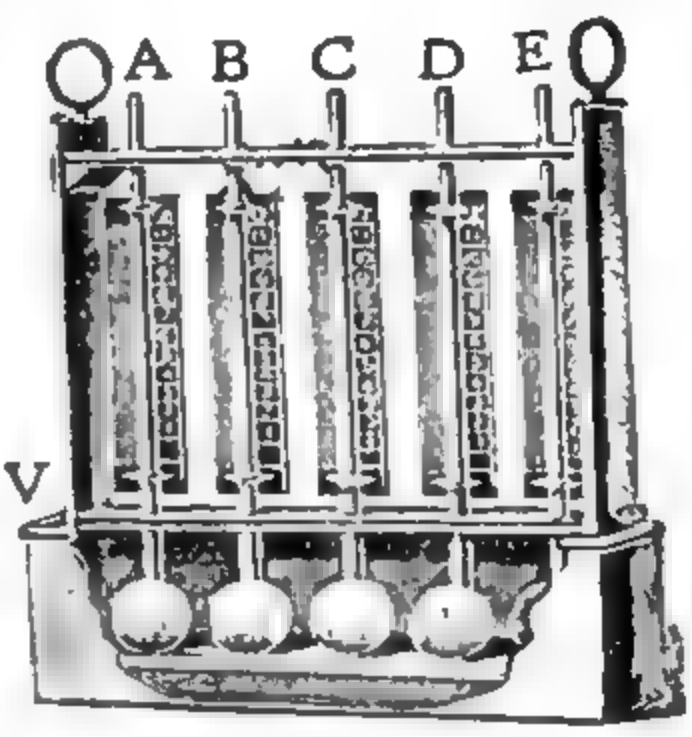


Fig. 20

Real and Apparent Expansion.—The expansion of the liquid due to a rise of a temperature is always accompanied with expansion of containing vessel. Let us take a large glass bulb provided with a long stem of uniform narrow bore (Fig. 21) and let the bulb and a part of the stem upto A, be filled with a liquid. If the bulb is now suddenly immersed into a bath of hot water, the level of the liquid momentarily comes down to a level B, and then gradually goes upto C higher than A. This happens because as the heat is supplied from outside, the vessel is warmed earlier than the liquid. The capacity of the vessel being increased, the level of the liquid falls. The heat then passes into the liquid which ultimately attains the temperature of the bath. As the liquid is more expansible than glass, its level finally rises above A. In fact we observe the expansion from A to C; the volume AC therefore measures the **apparent expansion** of the liquid. If we make a simple assumption that the vessel has first attained the temperature of the bath without imparting any heat to the liquid contained and then the liquid has been warmed up to the temperature of bath, then the volume BC measures the **real expansion** and AB measures that of the vessel, for the given rise of temperature. It is evident from Fig. 21, that AC = BC − AB.

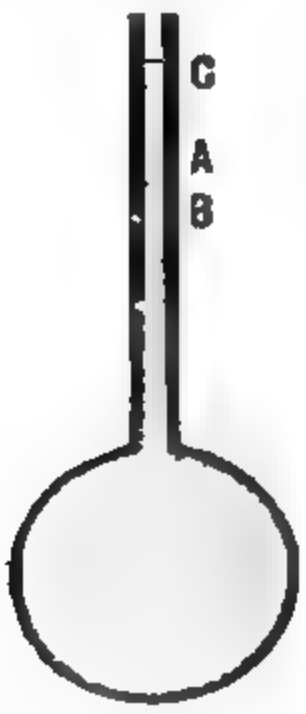


Fig. 21

∴ Apparent expansion = Real expansion − Expansion of the containing vessel.

i.e. *Real expansion = Apparent expansion + expansion of the containing vessel.*

23. Co-efficients of Real and Apparent Expansions.—The *coefficient of real or absolute expansion of a liquid* is defined to be the ratio of the increase in volume of a liquid for 1°C rise to the original volume. If γ be the coefficient of absolute expansion, then

$$\gamma = \frac{\text{real expansion}}{\text{original volume} \times \text{rise of temperature}}$$

The *coefficient of apparent expansion of a liquid* is given by

$$\gamma' = \frac{\text{observed expansion of the liquid}}{\text{original volume} \times \text{rise of temperature}}$$

Relation between γ and γ'.—Let us suppose that the stem of the bulb in Fig. 21 is graduated. Let the liquid stand at A when its temperatures is 0°C. and let the level come to C when the temperature rises by t°C. Let V_0 be the volume of the liquid upto A at 0°C and v, v_1, v_2 be the volumes represented by BC, AC and AB respectively. Let γ, γ' be the coefficients of absolute and apparent expansions of the liquid respectively and γ_g the coefficient of cubical expansion of the material of the vessel.

Then for a rise of t°C., $v = V_0\gamma t$; $v_1 = V_0\gamma' t$; and $v_2 = V_0\gamma_g t$.
Since $v = v_1 + v_2$; $V_0\gamma t = V_0\gamma' t + V_0\gamma_g t$

Hence, $\gamma = \gamma' + \gamma_g$ ... (23,1)

Thus coefficient of real expansion = coefficient of apparent expansion + coefficient of cubical expansion of the material of the vessel.

24. Determination of the Co-efficient of Apparent Expansion of a liquid.—

The Volume Dilatometer method.—Suppose the volume of the bulb in fig. 22 is known and the stem is graduated in equal divisions. The bulb and a part of the stem are filled with the given liquid. The bulb is then kept for some time in a bath at 0°C. and from the position of the liquid surface, volume at 0°C. is obtained. The bath is then raised to a constant higher temperature, say, t°C. and the position of the liquid level at this temperature is noted again.

If a be the area of cross-section of the stem in sq. cm. and l cm. be the distance through which the liquid column moves, then the increase in volume is la c.c.

Neglecting the expansion of the bulb, $\gamma' = \frac{la}{V_0 t}$

The volume of the bulb can be measured at any temperature by

weighing the vessel empty and then again weighing the bulb filled with mercury at the same temperature. The difference of weights gives the mass of mercury occupying the volume of the bulb. Dividing this by the density of mercury at the corresponding temperature, the volume of the bulb is obtained.

Fig. 22

The Weight Thermometer Method.—The weight thermometer consists of a glass bulb, provided with a short bent capillary stem (Fig. 22). The bulb is at first cleaned, dried and carefully weight empty. The nozzle of the bent tube is kept under the liquid under experiment. By a process of alternate heating and cooling, it is completely filled with the given liquid. When quite full the bulb is kept immersed for sometime in a water bath, say at the room temperature, with the open end still under the surface of the given liquid, until the contents of the bulb acquire the temperature t_1°C of the water as noted by a thermometer. The weight thermometer is then removed, gently wiped dry and weighed again. It is now placed in the water bath which is kept well stirred and gradually raised to a higher temperature. The temperature of the bath is kept constant for sometime at some higher temperature t_2°C. The liquid in the bulb expands and some of it is forced out. The apparatus is then removed from the bath, gently wiped dry, cooled to the initial temperature and weighed again ; liquid remaining in the bulb, of course, contracts to a smaller volume.

Let the mass of the weight thermometer, when empty $= W$ gm.

its mass when filled with liquid at t_1°C. $= W_1$ gm.

its mass with the liquid left at t_2°C. $= W_2$ gm.

The mass of the liquid filling the thermometer at t°C.

$$= W_1 - W = m_1 \text{ gm., say.}$$

Also mass of the liquid left in the thermometer at t_2°C.

$$= W_2 - W = m_2 \text{ gm., say.}$$

Neglecting the expansion of the weight thermometer itself, we find that the volume occupied by a mass of m_1 gm. of the liquid at t_1°C. is the same as the volume occupied by m_2 gm. at t_2°C.

Let the density of the liquid at t_1°C $= \rho$

Then volume of m_1 gm. of the liquid at t_1°C $= m_1/\rho$

Also volume of m_2 gm. of the liquid at t_1°C $= m_2/\rho$

Hence a volume m_2/ρ of the liquid at t_1°C occupies a volume m_1/ρ when raised to t_2°C.

$\therefore$ The apparent expansion between t_1°C and t_2°C $= \frac{m_1}{\rho} - \frac{m_2}{\rho}$

Hence the coeff. of apparent expansion of the liquid is given by

$$\gamma' = \frac{\frac{m_1}{\rho} - \frac{m_2}{\rho}}{\frac{m_2}{\rho}(t_2 - t_1)} = \frac{m_1 - m_2}{m_2(t_2 - t_1)} \qquad \ldots (24,1)$$

Thus, $\gamma' = \dfrac{\text{mass of liquid expelled}}{\text{mass remaining} \times \text{rise of temperature}}$

As the co-efficient of expansion is obtained directly from different weights, the method is known as that of *weight* thermometer. This method is not suitable in case of volatile liquids.

Examples :—

1. A weight thermometer containing 100 gm. of mercury at 0°C is surrounded by liquid in a bath when 4 gm. of mercury flow out. What is the temperature of the bath if the apparent coefficient of expansion of mercury is ·00018 ?
(E. P. U.—1952)

A. We know, $\gamma' = \dfrac{m_1 - m_2}{m_2(t_2 - t_1)} = \dfrac{4}{(100-4)(t_2 - 0)}$ and $\gamma' = ·00018$

$\therefore \quad ·00018 = \dfrac{4}{96 \times t_2} \quad$ or, $\quad t_2 = \dfrac{4}{96 \times ·00018}$

Using logarithms, log 4 = 0·6021 ;

log 96 =	1·9823	0·6021	antilog 2·3645 = 231·5
log ·00018 =	$\bar{4}$·2553	$\bar{2}$·2876	
	$\bar{2}$·2376	2·3645	$\therefore$ t_2 = temp. of bath = 231·5°C

2. A glass weight thermometer has a mass of 6·34 gm. when empty and 153·81 gm. when filled with mercury at 0°C. If 2·08 gm. are expelled when it is heated to 100°C, find the co-efficient of relative expansion of mercury in glass.
(R. P. B.—1952)

A. Mass of mercury with the buib at 0°C = 153·81 — 6·34 = 147·47 gm.
Mass of mercury expelled at 100°C = 2·08 gm.
Mass of mercury remaining within bulb- = 147·47 — 2·08 = 145·39 gm.
Hence from eqn. (24,1), the apparent expansion γ' for mercury

$$= \frac{2·08}{145·39 \times 100} = ·000143 \text{ per } °C.$$

2. The co-efficient of cubical expansion of glass and mercury being 25×10^{-6} and 18×10^{-5} per degree centigrade respectively, what fraction or the whole volume of a glass vessel should be filled with mercury in order that the volume of the empty part should remain constant when the glass and mercury are heated to the same temperature ?
(*Cf.* Gau. U.—1952)

A. Let V_0 be the total internal volume of the vessel at 0°C and v the volume filled with mercury.

Then $\frac{v}{V_o}$ — is the required fraction

At t°C. the volume of the vessel $= V_o (1 + \gamma t) = V_o (1 + 25 \times 10^{-6} \times t)$.

At t°C the volume of the mercury $= v (1 + 18 \times 10^{-6} \times t)$

$\therefore$ Volume of the empty part $= V_o (1 + 25 \times 10^{-6} \times t) - v (1 + 18 \times 10^{-6} \times t)$

$= V_o - v + (V_o \times 25 \times 10^{-6} - v \times 18 \times 10^{-6}) t$

As the volume of the empty part is to remain constant $(= V_o - v)$

$V_o \times 25 \times 10^{-6} - v \times 18 \times 10^{-5} = 0$. Whence $\frac{v}{V_o} = \frac{2 \cdot 5}{18} = \frac{5}{36}$

25. Effect of Heat on Density.—The density of a body is its mass per unit volume. When a body is heated, it generally expands in volume. As the mass remains constant, the density must decrease with a rise of temperature.

Let the volume and density of a mass M of a liquid be V_o and ρ_o at 0°C. and V_t and ρ_t when rises to t°C.

Then $M = V_o \rho_o = V_t \rho_t = V_o (1 + \gamma t) \rho_t$

$$\rho_o = \rho_t (1 + \gamma t) \quad \ldots \quad (25,1$$

whence $\rho_t = \rho_o (1 + \gamma t)^{-1} = \rho_o (1 - \gamma t)$ neglecting terms containing higher powers of γ.

Thus $$\rho_t = \rho_o (1 - \gamma t) \quad \ldots \quad (25,2)$$

The equation shows that *density decreases as temperature rises*.

From eqn. (25,2) $$\gamma = \frac{\rho_o - \rho_t}{\rho_o \times t}$$

If the temperature rises from t_1 to t_2 and ρ_1 and ρ_2 be the corresponding densities, then

$\rho_2 = \rho_1 \{1 - \gamma (t_2 - t_1)\}$ whence $\gamma = \frac{\rho_1 - \rho_2}{\rho_1 (t_2 - t_1)}$ ... (25,3)

Hence by knowing the densities of a liquid at two different temperatures t_1 and t_2, the value of the coefficient of cubical expansion γ can be *indirectly* determined.

Examples :—

1. The density of mercury at 20°C. is 13·546 and its co-efficient of cubical expansion is 0·000182. Find the mass of 500 c.c, of mercury at 80°C. Also find the volume of 500 gm. of mercury at this temperature.

A. Let ρ' be the density of mercury at 80°C.

From eqn. (25,3) $0 \cdot 000182 = \frac{13 \cdot 546 - \rho'}{13 \cdot 546 \ (80 - 20)}$ whence $\rho' = 13 \cdot 308$ gm./c.c.

$\therefore$ mass of 500 c.c. of mercury at 80°C $= 13 \cdot 308 \times 500 = 6699$ gm.

Again volume of 500 gm. of mercury at 80°C. $= 500/13 \cdot 4 = 37 \cdot 3$ c.c.

2. The density of mercury is 13·6 gm./c.c. at 0°C and at 100°C it is 13·35 gm./c.c. Calculate the co-efficient at absolute expansion of mercury.

(Utkal. U.—1949)

A. Let $\rho_o = 13 \cdot 6$ and $\rho_1 = 13 \cdot 35$ gm/c.c. Then according to eqn. (25,2)

$$\gamma = \frac{\rho_0 - \rho_1}{\rho_0 \times t} = \frac{13{\cdot}6 - 13{\cdot}35}{13{\cdot}6 \times 100} = \frac{{\cdot}25}{1360} = 1{\cdot}84 \times 10^{-4} \text{ per } ^\circ\text{C}.$$

26. Determination of the Co-efficient of Real or Absolute Expansion of a liquid.—(1) *Indirect Method.*—If γ and γ' be the coefficients of absolute and of apparent expansions of a liquid and if γ_g be the coefficient of cubical expansion of the material of the containing vessel, then from eqn. (23,1).

$$\gamma = \gamma' + \gamma_g$$

Finding γ' by any of the methods given in the last article and taking γ_g as three times the coefficient of linear expansion of the material of the vessel, the value of γ', can be calculated. Since different samples of glass have different expansibilities, the value of γ as calculated from γ_g is not very accurate.

(2) **Dulong and Petit's method.**—The coefficient of absolute expansion of mercury can be directly determined by this method.

The apparatus in its simplest form, consists of a glass tube open at both ends and bent twice at right angles in the form of a U-tube ABCD (Fig. 23) of which the limbs AB and CD are vertical and the portion BC is horizontal. Each of the vertical limbs is jacketed by a wider glass tube with its ends closed by corks through which the vertical tube passes. The jacket tubes are provided with outlets at the top and at the bottom. Thermometers are inserted through holes in the upper cork.

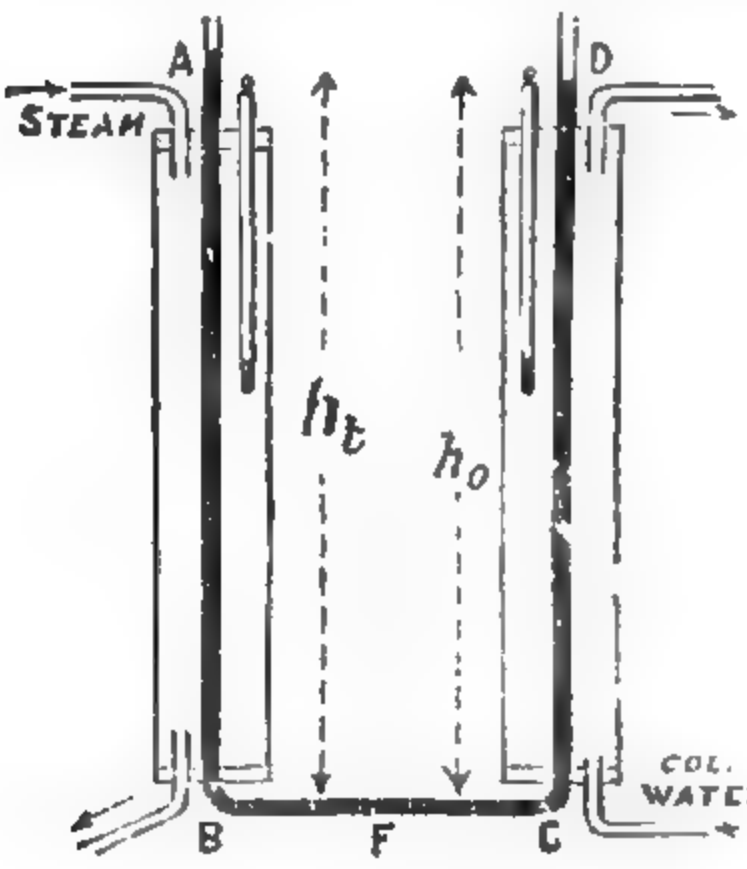


Fig. 23

The given liquid is poured into the U-tube, so that its level, equal in each limb, is just visible above the jacket tube. One of the jackets, say that of CD, is kept filled with a stream of ice-cold water, entering at the lower end and going out through the upper outlet. At the same time steam is passed into the other jacket, which enters at the upper end and goes out through the lower outlet. To prevent the passage of heat from the hot limb to the colder one, BC is kept covered with piece of cloth or blotting paper moistened with cold water.

A difference of temperatures is set up in the liquid in the two limbs and the level of mercury in the limb AB, is found to rise above that in CD. When the liquid columns have attained steady positions, the heights of the mercury levels in AB and in CD above the axis of the horizontal tube BC are measured and temperatures in the two limbs are noted.

Let heights of liquid columns in AB and CD $= h_t$ and h_0 respectively
temperatures of liquid in AB and CD $= t$ and 0°C ,,
densities of liquid at t°C. and 0°C $= \rho_t$ and ρ_0 ,,
coefficient of absolute expansion of mercury $= \gamma$

Then pressure at B $= h_t \rho_t g + P$ and pressure at C $= h_0 \rho_0 g_0 + P$ where P is the atmospheric pressure.

As BC is horizontal, pressure at B = pressure at C

$\therefore \quad h_t \rho_t g + P = h_0 \rho_0 g + P \quad$ or $\quad h_t \rho_t = h_0 \rho_0$

Since from eqn. (25,1), $\rho_0 = \rho_t(1 + \gamma t)$, and $h_t \rho_t = h_0 \rho_t (1 + \gamma t)$

whece $h_0 \gamma t = h_t - h_0 \quad$ or $\quad \gamma = \dfrac{h_t - h_0}{h_0 t}$ ... (26,1)

The increase of level is independent of the dimension or the expansion of the material of the vessel. In the laboratory, instead of water at 0°C. generally water at room temperature is used and the calculation is thus modified. Let h_2 and h_1 be the heights of the cold and the hot columns respectively and let t_2 and t_1 be their respective temperatures at which the densities of mercury are ρ_1 and ρ_2 respectively.

Then $h_1 \rho_1 g = h_2 \rho_2 g \quad$ or $\quad \dfrac{h_1 \rho_0}{1 + \gamma t_1} = \dfrac{h_2 \rho_0}{1 + \gamma t_2}$

$\therefore \quad h_2(1 + \gamma t_1) = h_1 (1 + \gamma t_2) \quad$ or $\quad \gamma(h_1 t_2 - h_2 t_1) = h_2 - h_1$

whence $\qquad \gamma = \dfrac{h_2 - h_1}{h_1 t_2 - h_2 t_1}$ ... (26,2)

It is to be noted that Dulong and Petit's method gives the *real* expansion of a liquid, while weight thermometer or a dilatometer gives the *apparent* expansion.

Example :—

1. In an experiment performed by Dulong and Petit's method, the heights of the cold and hot columns of mercury are found to be 90·6 cm. and 92·2 cm. respectively. If the first column is at 0°C. find the temperature of the other (γ for mercury $= 18{\cdot}2 \times 10^{-5}$).

A. Let t°C, be the required temperature.

From eqn. (26,1) $\quad 18{\cdot}2 \times 10^{-5} = \dfrac{92{\cdot}2 - 90{\cdot}6}{90{\cdot}6 \times t}$ whence $t = 97$°C nearly.

Table of Co-efficients of Real Expansion of Liquids

(Mean value between 10°C and 30°C.)

Water	(40° — 100°) ..	0·00058	Aniline	..	0·00085
Mercury	..	0·00018	Terpentine	..	0·00094
Glycerine	..	0·00053	Ethyl Alcohol	..	0·00110
Olive oil	..	0·00070	Methyl Alcohol	..	0·00122
Benzene	..	0·00124	Paraffin Oil	..	0·0009

27. Anomalous Expansion of Water.—We know that liquids expand on heating and contract on cooling. But in the case of

water some peculiarity is observed. If a quantity of water, from the room temperature, be gradually cooled, the volume decreases until the temperature falls to 4°C. If the temperature is further lowered the volume instead of diminishing increases. Water, therefore, occupies the smallest volume at 4°C and has consequently the *maximum density* at this temperature. This can be shown by any one of the following methods.—

Constant Volume Dilatometer.—This consists of a glass vessel with a cylindrical bulb and a long graduated stem of uniform narrow bore (Fig. 24). In the bulb is put a quantity of mercury whose volume is one-seventh the capacity of the bulb. As the coefficient of cubical expansion of mercury is nearly seven time that of glass, then for any change of temperature, the expansion of the vessel is equal to the expansion of mercury contained. Hence the open space inside the vessel remains constant and is independent of a change of temperature.

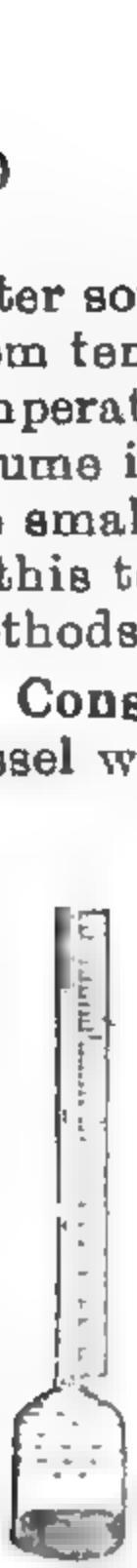


Fig. 24

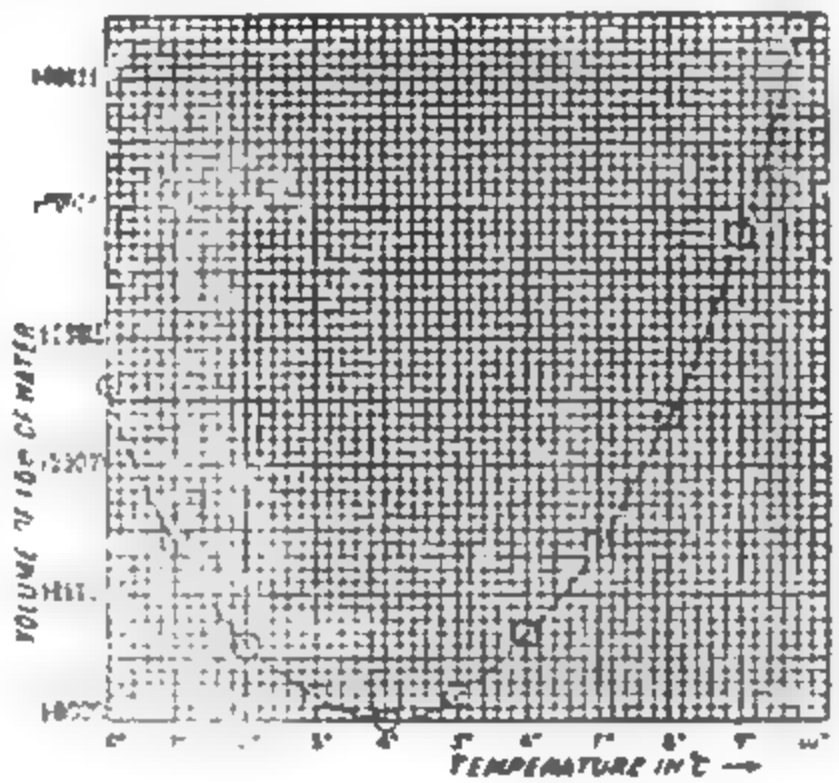


Fig. 25

A quantity of water is put into the dilatometer so as to occupy a certain level in the stem. The bulb is now placed in a water-bath maintained at 0°C. the temperature being noted by a sensitive thermometer placed in the bath. When the position of the water column in the stem becomes steady, the volume of the water is noted. The temperature of the bath is then raised slowly, the water being stirred well. From the position of the water level in the stem the volume of the water at the corresponding temperature is noted.

If the volumes be plotted in a graph against corresponding temperatures, we get a curve, as in fig. 25. It is a evident from the graph that as the temperature rises from 0°C, the volume of the water at first decreases, becomes *minimum* at 4°C. and then goes on increasing for further rise of temperature. At 4°C, therefore, the volume of a given mass of water is minimum and so the density is maximum. It may be noted that exactly 1 c.c. of water at 4°C. has a volume of 1·000127 c.c. at 0°C. and 1·000265 c.c. at 10°C. Hence the variation of volume with a change of temperature is very small.

Hope's Experiment.—The apparatus consists of a tall glass cylinder G (Fig. 26), closed at the bottom and surrounded at the middle by a wide circular trough J. The cylinder has two lateral openings, one near the top and the other near the bottom, closed by corks through

which thermometers T, T are inserted horizontally. The cylinder is filled with water previously cooled to about 10°C, and the trough J is packed with a freezing mixture of ice and salt.

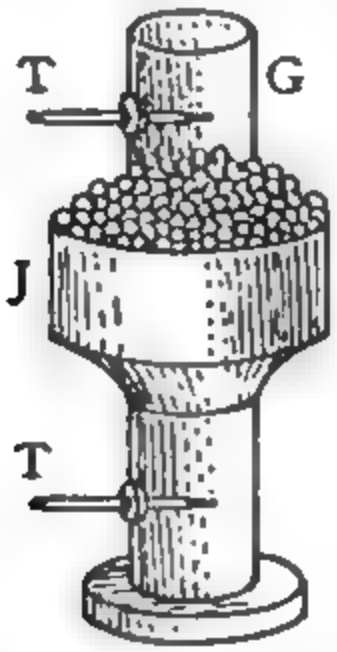


Fig. 26—Hope's apparatus

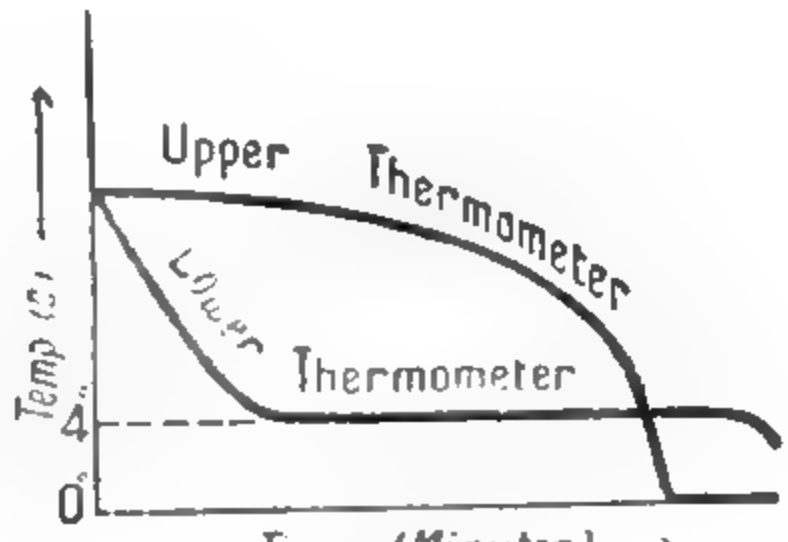


Fig. 27

The thermometers T, T are observed from time to time. Just at the beginning, the two give practically the same reading The lower one then starts to fall and gradually comes down to 4°C. at which it remains stationary. During all this time reading of the upper thermometer remains practically unchanged. After sometime it begins to fall and gradually comes to 0°C. If the cooling is continued for a long time small crystals of ice are found floating on the surface of the water while the lower thermometer remains steady at 4°C. After sufficiently long interval the lower thermometer records a fall of temperature from 4°C. This is an indication that water possesses the maximum density at 4°C.

Let us start with the assumption that water within the cylinder is initially at a uniform temperature of 10°C. when the freezing mixture is applied at its central part. Owing to the action of freezing mixture, the water in the central part of G cools, becomes denser and goes down ; and at the same time the warmer and lighter water from the bottom goes up, gets cooled and goes down again. Thus the water at the bottom gradually falls in temperature, as is shown by the lower thermometer. This continues till the whole mass of water in the lower part has come to 4°C. On further cooling, the water near the middle cools below 4°C, but does not sink down as is evident from the constant reading of 4°C. given by the lower thermometer. The water tends to ascend but cannot do so as it is heavier than the water in the upper portion. As the cooling continues, the water near the middle gradually falls to 0°C, and small crystals of ice are then formed ; these tend to rise to the surface, melt and cool the water in the upper part, as is shown by a rapid fall in the upper thermometer. This continues till the whole mass of water in the upper portion comes to 0°C., as indicated by the upper thermometer T, while all this time water in the lower part remains at 4°C. The crystal of ice which are now formed near the middle, being lighter than water at 0°C., float on its surface.

As the densest liquid occupies the lowest position and as the lower thermometer indicates a temperature of 4°C. it is concluded that water at 4°C. *has the maximum density* ; in other words, *a given mass of water occupies the smallest volume at* 4°C. The density of water at 4°C, is taken to be unity.

If the readings of the two thermometers, taken at regular intervals, are plotted against time, curves as in fig. 27, are obtained. The fact that water has its maximum density at 4°C is of great practical importance in nature. During frosty weather in cold countries, the water in lakes, river, etc. gradually falls to 4°C., on further cooling, the water in the upper layers becomes lighter and so remains on the surface. The water on the surface thus gradually cools to 0°C. and finally freezes while the water below remains practically at 4°C. The ice floats on the surface and thus lives of fish and other aquatic animals are saved. If the density of water would have continued to decrease below 4°C., the water at 0°C, would occupy the lowest layer and ice would be formed from below upwards so that the whole mass of water should be changed into a solid block of ice.

***28. Thermostat.**—One of the most useful practical applications of the expansion of a solid or liquid is the construction of an instrument which maintains a substance at a known constant temperature. Such an apparatus is called a *thermostat* or a thermo-regulator.

Fig. 28 represents a liquid thermostat, in which the glass bulb B coutains some such liquid as toluene, benzene or alcohol possessing large co-efficients of expansion. A glass tube P which is sealed into the bulb nearly reaches its bottom and is provided with a side tube N having a stop cock. The tube P is bent twice at right angles and is terminated with a bulb C having an opening. A short glass pipe is fused at the top of C reaching nearly to its bottom. A H-piece glass tube with an inter-connecting adjustable tap S is very often fitted as shown. The end F is connected to a gas burner.

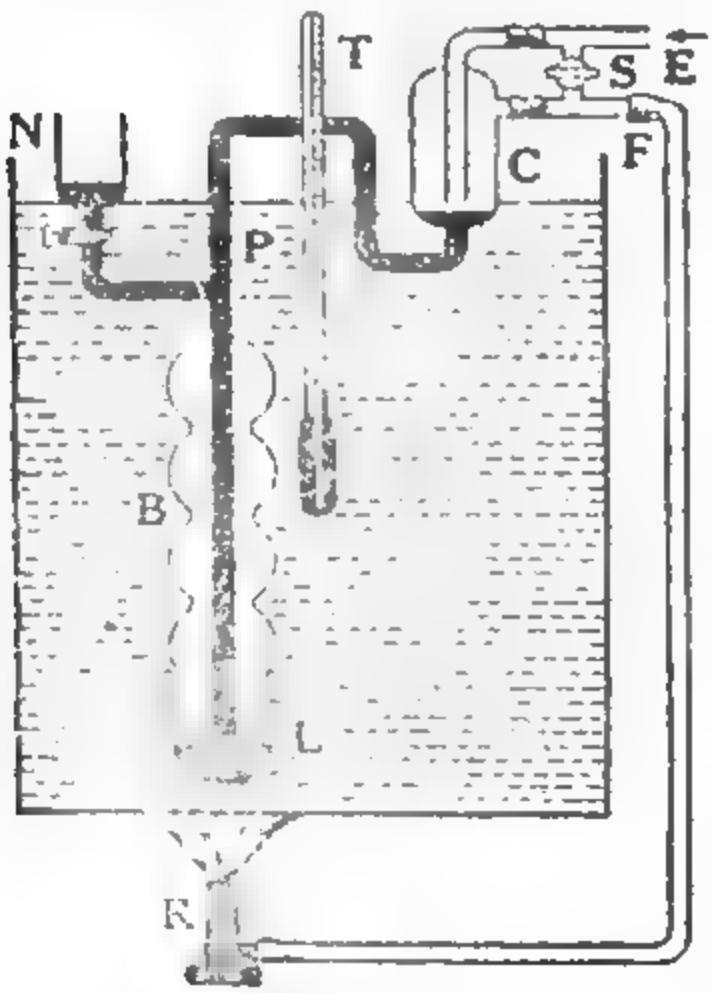


Fig. 28

When the temperature of the bath exceeds a certain critical limit, the liquid inside forces the mercury column contained in P through the pipe so as to close the end of the pipe E and hence the supply of gas to the burner through the pipe is stopped temporarily. The gas supplied through the by-pass tube is just sufficient to maintain a small flame at the burner. The heat supplied by the burner to the bath becomes insufficient and the temperature of the bath tends to fall producing a small decrease in volume of toluene. The mercury column recedes opening the gas supply. This explains how the temperature of the bath is not allowed

to cross an assigned limit. The critical temperature may be adjusted by the mercury column within the funnel N.

***29. Correction of Barometric reading for Temperatures.**—In a mercury barometer, the pressure of the atmosphere is expressed in terms of the height of a column of mercury, which is measured usually by a brass scale. The scale is correct at 0°C., and the liquid column is initially adjusted at 0°C. Hence the *observed* height of a barometer at t°C is to be reduced to the *equivalent* height at 0°C. To effect this, it is necessary,—

(*i*) to correct for expansion of the scale between 0°C and t°C.

(*ii*) to correct for the change in density of mercury.

Let H = the observed height of the barometer at t°C.

H_0 = the equivalent *corrected* height at 0°C.

ρ_0 and ρ = densities of mercury at 0°C and t°C. respectively.

α = coefficient of linear expansion of the material of the scale.

γ = coefficient of real expansion of mercury.

(*i*) *To correct for the Expansion of the Scale.*—At t°C the scale increases in length, so that each centimetre of the correct scale becomes $(1+\alpha t)$ cm. So the observed reading H, recorded by this increased scale is less than the reading h, given by the correct scale at 0°C. Then

$$h = H(1+\alpha t) \quad \ldots (29,1)$$

(*ii*) *To correct for the change in Density of Mercury.*—The reading h requires correction for the change in density of mercury. As the atmospheric pressure remains the same,

$$h\rho g = H_0\rho_0 g \text{ or } h\rho = H_0\rho(1+\gamma t)$$

$$\text{whence} \quad H_0 = \frac{h}{1+\gamma t} = h(1+\gamma t)^{-1} = h(1-\gamma t) \quad \ldots (29,2)$$

neglecting higher power of γ.

From (1) & (2)

$$H_0 = H(1+\alpha t)(1-\gamma t) = H\{1-(\gamma-\alpha)t\} \quad \ldots (29,3)$$

neglecting $\gamma\alpha t^2$ which is small in comparison to αt or γt.

The following Table supplies the variation of density of water with temperature.

Density of Water with Temperature.

Temp. °C	Density gm/c.c.	Temp °C	Density gm/c.c.	Temp. °C	Density gm/c.c.
0	·99987	10	·99978	24	·99732
1	·99993	12	·99958	26	·99681
2	·99997	14	·99927	28	·99626
3	·99999	16	·99897	30	·99567
4	1·00000	18	·99862	32	·99505
6	·99997	20	·99823	34	·99440
8	·99988	22	·99780	36	·99371

Example :—

1. A barometer with a brass scale, correct at 62°F, reads 30 in. at 50°F. Find the reading of the barometer when corrected to 32°F. (Given that α for brass $= 18 \times 10^{-6}$, and γ for mercury $= 18 \times 10^{-5}$ per 1°C.).

A. α for brass per 1°F. $= \frac{5}{9} \times 0{\cdot}000018 = 10^{-6}$.

Also γ for mercury per 1°F $= 10^{-4}$.

Correction of scale.—As the scale is below the standard temperature the reading h given by the correct scale at 62°F, *is less* than the observed reading at 50°F.

Then $h = 30 \{1 + (50 - 62) \times 0{\cdot}00001\} = 29{\cdot}964$ in.

Correction for change of density of mercury.—The height of the mercury column at 50°F, is given by h in. If H_o be the correct height at 32°F. and ρ_o and ρ be the densities of mercury at 32°F, and 50°F, respectively.

$H_o\rho_o = h\rho$ and $\rho_o = \rho\{1 + (50 - 32)\gamma\} = \rho(1 + 18 \times 0{\cdot}0001)$.

whence $H_o\ (1 + 0{\cdot}0018) = h = 29{\cdot}964$ $\therefore$ $H_o = 29{\cdot}91$ inches.

***30. Correction for exposed stem in a Thermometer.**—When a thermometer is used to measure the temperature of a body, the upper part of the stem is at a temperature considerably lower than that of the bulb. The divisions of the scale in the lower part do not, therefore, expand equally and so the observed temperature requires correction.

Let t_1 be the *observed* temperature of a body whose *real* temperature is t. Let there be n divisions of the temperature exposed to air at an average temperature t_2. If the thermometer were throughout at $t°$, these n divisions would cover a space equivalent to $n\{1 + \gamma(t - t_2)\}$ divisions, γ being the coefficient of apparent expansion of mercury in glass.

Hence the correct number of divisions in the exposed column

$$n\{1 + \gamma(t - t_2)\} - n = n(t - t_2)\gamma$$

$\therefore$ The true temperature of the body is given by

$$t = t_1 + n(t - t_2)\gamma \quad \ldots \quad (30,1)$$

In practice, however, $n(t_1 - t_2)$ will differ very slightly from $n(t - t_2)$. Hence the true temperature of the body is given by

$$t = t_1 + n(t_1 - t_2)\gamma \quad \ldots \quad (30,2)$$

THOMAS CHARLES HOPE (1766—1844)

Hope was born at Edinburgh in 1766. He was educated at the High School at the University of his native town. After finishing a brilliant career at the University, he was appointed to the Chair of Chemistry at the Glasgow University in 1787. Later in 1795 he was appointed the joint Professor of Chemistry with Prof Black at Edinburgh. After the death of Prof. Black in 1799, he took sole charge of the Chemistry department.

He had many researches of which the two outstanding are the discovery of strontium in 1792 and of water attaining the maximum density at 4°C. Due to old age he resigned his professorship in 1843 and the following year he died.

Summary

Coeff. of real expansion of a liquid = Coeff. of the apparent expansion + Coeff. of expansion of the material of the vessel.

Determination of the co-efficient of apparent expansion.—

(1) *By Volume Dilatometer.* (2) *By Weight Thermometer.*

Determination of the co-efficient of real Expansion of mercury.—

(1) Indirect method. $\gamma = \gamma' + \gamma_s$.

(2) Dulong and Petit's method. $\gamma = \frac{h_1 - h_0}{h_0 t}$ where h_0 and h_1 are the heights of two columns at 0°C. and t°C. respectively.

Expansion of water.—A given mass of water occupies the smallest volume at 4°C. and so has the *maximum density* at that temperature. This can be verified by (1) *Constant volume Dilatometer* (2) *Hope's experiment.*

Exercises on Chapter III

	Reference
1. Distinguish between the apparent and real expansions of a liquid, and describe an experiment to determine the apparent expansion of a liquid. (P. U.—1950; C. U.—1943; *Cf.* Del. H. S.—1954; R. P. B.—1945; Anna. U.—1951)	Arts. 22 & 24
2. Show that the absolute co-efficient of cubical expansion of a liquid is the sum of its apparent co-efficient of expansion and the co-efficient of cubical expansion of the material of the container. (*Cf.* Gau. U.—1952; Utk. U.—1953; P. U.—1952; Pat. U.—1952)	Arts. 22 & 23
3. A weight thermometer weighs 1·8760 gm. when empty. It is filled with mercury at 0°C and then heated to 100°C. when 0·2351 gm. of mercury is expelled. The thermometer with the remaining mercury weighs 16·8025 gm. Calculate the co-efficient of apparent expansion of of mercury. (Utk.—U.—1953)	Arts. 22 & 24
A. 16×10^{-5} per °C.	
4. A specific gravity bottle holds 50 gm. of glycerine at 30°C. How much will it hold at 100°C, assuming the co-efficient of cubical expansion of glycerine to be ·00051 and that of glass to be ·00003. (R. P. B.—1947)	Art. 24
A. 47·03 gm.	
5. Describe the weight thermometer method of determing the apparent expansion of a liquid. (E. P. U.—1950; Dac. U.—1943; R. P. B.—1947; U. P. B.—1952)	Art. 24

Reference

Art. 24 6. If the co-efficient of apparent expansion of mercury in glass be 15×10^{-5}, what mass of mercury will overflow from a weight thermometer which contains 300 gm. of mercury at 0°C when raised to 83°C? (Dac. U.—1943)

A. 3·69 gm.

Art. 24 7. A glass weight thermometer has a mass of 6·34 gm. when empty and 151·73 gm. when filled with mercury at 99°*C*. If 2·08 gm. have been expelled in changing the temperature from 0°*C* to 99°*C*. determine the co-efficient of relative expansion of mercury in glass. (U. P. B.—1952)

A. $14{\cdot}6 \times 10^{-5}$ per °*C*.

Art. 24 8. A weight thermometer weighs 25 gm. when empty and 475 gm. when filled with mercury at 0°C. On heating it to 100°C, 6·55 gm. of mercury is expelled. Calculate the co-efficient of linear expansion of glass, the co-efficient of real expansion of mercury being ·000182 per °C. (Pat. U.—1952)

A. ·000011 per °C.

Art. 24 9. A weight thermometer contains 82 gm. of a liquid at 10°C and on heating it to 85°C, 2 gm. of liquid are expelled. Find the co-efficient of absolute expansion of the liquid, if the co-efficient of linear expansion of glass is ·000009. (E. P. U.—1950)

A. ·00036 per °C.

Art. 24 10. Explain how to find the co-efficient of real expansion of a liquid by weight thermometer. Why is it called a thermometer. (E. P. U.—1952)

Art. 24 11. A thermometer is made by forming a bulb at the end of a glass capillary tube, of which the area of cross-section of the bore is 0·0304 sq. cm. The apparent co-efficient of expansion of mercury in glass being $15{\cdot}5 \times 10^{-5}$, per degree centigrade, what must be the volume of the bulb if the degree marks are to be 0·2 cm. apart?

A. 39·2 c.c.

Art. 24 12. A glass bottle with a fine stem, when immersed in melting ice just contains 300 gm. of mercury. Calculate the amount of mercury that will overflow if the bottle is kept sufficiently long time in boiling water, the barometric pressure being 76 cm. Would the amount of overflow have been different if the pressure had been considerably lowered? The co-efficient of expansion of mercury in glass = 1/6500. (C. U.—1943)

A. 4·55 gm.

Art. 24 13. A weight thermometer weighs 40 gm. when empty and 490 gm. when filled with mercury at 0°C. On heating to 100°C, 0·85 gm. of mercury escape. Calculate the co-efficient of linear expansion of glass, the co-efficient of real expansion of mercury being 0·000182. (R. P. B.—1942, '50; U. P. B.—1948)

A. 9×10^{-6} per degree C.

Art. 24 14. Find the mass of mercury which must be introduced into a glass bulb of 20 c.c. capacity in order to make the volume of the unoccupied portion in the bulb constant between 20°C. and 100°C. (α for glass $= 8 \times 10^{-6}$; ρ of mercury at 20°C, and 100°C. = 13·546 and 13·352 respectively).

A. 35·5 gm. nearly.

Art. 24 15. A graduated glass tube of uniform bore contains mercury upto 100 divisions at 0°C. Find the temperature at which the mercury

Reference

occupies 101·5 divisions? (coeff. of absolute expansion of mercury 0·00018; α for glass 8×10^{-6}).

A. 96·15°C.

Art. 24

16. The co-efficient of linear expansion of glass is 8×10^{-6} and the co-efficient of cubical expansion of mercury is $1{\cdot}8 \times 10^{-4}$ per deg C. What volume of mercury must be placed in a specific gravity bottle in order that the volume of the bottle not occupied by the mercury shall be the same at all temperatures? (Sen. Camb.)

A. $\frac{9}{17}$ of the volume of the bottle.

Art. 24

17. The bulb of a dilatometer has volume of 19·8 c.c. What volume of mercury must be introduced into it, so that the volume of the free portion becomes independent of changes in temperature? (α for glass $= 8 \times 10^{-6}$ and γ for the mercury $= 18 \times 10^{-5}$).

A. 2·64 c.c.

Art. 25

18. How does the density of a substance change with temperature. Find the relation between the two. (E. P. U.—1953)

Art. 26

19. Explain how to determine the co-efficient of absolute expansion of a liquid by Dulong and Petit's method. (E. P. U.—1952)

Art. 26

20. In an experiment with a liquid the column stood at 60 cm. at 32°F and stood 5 cm. higher at 212°F. Find the co-efficient of real expansion of the liquid in degrees centigrade. (E. P. U.—1952)

A. ·00083 per °C.

Art. 25

21. 1 c.c. of water weighs 0·999874 gm. at 0°C. and 1 gm. at 4°C. Find the mean co-efficient of absolute expansion of water between 0°C. and 4°C.

A. 32×10^{-5} per degree C.

Art. 26

22, In an experiment to find the real expansion of mercury with Dulong and Petit's apparatus, the temperature of hot column was 100°C and that of the cold column was 0°C. The heights of the mercury columns were found to be 51 cm. and 50·1 cm. respectively. Calculate the co-efficient of real expansion of mercury.

A. $17{\cdot}9 \times 10^{-5}$ per degree C.

Art 26

23. In an experiment two columns of mercury, one at the temperature of 14°C. and the other at a temperature of 100°C. balance each other. If the length of the shorter column is 86·4 cm., find the length of the other. (co-efficient of cubical expansion of mercury $= 0{\cdot}00018$.).

A. 87·7 cm.

Art. 27

24. Water is said to have its maximum density at 4°C. Explain what this means. In what respects is the behaviour of mercury different from that of water when both are gradually warmed from 0°C. (C. U.—1947)

Art. 29

25. Show how would you correct the reading of a barometer for the expansion of the mercury and the scale. (R. P. B.—1943, '45, 52; Pat. U.—1926)

Art. 29

26. A mercury barometer with brass scale and correct at 0°C. reads 76·69 cm. at 17·8°C. Reduce the reading to 0°C. (α for brass $= 19 \times 10^{-6}$., γ for mercury $= 18 \times 10^{-6}$.).

A. 76·47 cm.

Art. 29

27. A brass scale of a barometer was correctly graduated at 15°C. At what temperature would the observed reading require no temperature correction? Co-efficient of linear expansion of brass $=$ ·000019; co-efficient of absolute expansion of mercury $=$ ·00018. (R. P. B.—1946)

A. $-1{\cdot}8$°C nearly.

CHAPTER IV

EXPANSION OF GASES

31. Expansion of Gases.—Gases like solids and liquids change in volume due to changes in temperature but at a much greater rate. In considering the changes of volume of solids or liquids, the effect of pressure is not taken into account ; for even with a great change of pressure, the change in their volumes is negligibly small. But in the case of gases change in pressure produces an appreciable change in volume. To state the condition of a gas, it is necessary, therefore, to mention its *volume* (v), *pressure* (p) and *temperature* (t). A change in any one of these affects the others to a marked degree. Such changes are governed by the *Gas Laws* as given below.—

(1) Change of volume by change of pressure, temperature remaining constant (*Boyle's Law*).

(2) Change of volume by change of temperature, pressure remaining constant (*Charle's Law*).

(3) Change of pressure by change of temperature, volume remaining constant (*Law of Pressure*).

The first of these showing the relation between the volume (v) and the pressure (p) under constant temperature (t) is given by **Boyle's** law, which states that ***at constant temperature, the volume of a given mass of any gas varies inversely as the pressure to which it is subjected.***

Thus $v \propto \frac{1}{p}$ when t remains constant.

or $pv = k$, where k is a constant, depending on the mass and temperature of gas. This equation is called the isothermal equation of state. This has been dealt with in General Physics (Chap. XI).

If no heat is allowed to enter into or leave the mass of the gas during a change of volume, the temperature does not remain constant and the changes then are known as **adiabatic changes** of volume. The relation between pressure and volume under an adiabatic condition is given by the equation,

$$pv^{\gamma} = \text{a constant} \qquad \ldots \quad (31,1)$$

where γ is a number equal to the ratio of the specific heats of the gas at constant pressure to that at constant volume.

32. **Expansion of Gases at Constant Pressure.**—John Dalton first proved in 1801 that all gases when heated expand equally. It was found by Charles' that at constant pressure, the volume of a given mass of any gas increases for each degree centigrade rise of temperature by a constant fraction of its volume at 0°C. This is known as **Charles'**

Law. This relation was afterwards verified by Gay Lussac and more accurately by Regnault and is sometimes called Gay Lussac's Law.

If the volume of a given mass of gas at 0°C. be V_o c.c., and if γ_p be the fractional increase of volume per °C, then for a rise of temperature of t°C the increase in the volume is $V_o \gamma_p t$. Hence the volume of the gas at t°C. is given by

$$V_t = V_o + V_o \gamma_p t = V_o (1 + \gamma_p t) \quad \ldots \quad (32,1)$$

Whence $$\gamma_p = \frac{V_t - V_o}{V_o t} \quad \ldots \quad (32,2)$$

This constant fraction γ_p may, therefore, be taken as *the coefficient of cubical expansion of any gas at constant pressure* or simply as *the Volume Coefficient of the gas.* The volume coefficient has been experimentally found to be very nearly $\frac{1}{273}$ or 0·00366 for real gases.

Hence from eqn. (32,1) $$V_t = V_o\left(1 + \frac{t}{273}\right) \quad \ldots \quad (32,3)$$

The above relation is similar to that in the case of solids and liquids. But the following important points of difference are to be noted.—

(1) As a change of pressure considerably affects the volume of gas, the presure must be kept constant.

(2) The coefficient of expansion for gases is much greater than that for solids or liquids.

(3) The value of this coefficient is approximately *the same* for *all* gases and not appreciably different from one another as in the case of solids or liquids.

(4) As the expansion in the case of gases for slight change of temperature is much bigger than that in the case of solids or liquids, *the volume* at 0°C must always be taken, in applying the formula for expansion.

It is interesting to study the eqn. (32,3) ; for if we have a fixed mass of gas, then its volume V_o at 0°C is constant. If the temperature t of the gas is continuously increased, its volume V_t also continuously increases. The relation between V_t and t is that of a *straight line*, if their values are plotted on a graph paper.

33. Determination of the Co-efficient of Expansion of a Gas at Constant Pressure.—The coefficient of expansion of a gas at constant pressure can be determined by Constant Pressure Thermometers of different forms.

(1) Constant Pressure Thermometer of a Simple form.—This consists of a glass tube, about 50 cm. long and with a uniform bore of about 1 mm. in diameter. Dry air is passed through the tube for some time, after which one of the ends is sealed with a blow-pipe flame. The tube is then heated gently with the open end dipping in pure dry mercury. As the tube cools, the air inside contracts and small pellet H of mercury (Fig. 29) is sucked in, which serves as an index. The amount of the air expelled from inside should be such that the mercury

pellet stands about midway along the tube at the room temperature. A thermometer T is tied along the body of the tube. The tube A is now held within a wide jacket tube G of glass, with its open end protruding out of the jacket. The two ends of the jacket are also closed by stoppers and through each one passes a short glass tube to serve as inlet or outlet tube.

Fig 29—Expansion of a gas at constant pressure

Ice-cold water is at first circulated through the jacket, till the thermometer T records a constant temperature of 0°C. The gas within the glass tube also attains the temperature of 0°C and its volume becomes constant and so the pellet P comes to a steady position. After waiting for some time, the distance of the inner end of the pellet from the closed end is measured. Let l be the length of the air column at this state. The water is let out and steam is now passed through the jacket; the temperature rises and ultimately becomes constant. After sometime the position of the inner end of the pellet is noted again ; let the length of the air column be now l' cm. and let the temperature be t°C.

If a be the internal cross-section of the tube, the volumes of the enclossed air at 0°C and t°C are respectively la and $l'a$, neglecting the small expansion of the glass tube. If the inclination of the tube is kept fixed during the course of the experiment, its mouth being always open to the atmosphere, the pressure of the air inside remains equal during the period of experiment.

Hence the coefficient of cubical expansion of the air at constant pressure is given by

$$\gamma_p = \frac{l'a - la}{la(t-0)} = \frac{l'-l}{lt} \quad \ldots \quad (33,1)$$

The mean value of the coefficient γ_p is found to be the same at all constant pressure. The air in the tube may be replaced by any other gas, and the coefficient of expansion for the gas may be determined as above. The coefficient of expansion as found above is really the coefficient of apparent expansion of the gas with respect to glass. But as the expansion of the glass is very small in comparison to that of the gas, it is not ordinarily necessary to apply a correction for this.

(2) Constant Pressure Air Thermometer (*Regnault's apparatus*).—This consists of a U-tube AB (Fig. 30) with one limb graduated and ending in the bulb A while the limb B is open to the atmosphere. At the bottom of the U-tube a short tube is attached, having a stopcock C at its end. The U-tube is placed within a wide glass jacket J, the bottom of which is closed by a rubber cork through which the short tube just mentioned passes. There is a copper tube S, bent in the form of an inverted U, passing through the cork with both ends outside.

The jacket is filled with water so as to keep the bulb immersed. A

thermometer T is suspended in the water close to the bulb There is also a stirrer placed inside the jacket. Sulphuric acid is contained in the lower part of AB while the bulb contains some air, the volume of which can be directly read from the graduations on the stem attached to the bulb. By pouring in more of the liquid through the open limb or letting out some through opening C, the liquid in both the limbs can always be brought to the same level. The pressure of the enclosed air in A is then equal to the atmospheric pressure at the time.

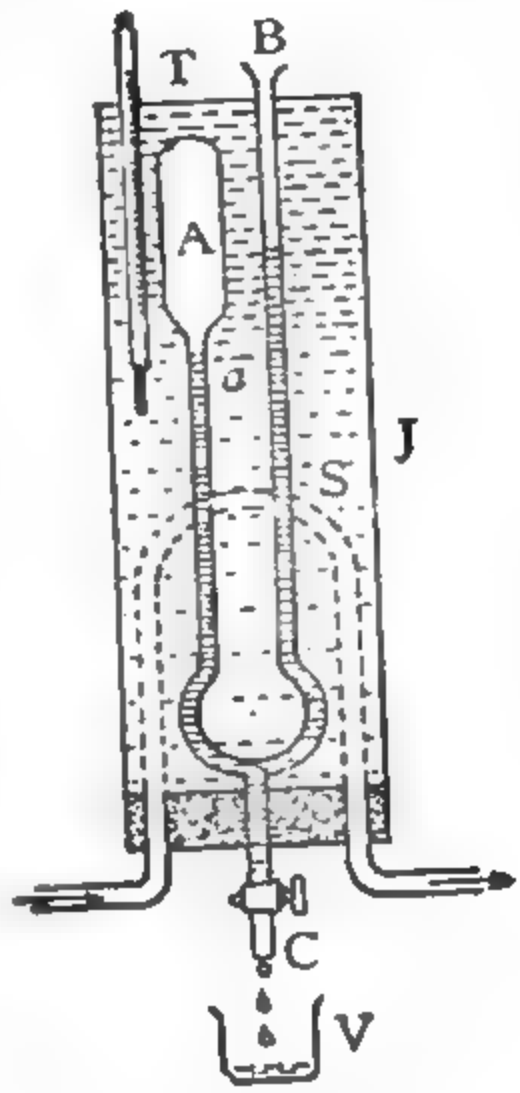


Fig. 30

At the room temperature t_1, as given by the thermometer T, sulphuric acid in the U-tube is brought to the same level and the volume of air in A is noted. Let the volume at the room temperature be V_1. Steam is now passed through the copper tube S which heats water in the jacket. To keep uniformity of temperature, water is kept constantly stirred. The temperature rises, the air expands and forces down the liquid which therefore, rises in the other limb. By regulating the steam, temperature of water at some higher value is maintained constant for some time, during which air within the bulb acquires the temperature of hot water. The stop cock is opened and some liquid is let out, till the level again becomes the same in both the limbs. The volume of the air is noted. Let the volume be V_2 and its temperature be t_2.

Then $V_1 = V_0(1 + \gamma_p t_1)$ and $V_2 = V_0(1 + \gamma_p t_2)$

$$\therefore \quad \frac{V_2}{V_1} = \frac{1 + \gamma_p t_2}{1 + \gamma_p t_1} \quad \text{or,} \quad \gamma_p = \frac{V_2 - V_1}{V_1 t_2 - V_2 t_1} \quad \ldots (33,2)$$

thus by knowing V_1, V_2, t_1, and t_2, the value of γ_p can be calculated.

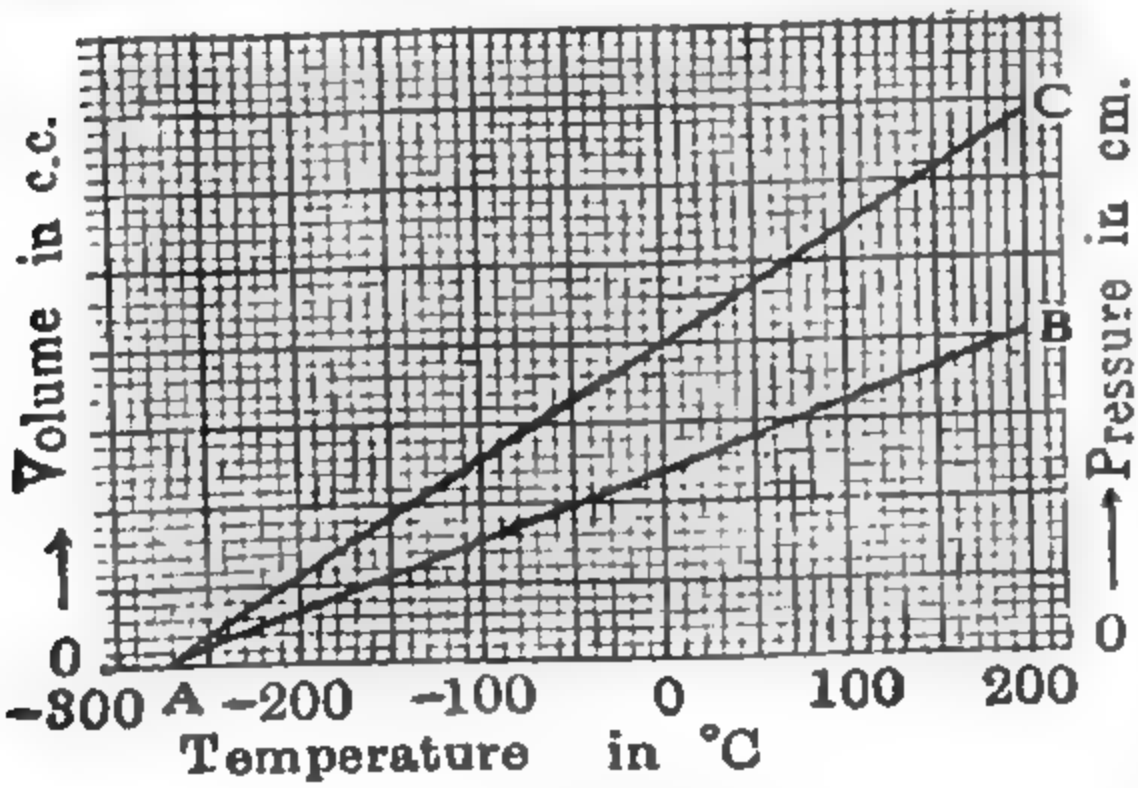


Fig. 31

For greater accuracy in result, the temperature of the bath is increased at successive steps and the corresponding volumes are noted. A *graph* is then plotted, taking temperature as abscissa and the corresponding volume as the ordinate. The graph is found to be a *straight line*. In Fig. 31, the straight line such as AB represents the relation between the volumes of a given mass of a gass and corresponding temperatures. The straight line is of course produced much on the negative side of the

temperature. The volume at 0°C and that at the some other temtemperature t°C are found from the graph and from these γ_p can be calculated from the eqn. (32,2).

34. Increase of Pressure at Constant Volume.—If the volume of a given mass of a gas is kept constant while it is heated, its pressure increases ; for due to a rise of temperature, the gas tends to expand, which being prevented, an increase of pressure occurs. It is found that pressure increases uniformly with rise of temperature according to the following law :—

The pressure of a given mass of any gas at constant volume increases for each degree centigrade rise of temperature by a constant fraction of its pressure at 0°C.

Hence if P_o be the pressure at 0°C. P_t that at t°C, and γ_v, the contant fraction, the increase in pressure is $P_o\gamma_v t$ and so

$$P_t = P_o + P_o\gamma_v t = P_o(1 + \gamma_v t$$

when $$\gamma_v = \frac{P_t - P_o}{P_o t} \qquad \ldots(34,1)$$

The constant fraction is called the **pressure co-efficient of a gas** and *may be defined as the ratio of the increase of pressure of the gas at constant volume due to a rise of temperature of 1°C to its pressure at 0°C.*

Constant Volume Air Thermometer.—The relation between the temperature and the pressure of a gas at constant volume can be studied by an apparatus, known as the *constant volume air thermometer*. It consists of a large glass bulb A (Fig. 32) at the end of thick-walled capillary tube BCD, bent twice at right angles and enlarged into a wider tube DE at the other end. The tube DE is again connected by means of a long piece of stout flexible rubber tubing FG to a wide glass tube GH open to the atmosphere. The tubes DE and GH can be separately moved along vertical up-rights on each side of wooden stand and can be clamped at any position. A vertical scale S is fixed on the wooden stand. There are three levelling screws at the base of the stand. At D there is a fine mark engraved on the tube or sometimes an ivory pointer is fixed. At C there is a three-way tap through which the bulb A can be exhausted of air and filled with any

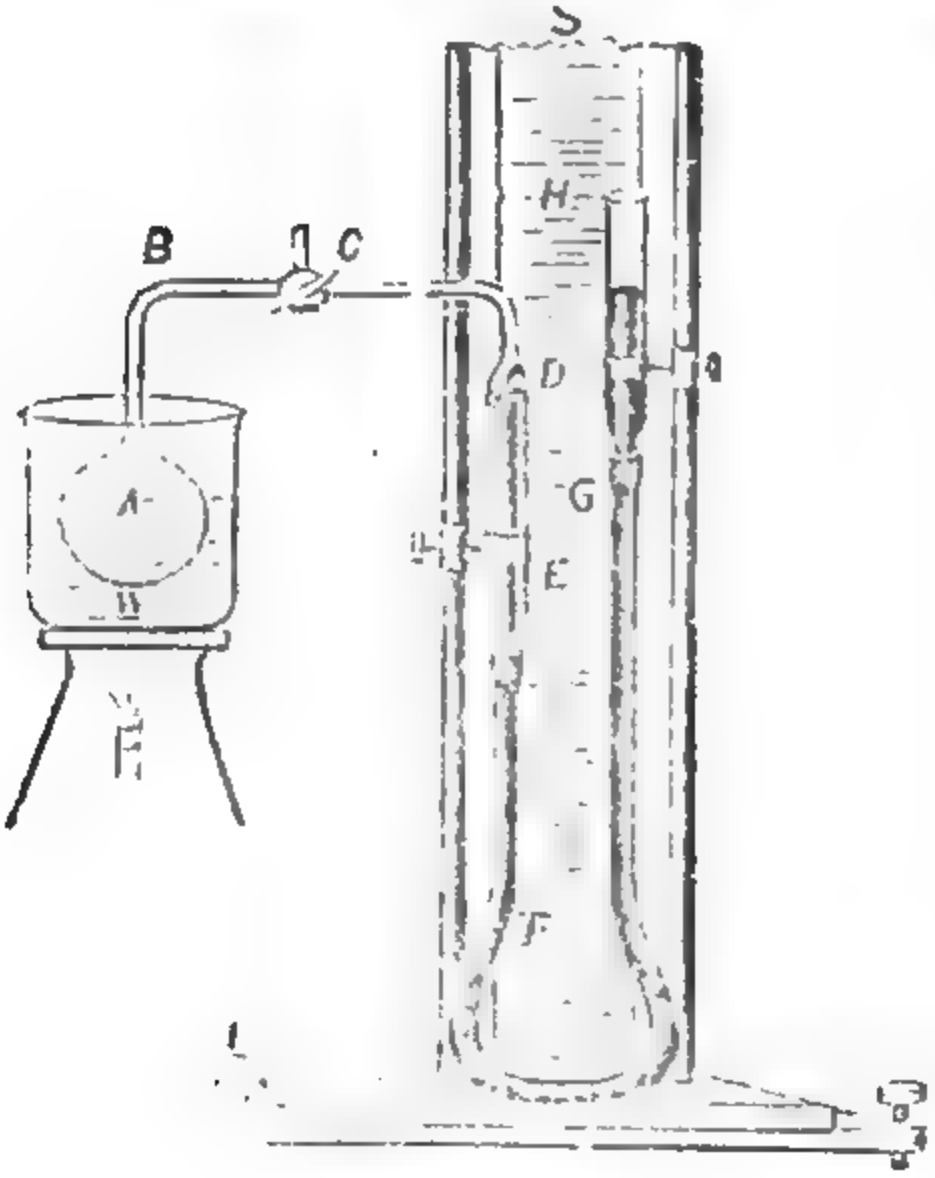


Fig. 32

other gas. A part of the tubes DE and GH and the whole of the rubber tube are filled with pure dry mercury while the bulb A and the capillary tube upto D contain dry air or any other gas.

The bulb A is now kept immersed into bath of ice-cold water and the tube DE is clamped at any position. By raising or lowering the tube GH, the level of the mercury in DE is adjusted exactly to the mark D. The difference of mercury levels in the two tubes is noted. By adding to this difference the barometric pressure at the time, the pressure of the enclosed air is obtained ; let it be p_o.

The water bath is now heated and is kept stirring well. The temperature is raised to some higher value which is read by a thermometer held in the bath. By adjusting the flame, the temperature of the bath is kept constant for sometime when the gas attains that temperature. The pressure of the enclosed air is then read. Let the pressure at higher temperature $t°$C. be p. Then the value of the pressure coefficient is given by,—

$$\gamma_o = \frac{p - p_o}{p_o \times t} \qquad \ldots(34,2)$$

The value of γ_o for air is found to be 0·00366. In the laboratory, temperature of the enclosed gas is raised usually from room temperature in steps and the corresponding pressures are noted. A *graph* is plotted taking temperature as abscissa and the corresponding pressure as ordinate. The graph is found to be a *straight line*, (Fig. 31. AC) which shows that *the pressure increases uniformly with temperature.* From the graph, the pressure p_o at 0°C, and that at some convenient higher temperature may be obtained ; from these the value of the pressure coefficient is calculated according to the eqn. (34,2).

Advantages of a Gas Thermometer.

(*i*) As gases are more expansible, a gas thermometer is much *more sensitive* than a liquid thermometer. Further, the effect due to the expansion of the containing vessel need not be taken into consideration except for a very great accuracy.

(*ii*) The rate of *expansion* of all gases is practically the *same* over a large range of temperature under the same condition.

(*iii*) The *expansion* of all gases is very approximately *uniform* and *regular*.

(*iv*) A gas thermometer can be used for a *much wider range* of temperatures, so as to record very low as well as very high temperatures.

Disadvantages of a Gas Thermometer.

(*i*) A gas thermometer is bulky and so is not comfortably portable.

(*ii*) As a change of atmospheric pressure affects the readings of the instrument, a barometer is also to be carried along with the gas thermometer.

(*iii*) It is too large to be used in calorimetric work.

(*iv*) The instrument cannot be furnished with fixed scales as the atmospheric pressure varies from time to time.

Examples :—

1. A uniform vertical glass tube, open at the top and closed at the bottom contains air and a pellet of mercury 3·0 cm. long. The lower end of the pellet is 30·5 cm. above the bottom of the tube when the tube is at temperature of 5°*C*. How far will the pellet rise if the tube is heated to 100°C? (Utkal. U.—1952)

A. Since the volume co-efficient of air is considerably larger than the cubical expansion of glass, the latter may be neglected in comparison to the former. Hence bore of the glass tube may be assumed not to change with temperature in so far as the problem is concerned and so the length of the air column within the glass tube at any temperature is proportional to the volume of air at that temperature. Further the pressure on the air column is due to the atmospheric pressure and the pellet of mercury, and consequently if the inclination of the tube is not changed during the course of the experiment, the *pressure* on the air column remains *constant* whatever might be the inclination of the tube and the length of the mercury pellet.

Let l_o be the length of the air column at 0°*C*; then volume of air column at 0°*C* is kl_o, where k is a constant. At 5°*C*, the volume is k 30·5.

Then $k\ 30{\cdot}5 = kl_o\ (1 + {\cdot}00366 \times 5)$ whence $l_o = 29{\cdot}89$ cm.

At 100°C, if the length of the column be l cm.

Then $kl = k\ 29{\cdot}89\ (1 + {\cdot}00366 \times 100)$ whence $l = 40{\cdot}83$ cm.

2. Ten million cubic feet of gas are supplied to a town per week under a pressure of 3 inches of water over the atmospheric pressure at 5 *s*. per 1000 cubic ft. When the barometer reads·31 inches, gas company can make no profit at all. Find the profit per week when the average height of the barometer is 29 inches. (Density of mercury = 13·6).

A. Let v cu. ft. be the increase in volume of 10×10^6 cu. ft. of gas for the decrease of pressure. Then the company can save this v cu. ft. of gas per week.

The pressure due to 3 inches of water is equal to that of $\frac{3}{13{\cdot}6}$ inches of mercury. From $PV = P'V'$.

$$\left(31 + \frac{3}{13{\cdot}6}\right) \times 10^7 = \left(29 + \frac{3}{13{\cdot}6}\right)\left(10^7 + v\right)$$

Hence the profit made by the company $= \frac{6{\cdot}85 \times 10^5}{10^3} \times 5s. = £171\ 5s.$

35. Absolute Scale of Temperature.—The volume of a given mass of a gas at any temperature can be obtained from Charles' Law according to the equation.—

$$V_t = V_o\left(1 + \frac{t}{273}\right)$$

Thus the volume at −10°C, is $V_{-10} = V_o\left(1 - \frac{10}{273}\right)$.

And the volume at −273°C. $V_{-273} = V_o\left(1 - \frac{273}{273}\right) = 0$

Evidently −273°C. is the lowest temperature that can be indicated

by the gas thermometer. For at this temperature, the volume of any gas, obeying Charles' Law, appears to occupy no space. Indeed all known gases liquefy and even solidify before this temperature is reached. Further, the pressure co-efficient of a gas having also a value $\frac{1}{273}$, we can show from eqn. (34,1) that at $-273^{\circ}C$, a gas would have *no pressure*. The same conclusion may be derived from the volume-temperature (AB) and pressure-temperature (AC) graphs (Fig. 31). Both the straight lines when produced to meet the abscissa representing zero volume and zero pressure lines, cross at one point A which is $-273^{\circ}C$. We are thus led to consider $-273^{\circ}C$. as the lowest temperature conceivable and so it is called the **absolute zero** of the gas thermometer.

If we now have a scale of temperature starting from the absolute zero *i.e.*, from $-273^{\circ}C$ as its zero and having each degree equal to that on the Centigrade scale, that scale is called the **Absolute Scale of Temperature.** Temperatures when reckoned on the absolute scale are termed *absolute temperatures* (Degrees absolute). According to this scale the melting point of ice ($0^{\circ}C$) is $273^{\circ}A$ and the boiling point of water ($100^{\circ}C$.) under normal pressure is $373^{\circ}A$. In general, if t° of the centigrade scale corresponds to T° of the absolute scale, then

$$T^{\circ}A = t^{\circ}C + 273$$

i.e., Absolute Scale reading = Centigrade reading + 273

As Kelvin proposed the absolute scale, the temperatures on the absolute scale are sometimes termed degrees Kelvin in place of degrees absolute; thus $100^{\circ}C = 373^{\circ}K$. More accurately absolute zero is 273·18 degrees below centigrade zero.

36. Charles' Law and the Law of Pressure expressed in Different forms.— From Charles' Law, if V and V′ are the volumes of a gas at *centigrade temperatures* t and t', then

$$V = V_0\left(1 + \frac{t}{273}\right) \text{ and } V' = V_0\left(1 + \frac{t'}{273}\right)$$

$$\therefore \frac{V}{V'} = \frac{273 + t}{273 + t'} = \frac{T}{T'} \text{ or } \frac{V}{T} = \frac{V'}{T'} \quad \ldots (36,1)$$

when T and T′ are absolute temperature corresponding to $t^{\circ}C$, and $t'^{\circ}C$, respectively.

Hence $\frac{V}{T}$ *is constant, when* P *is constant.*

From this relation Charles' Law may be stated as follows :—

The volume of a given mass of gas at constant pressure is directly proportional to the absolute temperature of the gas.

Again from the relation $P = P_0\left(1 + \frac{t}{273}\right)$,

$$\frac{P}{P'} = \frac{273 + t}{273 + t'} = \frac{T}{T'} \quad \ldots (36,2)$$

Hence $\frac{P}{T}$ *is constant, when* V *is constant.*

In other words, *the pressure of a given mass of gas at constant volume is directly proportional to the absolute temperature of the gas.*

Gas Laws Combined.—A simple general relation can be established between the pressure, volume and temperature of a given mass of a gas. Let V be the volume of a given mass of a gas at pressure P and absolute temperature T. We are to find the volume V′ of the same mass at pressure P′ and absolute temperature T′. First let the pressure change from P to P′, the temperature remaining constant. If the volume thereby changes to V_1, then

$$\text{From Boyle's Law } PV = P'V_1 \text{ or } V_1 = \frac{PV}{P'}$$

Let the temperature now change from T to T′, the pressure remaining constant at P′. If the volume consequently changes from V_1 to V′, then from Charles' law,—

$$\frac{V_1}{T} = \frac{V'}{T'} \text{ or } \frac{PV}{P'T} = \frac{V'}{T'}$$

Therefore,
$$\frac{PV}{T} = \frac{P'V'}{T'} \quad \ldots (36,3)$$

Hence $\frac{PV}{T}$ = a constant, say K or $PV = KT$ ... (36,4)

Thus, *the product of the pressure and the volume of a given mass of a gas varies directly as its absolute temperature.*

The eqn. (36,4) is referred to as the *gas equation* or **equation of state,** for by knowing any two of the three quantities P, V and T, the third can be found and the state of the gas can be completely determined. The value of the constant K depends on the mass of the gas taken. If a gramme-molecule of a gas be taken, the value of K is then the same for all gases. It is, then called the universal gas constant and is denoted by R. Hence for a gram-molecule, usually called the *mole*, of a gas,—

$$PV = RT \quad \ldots (36,5)$$

If M be the mass of the gas occupying a volume V at pressure P and at absolute temperature T, then the volume of a gram-molecule of the gas, having a molecular weight *m*, under the same conditions of temperature and pressure is $\frac{mV}{M}$ and consequently,

$$P\frac{mV}{M} = RT \text{ whence } PV = \frac{M}{m}RT = nRT \quad \ldots (36,6)$$

when *n* is the ratio of the mass of the gas under examination to its molecular weight.

As the volume of a gas is affected by the pressure and temperature, a gas is always to be referred to a definite temperature and pressure. A volume of a gas at 0°C, at a pressure at 760 mm. of mercury is said to be at **normal temperature and pressure (N. T. P.).**

To find the value of the gas constant (R).—One litre of hydrogen at N. T. P. weighs 0·0896 gm. Find the value of R considering one gram-molecule of the gas. (Mys. U.—1952; R. P. B.—1949)

Now the volume of 0·0896 gm. of hydrogen at N. T. P. = 1000 c.c.

$\therefore$ „ „ „ 2 gm. „ „ $= \frac{2000}{0{\cdot}0896} = 22321$ c.c.

The normal atmospheric pressure $= 76 \times 13{\cdot}6 \times 981$

$\therefore$ $76 \times 13{\cdot}6 \times 981 \times 22321 = R \times 273$

when $R = 8{\cdot}3 \times 10^7$ ergs per degree °C.

37. Variation of the Density of a Gas.—Let ρ be the density and V the volume of a mass of a gas at pressure P and absolute temperature T; let ρ' be the density and V′ the volume of the same mass of the gas at pressure P′ and absolute temperature T′.

As the density of a gas at a given pressure and temperature varies inversely as its volume, we have from equation (36,4)

$$\frac{P}{\rho T} = \frac{P'}{\rho' T'} \quad \ldots (37,1)$$

If the temperature does not change, $T = T'$

$\therefore$ From (37,1) $$\frac{P}{\rho} = \frac{P'}{\rho'} \quad \ldots (37,2)$$

Hence the density of a gas at constant temperature varies directly as the pressure.

Again if the pressure remains constant, $P = P'$.

$\therefore$ From (37,1), $$\frac{1}{\rho T} = \frac{1}{\rho' T'} \text{ or } \rho T = \rho' T' \quad \ldots (37,3)$$

Hence the density of a gas at constant pressure varies inversely as the absolute temperature.

***38. Standard Gas Thermometer.**—An ideal thermometer is one in which equal rise of temperature would be indicated by graduations of its scale by equal increment. This can only be theoretically attained by the absolute Thermodynamic Scale as proposed by Lord Kelvin (Art. 11). No property of a substance varies *exactly uniformly* with temperature and as a result if any material property be utilised as a thermometric substance, then equal changes of magnitude of that property will not indicate *exactly equal* changes of temperature. The nearest approach to the Absolute Thermodynamic Scale is obtained by employing the volume coefficient or the pressure co-efficient of a real gas as a thermometric substance. Hence either a constant pressure or a constant volume gas thermometer may be used to indicate a temperature with least possible correction. The constant volume thermometer is however more preferable as it is easier to adjust and the volume of excess of gas within the connecting tube which always remains outside the temperature-bath is very small and constant. For this reason the International Committee of Weights and Measures decided, in 1887, to adopt as a practical standard the centigrade scale of a constant volume hydrogen thermometer.

Constant Volume Hydrogen Thermometer.—The constant volume hydrogen thermometer was originally designed by Chappuis in 1887. Fig 33 represents a hydrogen thermometer of a simpler form which consists of a bulb B about 110 cm long and of capacity 1 litre. The bulb is held horizontally and can be placed at any temperature bath. In the original form this bulb was made of platinum-iridium to withstand a high temperature and pressure. The bulb is connected by a narrow bore to a wider tube C to which it is sealed air-tight. A little ivory pointer p_1 projects vertically at the top of C. There is another vertical side tube D having communication with C at the lower part. A barometer tube E is kept above the open top of D and this tube is provided with an identical pointer p_2 near its top. The combination of the tubes C and D together with the barometer forms a manometer with which the gas pressure of the bulb can be measured. A mercury reservoir R is connected by air-tight pressure tubing to the lowest point of manometer. The barometer and the reservoir can be slided up and down a vertical metre scale and can be fixed at any position. The heights of mercury column in C, E and R can be accurately read with verniers attached to the main scale or with a cathetometer.

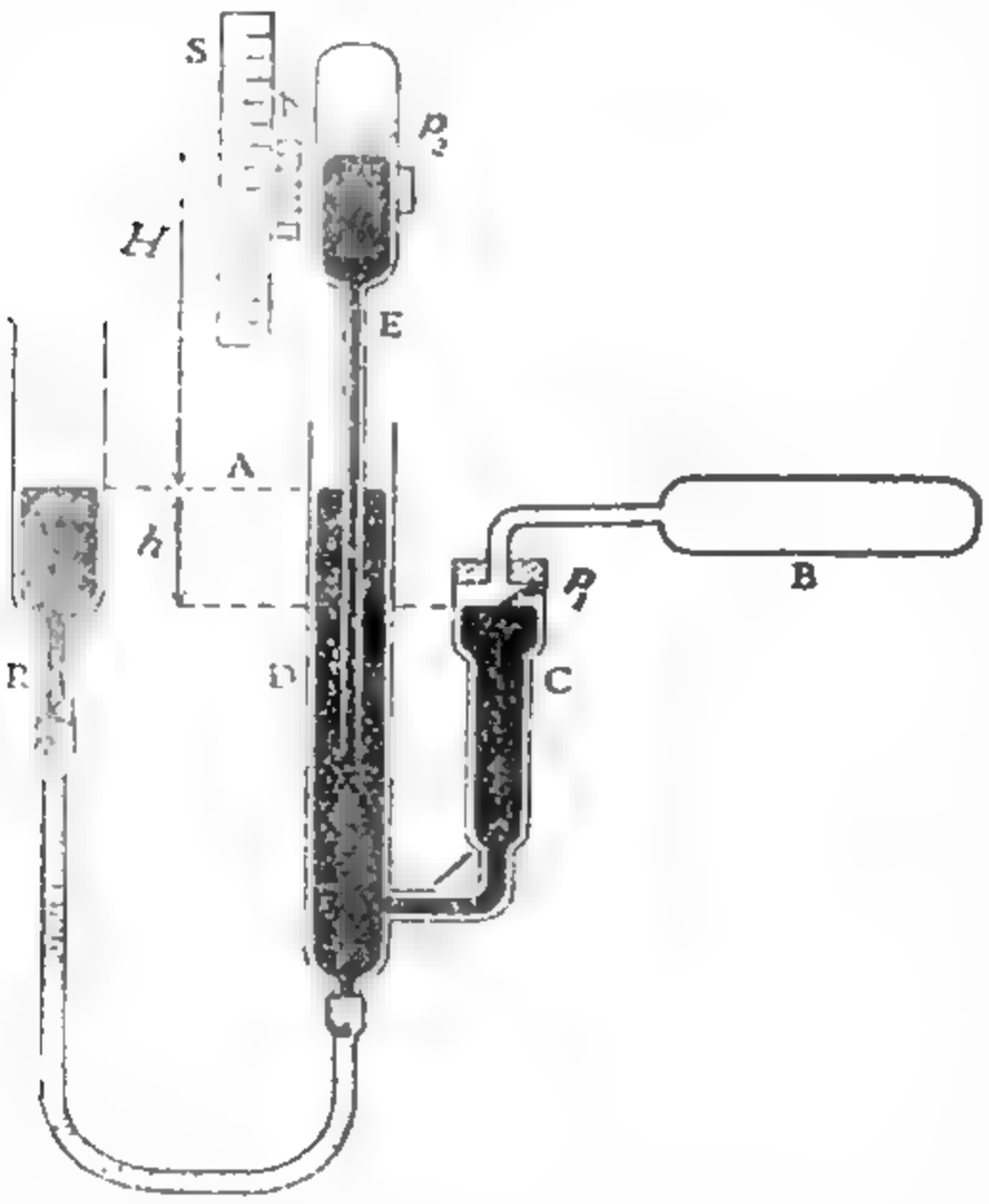


Fig. 88—Constant Volume Thermometer

The bulb B is kept within a temperature bath of melting ice for a considerable time so as to attain a temperature of 0°C. Then the reservoir R is raised or lowered until the mercury surface in C touches the pointer p_1. The barometer tube is then adjusted so that mercury surface in it touches p_2. Adjustment more than once may be necessary to make mercury surfaces in the two tubes exactly touch the pointers. When this is done, the volume of the enclosed gas becomes always constant and the pressure exerted by the gas is the mercury level differences in R and C plus the barometric pressure. Consequently the pressure P_0 is the difference of vertical heights of p_2 and p_1, which may be determined with reference to the vertical scale attached.

Next the bulb is kept in the steam of boiling water and when temperature becomes steady the difference of heights of p_2 and p_1 is found. Thus with proper correction of boiling point, the pressure

of the gas of 100°C is determined. Call this pressure P_{100}. If now the bulb is kept at an unknown temperature bath t°C, and when the temperature becomes steady, if the pressure recorded be P_t, then,

$$\frac{t}{100} = \frac{P_t - P_0}{P_{100} - P_0} \quad \text{whence } t = 100 \times \frac{P_t - P_0}{P_{100} - P_0} \qquad \ldots(38,1)$$

Hydrogen is kept within the bulb such that at 0°C, the pressure exerted by it is one metre of mercury. A hydrogen thermometer may be used from −200°C to 500°C. Above 500°C hydrogen diffuses through platinum and attacks glass or porcelain. Between 500°C to 1500°C nitrogen is used in place of hydrogen. For temperatures below −200°C hydrogen is replaced by helium.

39. Boyle's Law and Real Gases.—Boyle's law states that the temperature remaining constant, the product of the pressure and volume of a given mass of a gas is constant, no matter what the pressure on the gas might be. During Boyle's time there was no means to exert a large pressure on a gas and the law could be verified only for a moderate range of pressure over which $p \times v$ was found to be *very nearly constant*. Later on much improved apparatus, capable of exerting pressures upto two to three thousand atmospheres, were designed. With such apparatus volumes and corresponding pressures of gases such as air, hydrogen, nitrogen, and carbon dioxide at a constant temperature were recorded by Andrews, Amagat and Regnault. They found that at higher pressures, the product of the volume and the corresponding pressure of a gas is not constant but the product continuously varies as the pressure is changed The variation of the product pv with p depends upon the temperature at which the gas is maintained. For each gas there is a certain temperature at which the product pv is very nearly constant. A temperature peculiar to each gas for which the Boyle's law holds very closely even for a large variation of pressure is called the Boyle temperature for that gas. The Boyle temperature for hydrogen is 109°K and for nitrogen 323°K. So we can make a general conclusion that when pressure on a given mass of gas changes greatly at a constant temperature, except the Boyle temperature, the product of the volume and the corresponding pressure of the gas is not constant and this phenomenon is called the deviation of Boyle's law at high pressures.

Van der Waals' Equation.—The equation $PV = nRT$ holds good for a real gas so long as the pressure applied is not high. For higher pressures there is a considerable departure from Boyle's law.

It is found that considerable range of pressure the following equation due to Van der Waals holds good.

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT. \qquad \ldots(39,1)$$

where a and b are constant for a particular gas.

Examples :—

1. A quantity of gas collected over mercury in a graduated tube is found to

occupy 25 c.c. at 27°C. The level of the mercury inside stands 15 cm. higher than the level outside while the barometer stands at 75 cm. Find the volume that the mass of the gas would occupy at a pressure of 74·5 cm. of mercury and at a temperature of 32°C.

A. The initial pressure of the gas is $P = (75 - 15) = 60$ cm. of mercury

Let the required volume be V′ c.c.

From $\frac{PV}{T} = \frac{P'V'}{T'}$ or $\frac{60 \times 25}{27 + 273} = \frac{74{\cdot}5 \times V'}{32 + 273}$ whence $V' = 20{\cdot}5$ c.c.

2. At 22°C. and at pressure of 74 cm. the volume of a given mass of gas was found to be 54·02 c.c. On cooling to 0°C, the volume became 49·3 c.c., the pressure having risen to 75 cm. Find the co-efficient of expansion of the gas.

A. Let the co-efficient of expansion of the gas be γ.

The volume of the gas at 0°C. and 75 cm. pressure = 49·3 c.c.

From Boyle's law, the volume at 0°C at 74 cm. pressure $= \frac{49{\cdot}3 \times 75}{74}$

$= 49{\cdot}97$ c.c.

From Charles' law, the value at 22°C. and 74 cm. pressure $= 49{\cdot}97\ (1 + 22\gamma)$.

$\therefore \quad 49{\cdot}97\ (1 + 22\gamma) = 54{\cdot}02 \quad$ whence $\quad \gamma = 0{\cdot}00368$.

3. The internal volume of a glass vessel is 1 litre at 0°C. It is filled with hydrogen at N. T. P. and is then heated to 27°C. Find the mass of hydrogen that would escape, if the flask is now opened under a pressure of 74 cm. (1 litre of hydrogen at N. T. P. weighs 0·09 gm.)

A. Let V litres be the volume that 1 litre of hydrogen at N. T. P. would occupy at 27°C and 74 cm. pressure.

Then $\frac{76 \times 1}{273} = \frac{74 \times V}{27 + 273}$ whence $V = \frac{300 \times 76}{273 \times 74}$ litres

So the volume of hydrogen that escapes $= \left(\frac{300 \times 76}{273 \times 74} - 1\right)$ litres

At N. T. P. the above volume becomes $= \frac{2593}{22800}$ litres

The wt. of hydrogen that escapes $= \frac{2593}{22800} \times 0{\cdot}09 = 0{\cdot}0102$ gm.

4. What is the height of a barometer when a milligramme of air at 27°C. occupies a volume of 25 c.c. in a tube over mercury, the mercury standing 70 cm. higher inside the tube than outside? (1 litre of dry air at N. T. P. weighs 1·293 gm.)

A. Let the required height of the barometer $= h$ cm.

$\therefore$ The pressure of the air in the tube over mercury $= (h - 70)$ cm.

$\therefore$ Vol. of 1 milligram of air at this pressure and 27°C = 25 c.c.

Again the volume of 1·293 gm. at N. T. P. = 1000 c.c.

$\therefore$ The volume of 1 milligram of air at N. T. P. $= \frac{1000}{1293}$ c.c.

Hence from the relation $\frac{PV}{T} = \frac{P'V'}{T'}$ or $\frac{(h - 70) \times 25}{27 + 273} = \frac{76 \times 1000}{273 \times 1293}$

whence $h - 70 = 2{\cdot}58$ cm. or $h = 72{\cdot}58$ cm.

5. The density of dry air at N. T. P. is 0·001293 gm. per c.c. At what temperature will a litre of dry air weigh 1 gm. at a pressure of 75 cm.?

Let the required temperature be t°C. and let ρ be the density of dry air at t°C. at a pressure of 75 cm.

Then from $\frac{P}{\rho T} = \frac{P'}{\rho' T'}$ we have,

$$\frac{76}{0{\cdot}001293 \times 273} = \frac{75}{\rho(t+273)} \quad \text{whence } \rho = \frac{75 \times 0{\cdot}001293 \times 273}{76(t+273)}$$

$$\therefore \frac{75 \times 0{\cdot}001293 \times 273 \times 1000}{76(t+273)} = 1 \text{ whence } t = 75{\cdot}35°\text{C}.$$

***40. Kinetic Theory of Gases.**—A mass of gas consists of a vast assemblage of molecules, which are extremely small in dimension. An idea of smallness of a molecule may be made by imagining the molecules to be arranged side by side just as a row of balls of equal size. If this were so, then a hundred million molecules could be packed in a length of one centimetre. In fact, in proportion to the volume occupied by a quantity of gas, the volume occupied actually by the molecules is so small that there is enough of space around any molecule for its free movement. The space between any two molecules is called the *intermolecular space* and it is in these spaces that the molecules keep on moving continuously in different directions. Fig. 34 represents a cubical box having walls A, B and C and containing a number of molecules as shown by black dots. They are moving in various directions shown by arrow-heads. It is apparent that due to such motions some might collide with each other, while some other might strike the walls.

Molecules, however tiny they might be, have some mass and so they possess certain amount of inertia. Hence a molecule continues to move in a straight line until it knocks against the walls of the containing vessel or with another molecule. It is only when a collision occurs that there is a deflection of the path of a molecule. A large number of knocks thus occur every second against the walls of the containing vessel, and this continual hail storm of particles tends to push the wall out. This results in a *steady pressure*, which is the pressure exerted by the gas. The kinetic energy possessed by the molecules by virtue of being in continuous motion manifests itself as the temperature of the gas. Such an idea regarding the mechanism of gas pressure was advanced by Joule in 1853, which was later on modified by Maxwell.

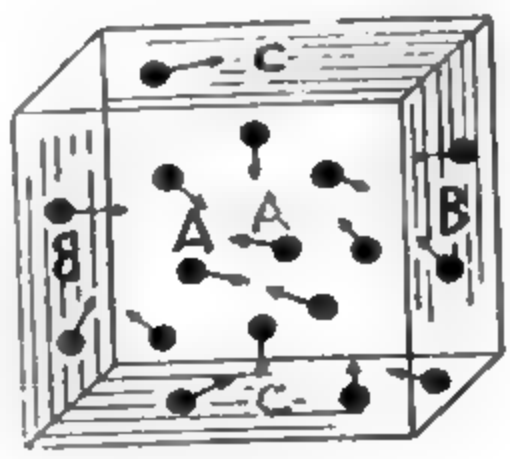


Fig. 34

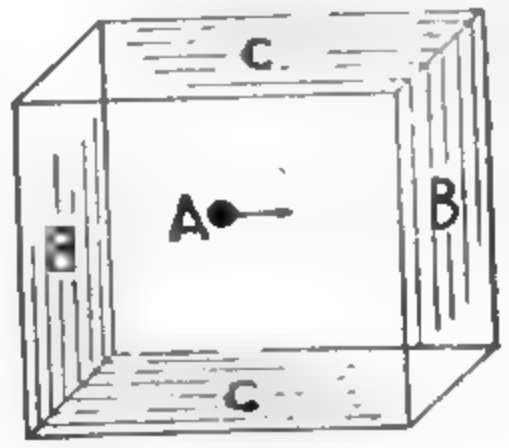


Fig. 35

Let us suppose that there are N molecules enclosed in a cubical box, the length of each edge being a. For simplicity let us assume that all the molecules have an equal speed c and each one of them behaves as a perfectly elastic sphere of mass m. Since the number of molecules is extremely large, we can imagine that at any instant

one-third of the total number of molecules move perpendicular to the pair of faces A, one-third to faces B and one-third to faces C. Take the case of a molecule which is moving at right angles to the pair of faces BB (Fig. 35) Starting from any wall of the pair, it must travel a distance $2a$ before impinging on the same wall. Hence the time taken between any two consecutive hits on the same wall is $2a/c$ and so the number of hits per second on any wall is $c/2a$.

Again due to impact on any wall, the velocity of a molecule changes from $+c$ to $-c$. Thus change in velocity is $2c$ considering the impact to be perfectly elastic and the change of momentum of any molecule for one impact is $2mc$. In unit time the change of momentum for a *molecule* is

$$\frac{c}{2a} \times 2mc = \frac{mc^2}{a}$$

Since we assume that $\frac{1}{3}$N molecules move perpendicular to any pair of faces, the rate of change of momentum for such molecules is

$$\tfrac{1}{3}\mathrm{N}\frac{mc^2}{a}$$

which represents the force on an area a^2.

Thus pressure $p = \dfrac{\text{force}}{\text{area}} = \tfrac{1}{3}\dfrac{\mathrm{N}mc^2}{a^3} = \tfrac{1}{3}\dfrac{\mathrm{N}mc^2}{\mathrm{V}}$ ... (40,1)

where V represents the volume of the chamber.

But Vc c. of the gas contains N molecules, each of mass m. Thus we have $\dfrac{\mathrm{N}m}{\mathrm{V}} = \rho$, where ρ is the density of the gas.

$$\text{or,} \qquad \mathrm{P} = \tfrac{1}{3}\rho c^2 \qquad \text{... (40,2)}$$

Boyle's Law and Kinetic Theory of Gases—If the mass of a gas be compressed into a smaller volume, while the average kinetic energy of the molecules remains unchanged, a larger number of impacts takes place per second against the walls of the containing vessel, causing an increase of pressure If the volume is increased, the number of impacts per second naturally decreases and so the pressure decreases. At constant temperature, c^2 remains constant and so from eqn. (40,2), P ρ is a constant But since ρ varies as 1/V, PV *is constant*. This is an interpretation of Boyle's law from the standpoint of kinetic theory. If M be the mass of a gas occupying a volume V at the pressure P, then since $\rho = \mathrm{M}/\mathrm{V}$, eqn. (40,2) may be written in the form.

$$\mathrm{PV} = \tfrac{1}{3}\mathrm{M}c^2 \qquad \text{... (40,3)}$$

Physical Interpretation of the Absolute zero—If there are N molecules, each of mass m, contained in a volume V of a gas and if c^2 be their mean square velocity then the mean *kinetic energy* of this volume of the gas is $\frac{1}{2}\,\mathrm{N}mc^2 = \frac{1}{2}\mathrm{M}c^2$. But from eqns (40,3)

$$\text{Kinetic Energy} = \tfrac{1}{3}\mathrm{M}c^2 = \tfrac{3}{2}\,\mathrm{PV} = \tfrac{3}{2}n\mathrm{RT} \qquad \text{... (40,4)}$$

Hence kinetic energy of a given mass of gas is proportional to the absolute temperature. Thus at the absolute zero degree the kinetic

energy possessed by the molecules tends to vanish. This may be taken as the *physical interpretation* of the Absolute Zero.

JOSEPH LOUIS GAY-LUSSAC (1778—1850)

Gay-Lussac was born on December 6, 1778, at Limousin, in France. His father was a judge but during French revolution he was sent to prison for some time. Due to such internal troubles in the country, Joseph was sent to the Polytechnic school at Paris, rather late at the age of seventeen. To give monetary relief to his father, he used to earn money by teaching lower class boys during day time and attend night classes. In 1798 he came out from the school as an engineer but with a strong passion for Chemistry.

Gay-Lussac then became a research student under Berthollet and in a few years he discovered the law of expansion of gases with temperature. On two occasions he ascended to altitudes of 13,000 ft. and 23,000 ft. to study the variation of magnetic fields and temperature. He also formulated the theory of temperature variation with altitude. In 1809 he was made the Professor of Chemistry at the Polytechnic. During his researches on Chemistry, he discovered iodine, cyanogen and prussic acid. He died on May 9, 1850.

Summary

Boyle's Law.—*At constant temperature, the volume of a given mass of any gas varies inversely as the pressure to which it is subjected.*

$V \propto \frac{1}{P}$ where t is constant, whence $PV =$ a constant.

Charles' Law.—*At a constant pressure, the volume of a given mass of any gas increases for each degree centigrade rise of temperature by a constant fraction* $(\frac{1}{273})$ *of its volume at 0°C.*

$$V_t = V_o \ (1 + \gamma_v t)$$

Law of Pressure.—*The pressure of a given mass of any gas at constant volume increases for each degree centigrade rise of temperature by a constant fraction* $(\frac{1}{273})$ *of its pressure at 0°C.*

$$P_t = P_o \ (1 + \gamma_v t)$$

Gas Thermometers—A *constant pressure* as also a *constant volume thermometer* may be used as an instrument for measuring temperatures.

Absolute Scale of Temperature.—The scale in which temperatures are reckoned from the absolute zero, *i.e.*, from — 273°C. as the zero is known as the Absolute Scale of temperature.

Charles' Law and the Law of Pressure expressed in different forms.—If V be the volume of a mass of a gas at pressure P and absolute temperature T.

$V/T =$ a constant when P is kept constant. (*Charles' Law*).

$P/T =$ a constant, when V is kept constant. (*Law of Pressure*).

The gas equation (*Gas laws combined*).

$\frac{PV}{T} = \text{constant}$; $PV = nRT$, when R is called the *gas constant.*

In case of real gases Van der Waals' Equation holds good *viz.*,

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

Exercises on Chapter IV

Reference

Art. 82 1. A flask with a capacity of 500 c.c. is heated to 87°C. at atmospheric pressure and then corked up. It is then inverted, immersed in water at 24°C. and the cork is removed. What weight of water will enter the flask if the pressure remains the same as above? Density of water at 24°C = 0·9973.

A. 87·26 gm.

Art. 32 2. Calculate the weight of 1000 litres of air at 120 atmospheric pressure at 30°C. given a litre weighs 1·3 gm. at 0°C. under one atmosphere. (Dac. U.—1940)

A. 140·55 kg.

Art. 88 3. Describe an experiment to find co-efficient of expansion of a gas at constant pressure. (C. U.—1940, '49; Pat. U.—1932; Nag. U.—1951)

Art. 83 4. Explain how thermal expansion of air can be utilised as a convenient means of measuring temperature. What are the advantages and disadvantages of a gas thermometer. (Gau. U.—1951)

Art. 35 5. Describe the constant volume air thermometer and explain how you will use it to find the temperature. (P. U.—1951; E. P. U.—1952; R. P. B.—1944, '46; Pat. U.—1923; *Cf.* Del. H. S.—1949)

Art. 85 6. Describe a form of an air thermometer and explain how this could be used to determine the melting point of napthalene. Why have gas thermometres be chosen as standards? (R. P. B.—1941)

7. The pressure in the bulb of a constant volume air themometer is 74 cm. at 0°C, 101·2 cm. at 100°C and 52·8 cm. when the bulb is surrounded by solid carbon dioxide. Calculate the temperature of solid carbon dioxide. (Del. H. S.—1949)

A. −77·9°C.

Art. 85 8. What is meant by Absolute temperature? Find the value of Absolute zero on the Fahrenheit Scale.
(Gau. U.—1951; C. U.—1938, '49)

A. −459·4°F.

Art. 86 9. Establish $PV = RT$ for a perfect gas. (E. P. U.—1953; R. P. B.—1945, '49; C. U.—1950; U. P. B.—1951, '53; Mys. U.—1952; Nag. U.—1952, '53)

Art. 86 10. Calculate the value of the constant for carbon dioxide given that 22·4 litres of the gas at N. T. P. weighs 44 gm.
(U. P. B.—1947; R. P. B.—1945)

A. $8{\cdot}32 \times 10^7$ ergs/degree.

Reference

Art. 86 **11. A gas at 13°C. has its temperature raised, so that its volume is doubled, the pressure remaining constant. What is its final temperature?** (Dac. U.—1932)

A. 299°C.

Art. 86 **12. A bicycle is standing in shade where the temperature is 86°F. It is then taken out into the sun where the temperature is 140°F. Find the percentage increase of pressure within the tyre.**

A. 9·9%.

Art. 86 **13. A litre of hydrogen at N. T. P. weighs 0·9 gm. What is the weight of a litre of this gas at 27°C and 75 cm. pressure** (E. P. U.—1952)

A. ·8 gm.

Art. 86 **14. One litre of air at N. T. P. weighs 1·293 gm. Find the value of R for air. (g = 980 cm./sec.²; density of mercury = 13·6).** (U. P. B.—1951)

A. $8{\cdot}03 \times 10^7$ ergs/°C.

Art. 86 **15. A vessel of capacity 2100 litres contains gas at N. T. P. The vessel is heated to 26°C. but there is being a slight leak, the pressure remains constant. If the original mass of the gas were 184 gm. find the mass of gas remaining in the vessel. The volume of the vessel may be taken as unchanged.**

A. 168 gm.

Art. 86 **16. The measurement of a room is 10 ft.× 30 ft. × 25 ft. If the temperature of the room be increased from 20°C to 25°C, calculate what percentage of the original volume of air will be expelled from the room, the pressure remaining constant.** (U. P. B.—1949)

A. 17%.

Art. 86 **17. A barometer tube containing a little air at 27°C. is fixed inverted over deep cistern of mercury. The height of the mercury column in the tube is 61 cm. from the mercury surface in the cistern, while the barometer reads 76 cm. Find by how much the level of mercury in the cistern must be raised, so that the position of the mercury meniscus in the tube may remain the same when the temp. of the closed air is raised to 87°C.**

A. 3 cm.

Art. 89 **18. A corked bottle contains air under a pressure of 770 mm. of mercury at a temperature of 20°C. It stands near a fire and the temperature rises to 200°C. If the area of the cork exposed to the air in the bottle be 9 sq. cm. what will be the force on the cork urging it out? (g = 980 cm. per sec.² density of mercurry = 13·6).**

A. Force = pressure × area-= $1{\cdot}49 \times 10^7$ dynes.

Art. 86 **19. Given that the universal gas constant $R = 8{\cdot}315 \times 10^7$ ergs per gramme-molecule per centigrade degree, calculate the volume occupied by 6 gm. of oxygen at— 30°C. and 90 cm. Find also the molecular weight of a gas 60 gm. of which occupy a volume of 10 litres at 200°C. and 36 cm.**

A. 8150 c.c.; 491.

Art. 86 **20. Calculate the change in volume during expansion of 1 gm. of hydrogen when heated from 25°C to 26°C. against the atmospheric**

Reference

pressure of 1 million dynes per cm.^{2} Given gas constant $R = 8{\cdot}3 \times 10^7$ ergs./gm. molecule/degree centigrade. (R. P. B.—1951)

A. 318 c.c.

Art. 36 21. A cylinder contains 6 cu. ft. of gas at a pressure of 15 lb. wt. per sq. in. and 20°C. The amount of the gas in the cylinder is kept constant and the pressure is changed to 150 lbs. wt. per sq. in. What should be the temperature of the gas if the volume is found to be 2·5 cu. ft.?

A. 947·8°C.

Art. 36 22. A vessel contains 1000 litres of a gas at 0°C and at 740 mm. pressure. Find the pressure of the gas when temperature rises to 27°C. (U. P. B.—1953)

A. 813·2 mm.

Art. 36 23. 20 c.c. of hydrogen are at pressure equal to the pressure at 50 cm. below the surface of water exposed to the atmosphere when the barometer reads 75 cm. The temperature of the hydrogen is 27°C.; what would be the volume of the hydrogen under N. T. P.? (density of mercury = 13·6).

A. 18·84 c.c.

Art. 36 24. A litre of air at 10°C. is heated until both volume and pressure are doubled. What is the temperature then?

A. 859°C. (C. U.—1947)

Art. 36 25. A litre of oxygen at N. T. P. weighs 1·429 gm.; find the mass of 10 litres of oxygen at — 35°C. and 77 cm. pressure.

A. 16·59 gm.

Art. 37 26. Prove that for a perfect gas the volume and the pressure coefficient are equal. (Nag. U.—1953; R. P. B.—1940; U. P. B.—1947; Del. U.—1952)

Art. 37 27. Compare the density of air at 10°C and 750 mm. pressure with its density at 15°C and 760 mm. pressure.

A. 5400 : 5377.

Art. 37 28. On a certain day the barometer reads 76 cm. and the temperature is 10°C. On being taken to the bottom of a mine shaft, where the temperature is 27°C. the barometer reading increases by 4 cm. Find the ratio of the density of the air at the bottom of the shaft to that of air on the ground level.

A. 0·993 : 1.

Art. 38 29. Describe the standard constant volume hydrogen thermometer. Explain how it may be used to measure temperatures. Mention the advantages of a permanent gas over mercury as a thermometric substance. (R. P. B.—1952)

Art. 40 30. Write a note on behaviour of vapour towards Boyle's law and Charles' law of gases. (Raj. U.—1953)

Art. 40 31. How do you account for the pressure of a gas in a closed space and on what factors does it depend? (Pat. U.—1932)

Art. 40 32. Write short notes on the molecular motion in gases. (C. U.—1949)

CHAPTER V

CALORIMETRY

41. Quantity of Heat: Unit of Heat.—It has already been stated the heat is a form of energy and as such it is a *measurable quantity*. Thus when a body is heated, it gains heat from the heater and when cooled it parts with heat to the neighbouring bodies. Again a given mass of a substance requires a certain quantity of heat for a certain rise of temperature. If the mass taken is twice, the quantity of heat required for the same rise of temperature is also twice; here two quantities of heat are added. Hence quantities of heat obey arithmetical laws of addition and subtraction.

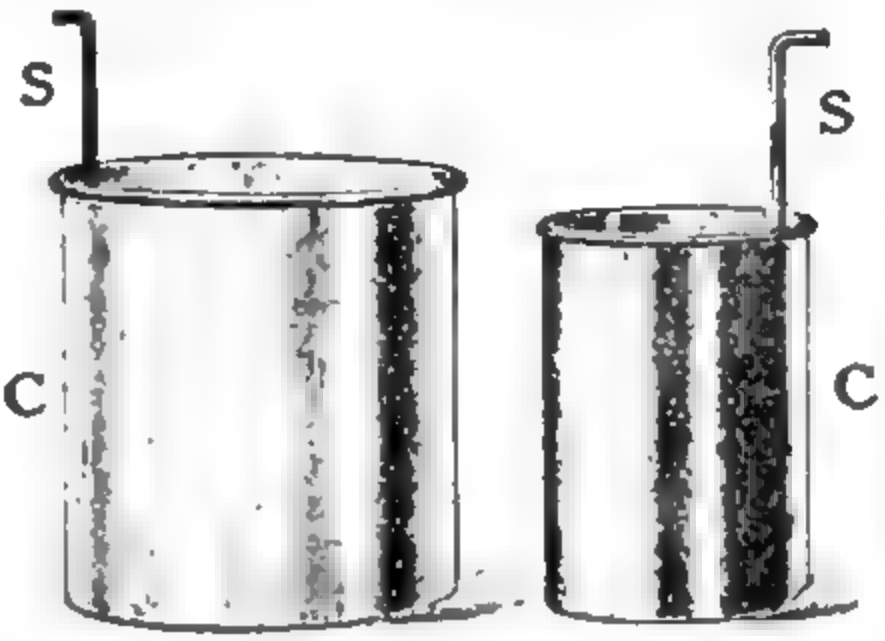


Fig. 36—Calorimeters

Calorimetry deals with the measurement of quantities of heat. The vessel which is largely used in such measurement is known as a *calorimeter*. It is a cylindrical vessel C usually made of copper and provided with a stirrer S of the same material. Fig. 36 represents two calorimeters of different sizes.

The **unit of heat** is taken to be *the quantity of heat required to raise one gramme of pure water through* 1°C. This unit is called the **calorie** or sometimes gramme degree calorie.

It is found experimentally that the quantity of heat required to raise one gramme of water through 1°C. is not *exactly* the same all over the scale of temperature. It decreases from 0°C. to a minimum value of about 40°C. and then increases again. As the variation over the range 0°C to 100°C.is extremely small, the difference need not be taken into consideration unless a great accuracy is required.

Although a calorie is the unit most commonly used, other units are sometimes employed. The Continental Engineers' Unit is the heat required to raise 1 kgm. of water through 1°C which is termed the *kilogram calorie* The British Thermal Unit (B. Th. U.) is the quantity of heat required to raise 1 lb. of water through 1°F. The *Therm* is a special unit used by the gas companies and is the heat required to raise 1000 lb. of water through 100°F. and is so equal to 100,000 B.Th. Units. The heat required to raise 1 lb. of water through 1°C. is also used as a unit in Great Britain and is known as the *Centigrade unit* (lb. deg. C.).

The quantity of heat required to raise m gm. of water through t°C. is, therefore, mt calories. Conversely, mt calories of heat are given out when m gm. of water cool through t°C.

Since 1 lb. = 453·6 gm. and 1°F. = $\frac{5}{9}$ of 1°C.

1 B. Th. U. = (453·6 × $\frac{5}{9}$) calories = 252 calories.

1 pound degree Centigrade = 1·8 B. Th. U. = 453·6 calories.

Principle of Measurement of Heat.—If two bodies at different temperatures be mixed, there will be a sharing of the heat possessed by the bodies until they attain a common temperature. The heat given out by one body during the period of its cooling is taken up by the other body during the same period in reaching the common temperature. If it is assumed that no heat is received from or given to any body outside the system and if there be no chemical action in the mixture, we may put,—

Heat lost = Heat gained

42. Specific Heat.—It is found that equal masses of the *same* substance require the same quantity of heat to be heated through the same range of temperature. Conversely, it is also true that equal masses of the same substance give out the same quantity of heat when cooled through the same range of temperature. But equal masses of *different* substance take in (or give out) *different* quantities of heat when heated (or cooled) through the same range of temperature. This can be shown by a simple experiment.

Take a number of small balls of different materials, say lead, tin, copper and iron and all of the *same* mass. Heat them to the same temperature either in steam or in any suitable bath. Remove the balls quickly by means of strings attached to them and place them simultaneously on a thick cake of paraffin wax (Fig. 37). Observe that each ball melts different amount of the wax and may consequently sink to different extent into the cake. It is clear that different balls, although of the same mass, give out different quantities of heat in cooling through the same range of temperature, namely, from the temperature of the bath to that at which wax melts.

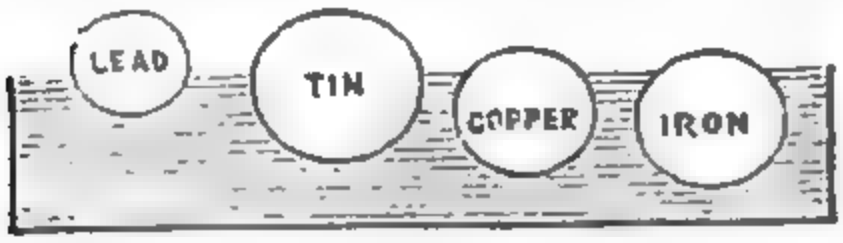


Fig. 37

The above fact is due to a difference in some specific property of the materials of the balls and is expressed by saying that different materials possess different *specific heats*.

The specific heat of a substance is *the ratio of the quantity of heat required to raise a given mass of the substance through any range of temperature to the quantity of heat required to raise an equal mass of water through the same range of temperature.*

Let s be the specific heat of a substance of mass m gm. Then,

$$s = \frac{\text{Heat reqd. to raise } m \text{ gm. of the substance } 1°\text{C.}}{\text{Heat required to raise } m \text{ gm. of water } 1°\text{C.}}$$

$$= \frac{\text{Heat reqd. to raise 1 gm. of the substance } 1°\text{C.}}{\text{Heat required to raise 1 gm. of water } 1°\text{C.}}$$

$$= \frac{\text{Heat reqd. to raise 1 gm. of the substance 1°C.}}{\text{1 calorie}}$$

Hence we can also state that *the specific heat of a substance is equal to the number of units of heat required to raise 1 gm. of the substance through* 1°C.

Thus, by the statement that the specific heat of copper is 0·1, we mean that the heat required to raise 1 gm. of copper through any range of temperature is $\frac{1}{10}$th of the quantity of heat required to raise 1 gm. of water through the same range of temperature ; in other words, the heat required to raise 1 gm. off copper through 1°C. is one-tenth of a calorie. As the specific heat is taken as a ratio, it is a mere **number** and is independent of the unit of heat chosen. However, the specific heat is sometimes expressed in calories per gm. per degree centigrade.

From the definition of specific heat,
heat required to raise m gm. of a substance through t°C.
$= s \times$ heat required to raise m gm. of water through t°C.
$= s \times mt$ calories $= mst$ calories.

Conversely, heat given out by m gm. of the substance in cooling through $t°C = mst$ calories. Similarly, the heat reqd. to raise m lb. of the substance through t°F. $= mst$ B. Th. U. Also the heat given out by m lb. of the substance in cooling through t°F $= mst$ B. Th. U.

∴ Heat taken in or given out.

$=$ ***Mass* × *Sp. ht.* × *Difference of Temperatures.***

43. Thermal Capacity.—The thermal capacity or the *capacity of heat* of a body is the quantity of heat required to raise the temperature of the body through 1°C.

Let the mass of a body $= m$ gm.
the sp. ht. of the material of the body $= s$
Then thermal crpacity of the body per 1°C $= ms$ calories.

Hence the Specific Heat of a substance is sometimes defined as its *Thermal capacity per unit mass* and is referred to as the *specific thermal capacity* of the substance.

Further, the *thermal capacity per unit volume* is the amount of heat required to raise unit volume of the substance through 1°C. and is, therefore, equal to the product of the density and the specific heat.

The **water equivalent** of a body is the quantity of water in grammes which would be raised through 1°C. by the amount of heat required to raise the body through 1°C.

Let the mass of the body $= m$ gm.
and the sp. ht. of the material of the body $= s$
Then heat reqd. to raise the temp. of the body 1°C $= ms$ calories

This quantity of heat will raise the temperature of ms gm. of water through 1°C.

∴ The water equivalent of the body $= ms$ gm.

Hence the *numerical portion* of the expression for the thermal capacity and the water equivalent of a body is the same.

Thus, for a body of mass 100 gm. and made of copper ($s = 0{\cdot}1$)

the water equivalent $= 100 \times 0{\cdot}1 = 10$ gm.

and the thermal capacity $= 100 \times 0{\cdot}1 = 10$ calories.

44. Regnault's Calorimeter.—In experiments on measurement of heat, a hot and a cold body at *known* temperatures are very often mixed together in a calorimeter and when the exchange of heat is complete, the resulting temperature is found. For this purpose an apparatus, originally designed by Regnault, is very useful. It consists

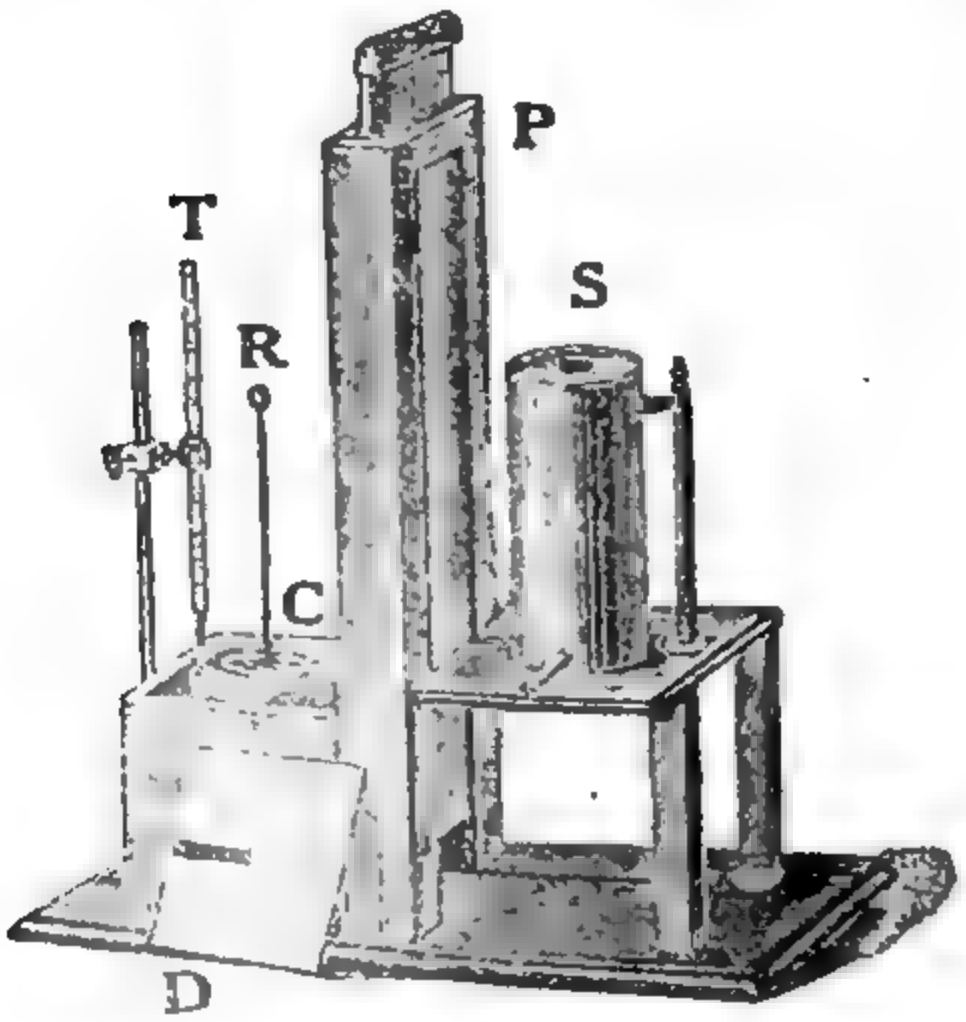


Fig. 38

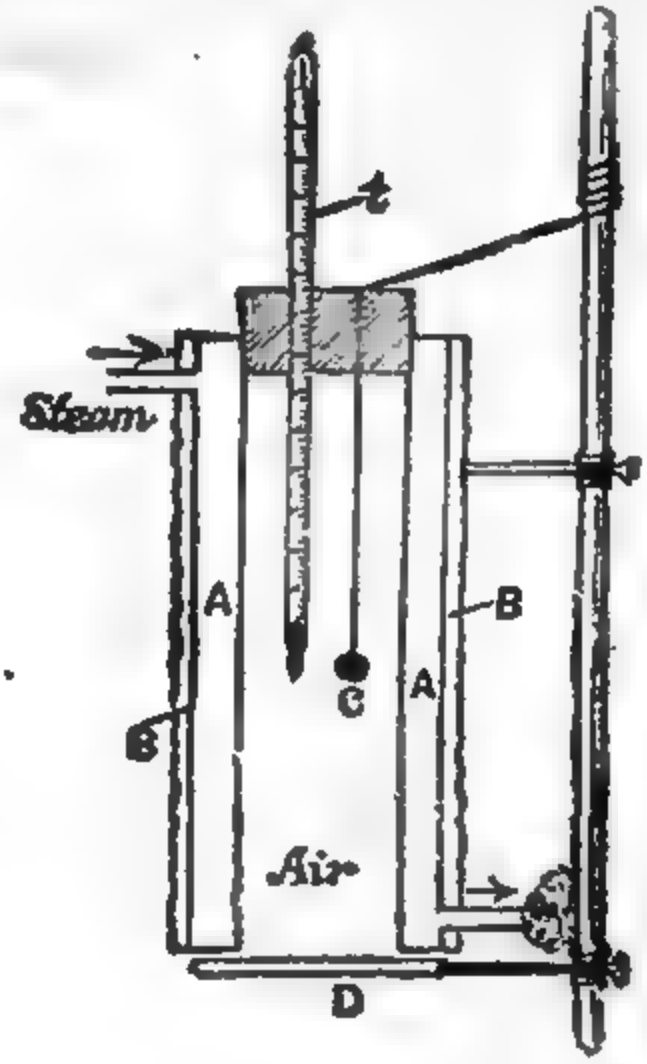


Fig. 39

of a calorimeter C placed on a non-conducting support inside a larger wooden vessel (Fig. 38). The calorimeter is provided with a stirrer R with which any liquid within it may be well stirred. A thermometer T fixed with a stand can be lowered into the calorimeter to read the temperature of its contents. The calorimeter can be covered with a suitable lid D with a slit for the insertion of thermometer and stirrer. The object of placing the calorimeter in an all round closed chamber is to prevent as much as possible the loss of heat from it.

A sliding gate P stands as a partition wall between the calorimeter and the steam-heater S. Whenever necessary, the gate may be slided up and the chamber containing the calorimeter may be brought below the stand just under the steam heater.

Steam Heater.—The steam heater S is a mechanism in which a small body may be heated by steam not by direct contact. It is represented in section in Fig. 39. It consists of two co-axial hollow metal cylinders A, B, usually of copper, the air space between them

being closed at both the ends. Two pipes, serving as the inlet and the outlet, are connected to the annular space. A boiler containing water is connected with a rubber tubing to the inlet pipe at the top. On heating, steam passes through the space between the two cylinders and escapes through the outlet pipe at the bottom. The inner cylinder is closed by a cork at the top and by a movable shutter O at the bottom. The body C is suspended inside the heater by a thread passing through a hole in the cork and the thermometer t is inserted through another hole, with its bulb close to the solid. The solid is thus indirectly heated by the steam.

45. Determination of the Water Equivalent of a Calorimeter.— Take a clean, dry calorimeter with a stirrer. Weigh the calorimeter with the stirrer. Fill the calorimeter more than half with cold water and weigh again; the difference between the two weights gives the mass of water taken. Note the temperature of the water by putting a thermometer into it. Place the calorimeter with its contents on non-conducting support inside a larger vessel, the thermometer being held immersed in water by a suitable clamp. The inside surface of the calorimeter and the inner surface of the surrounding vessel should be highly polished and the intervening space packed with cotton wool.

Boil some water in a vessel with a thermometer immersed in it. Note the temperature of the water and quickly pour off some quantity of it into the calorimeter. Stir the mixture well and note final temperature. Allow the calorimeter to cool and then weigh it again with its contents. The difference between this and the second weight gives the mass of hot water added.

Let water equivalent of the calorimeter (+ stirrer) $= W$ gm.

mass of cold water taken $= m$ gm.

initial temperature of water and calorimeter $= t_1$°C.

temperature of the hot water added $= t_2$°C.

final temperature of the mixture $= t$°C.

mass of hot water added $= m'$ gm.

Heat lost by m' gm. of water in cooling from t_2°C to t°C $= m'(t_2 - t)$ cal.

Heat gained by m gm. of water in rising from t_1°C. to t°C. $= m(t - t_1)$ cal.

Heat gained by calorimeter and stirrer in rising from t_1° to t°C $= W(t - t_1)$ cal.

Assuming that no heat is exchanged with any outside body

Heat lost = Heat gained

$$\therefore \quad m'(t_2 - t) = m(t - t_1) + W(t - t_1)$$

or $$W(t - t_1) = m'(t_2 - t) - m(t - t_1)$$

$$\therefore \quad W = m'\frac{t_2 - t}{t - t_1} - m$$

whence W can be calculated. The result can be verified from the relation, W = M.s., where M is the mass of calorimeter and s is the specific heat of the material of the calorimeter.

Examples :—

1. A calorimeter contains 30 gm. of water at 28°C. A mass of water at 57°C. is poured into the calorimeter and the contents are well stirred after which the resulting temperature is found to be 43°C. The mass of the water added is found to be 44 gm. Find the water-equivalent of the calorimeter.

A. Let the water equivalent of the calorimeter be W gm.

Heat lost by hot water $= 44 \times (57 - 43)$ calories.

Heat taken by calorimeter + water in it $= (W + 30) \times (43 - 28)$ calories.

$\therefore$ $44 \times (57 - 43) = (W + 30)\ (43 - 28)$ whence $W = 11{\cdot}07$ gm.

2. A Dewar flask is filled about three-quarters full of water at a high temperature. The water is rolled about the flask until a uniform temperature of 95°C is attained. The hot water is emptied away and 50 gm. of cold water at 15°C. are poured in. After shaking the resulting temperature is observed to be 31°C. Calculate the water equinalent of the flask. (Gau. U.—1952)

A. A Dewar flask is a contrivance wherein either a hot or a cold substance can be preserved without any appriciable loss or gain of heat from surrounding atmosphere (Art. 114). The object of rolling hot water inside the flask is to raise to temperature. When hot water is thrown away, the temperature of the flask is found to be 95°C, which is thus its *initial high temperature* when empty. If W be the water equivalent of the dask, then the heat lost by the flask $= W\ (95 - 31)$ cals.

Again, heat gained by 50 gm. of cold water in rising from 15°C to 31°C $= 50 \times (31 - 15)$ cals.

whence $W = 12{\cdot}5$ gm.

In determining **thermal capacity** of a body, the body need not be weighed but it is simply heated to a known high temperature and then dropped into a calorimeter containing a known mass of water.

3. A piece of lead at 99°C. is placed in a calorimeter containing 200 gm. of water at 15°C. The temperature after stirring is 21°C. The calorimeter weighs 40 gm. and is made of a material of specific heat 0·1. Calculate the thermal capacity of the piece of lead.

A. Let the required thermal capacity of the piece of lead be W calories.

Heat lost by lead in cooling from 99°C. to 21°C. $= W\ (99 - 21)$ cal.

Heat gained by the calorimeter $= 40 \times 0{\cdot}1 \times (21 - 15)$ cal.

Heat gained by the water in the calorimeter $= 200 \times (21 - 15)$ calories.

$W \times (99 - 21) = (200 + 4) \times (21 - 15)$ whence $W = 15{\cdot}7$ colories nearly.

4. The densities of two substance are as 2 : 3, and their specific heat are 0·12 and 0·09 respectively. Compare the thermal capacities per unit volume. (C. U.—1922 34)

A. Let densities of two substances respectively $= 2\rho$ and 3ρ

Then mass per unit volume of the first substance $= 2\rho$ gm.

And mass per unit volume of the second substance $= 3\rho$ gm.

Hence thermal capacity per unit vol. of the first substance $= 2\rho \times 0{\cdot}12$ cal.
And thermal capacity per unit vol. of second substance $= 8\rho \times 0{\cdot}09$ cal.

$$\therefore \quad \frac{\text{Thermal capacity per unit vol. of the first substance}}{\text{Thermal capacity per unit vol. of the second substance}} = \frac{2\rho \times 0{\cdot}12}{8\rho \times 0{\cdot}09} = \frac{8}{9}$$

46. Determination of the Specific Heat of a Solid.—*By the Method of Mixture.*—Take a clean, dry calorimeter and weigh it with a stirrer of the same material. Pour a suitable quantity of water into the calorimeter and weigh it again ; the difference between these two weights gives the mass of water taken. Note the temperature of the water.

Take a suitable piece of the soild whose specific heat is to be determined and weigh it. Heat the soild in a steam heater. Note the temperature of the inside space of the steam heater when it has become steady. Then bring the calorimeter under the heater, move away the shutter D, quickly drop the solid into the calorimeter and immediately remove the calorimeter. Stir the water well and note the final temperature of the contents of the calorimeter.

Let mass of the calorimeter and stirrer $= W$ gm.
sp. ht. of the material of the calorimeter $= s'$
mass of water taken $= m$ gm.
initial temperature of water and ca lorimeter $= t_1$°C
mass of solid $= M$ gm.
steady temperature of the solid in this heater $= t$°C.
final temperature of the mixture $= t_2$°C
The required specific heat of the solid $= s$.

Heat lost by the solid in cooling from t°C. to t_2°C.
$= Ms(t - t_2)$ calories.

Heat gained by calorimeter, stirrer and water in rising from t_1°C. to t_2°C $= Ws'(t_2 - t_1) + m(t_2 - t_1)$ calories.

Assuming that no heat was exchanged with any body outside.

Heat lost = Heat gained

$$\therefore \quad Ms(t - t_2) = Ws'(t_2 - t_1) + m'(t_2 - t_1)$$

whencee s can be calulated.

Sources of Errors and Precautions :—

(1) Some heat is lost by the solid in being transferred from the heater to the calorimeter which makes the observed specific heat *too low.* This is minimised by dropping the hot solid directly into the calorimeter by bringing the latter under the heater.

(2) Some heat passes by *conduction* and *radiation* from the calorimeter to surrounding media when the former is at higher temperature than the latter. This will make the observed specific heat *too low* and can be minimised in the following way :—

(*i*) The calorimeter is placed on *non-conducting* supports inside a deeper vessel the intervening space being sometimes packed with cotton-wool or some other *bad conducting* material, so that loss of heat by *conduction* is made very little.

(*ii*) The outer surface of the calorimeter and the inner surface of the outer vessel are *polished*, which minimises loss of heat by *radiation*.

The water in the calorimeter previously cooled as much below the room temperature as the final temperature is above it. By this means the average temperature of the calorimeter and its contents is nearly the same as the room temperature, so that the gain of heat by the calorimeter and its contents from outside due to *radiation* during the first half of the experiment is practically compensated by the loss during the second half. This is known as Rumford's *compensation method*. A detailed process of correcting for radiation is given in Intermediate Practical Physics (All India Edition) by J. Chatterjee.

(3) The changes of temperature should be very accurately and carefully determined by a very *sensitive thermometer*.

(4) In the case of a badly conducting solid, it should be broken into small fragments, so that the different pieces will be raised to the same temperature throughout the mass.

Examples :—

1. A body of mass 100 gm. at 120°C. is plunged into 300 gm. of water at 20°C. contained in a copper calorimeter of mass 50 gm. The final temperature attained is 30°C. Find the specific heat of the material of the body. (Sp. ht. of copper = 0·09).

A. Let the required specific heat be s.

Heat lost by the hot body in cooling from 120°C. to 30°C.

$$= 100 \times s \times (120 - 30) \text{ cal.}$$

Heat gained by calorimeter + water in warming from 20°C. to 30°C.

$$= [50 \times 0{\cdot}09 + 300] \ (30 - 20) \text{ cal.}$$

Since Heat lost = Heat gained

or $9000s = 3045$ whence $s = {\cdot}34$

2. Equal volume of mercury and glass have the same capacity for heat. Calculate the specific heat of a piece of gloss of specific gravity 2·5, if the specific heat of mercury is ·033 and its specific gravity 13·6.

A. Let the sp. ht. of glass be s and the vol. of glass piece be V c.c.

$\therefore$ Mass of the glass piece $= 2{\cdot}5$ V gm.

Also mass of mercury taken $= 13{\cdot}6$ V gm.

Hence capacity for heat of V c.c. of glass $= 2{\cdot}5 \text{ V} \times s$

Also capacity for heat of V c.c. of mercury $= 13{\cdot}6 \text{ V} \times {\cdot}033$

$\therefore$ $2{\cdot}5 \text{ V} \times s = 13{\cdot}6\text{V} \times {\cdot}033$ whence $s = 0{\cdot}18$

47. Determination of the Specific Heat of a liquid.—(*By the method of Mixture*).—Take a clean, dry calorimeter and weigh it together with a stirrer of the same material. Take some of the given liquid in the calorimeter and weigh again. The difference between these two weights gives the mass of the liquid taken. Note the temperature of the liquid and the calorimeter.

Take a piece of a solid of *known* specific heat and weigh it ; the solid must be insoluble and chemically unaffected by the given liquid.

Heat the solid in a steam heater to a steady high temperature and note the temperature. Bring the calorimeter with the liquid under the heater, drop the solid into liquid and quickly remove the calorimeter. Stir the liquid well and note the final temperature of the contents of the calorimeter.

Let the mass of the calorimeter and stirrer $= W$ gm.
specific heat of the material of the calorimeter $= s_1$
mass of the liquid taken $= m$ gm.
initial temperature of the liquid and calorimeter $= t_1$°C.
mass of the solid $= M$ gm.
specific heat of the material of the solid $= s_2$
steady temperature of solid in the steam heater $= t$°C.
final temp. of the contents of the calorimeter $= t_2$°C.
the required specific heat of the liquid $= s$

The heat lost by the solid in cooling from t°C to t_2°C.
$= Ms_2 (t - t_2)$ calories

Heat gained by calorimeter, stirrer and the liquid in warming from t_1°C. to t_2°C. $= Ws_1 (t_2 - t_1) + ms(t_2 - t_1)$ calories.

Assuming that no heat is exchanged with any outside body,

Heat lost = Heat gained

$$\therefore \quad Ms_2 (t - t_2) = Ws_1 (t_2 - t_1) + ms (t_2 - t_1)$$

or $$ms (t_2 - t_1) = Ms_2 (t - t_2) - Ws_1 (t_2 - t_1)$$

Whence s can be calculated.

In the case of a liquid which has no *chemical action with water*, the specific heat may be determined by heating a quantity of it and then pouring it into a known mass of water in a calorimeter. If *there be a chemical action between the liquid and water*, the hot liquid may be poured directly into a thin-walled metallic vessel (preferably of a good conductor) of known water equivalent, placed in a known quantity of water contained in a calorimeter.

In the case of a *volatile* liquid, such as, alcohol, a quantity of the liquid may be enclosed in a thin-walled vessel of a good conductor, the vessel is almost filled with the liquid and through the cork a thermometer is introduced. The vessel with its contents is heated to a known temperature and then transferred into water contained in a calorimeter. The vessel itself may be used as a stirrer and the final temperature is given by the thermometer. Then proceeding as usual, the specific heat of the liquid is obtained. The specific heat may also be conveniently determined by previously cooling the liquid to known low temperature and then dropping a piece of a heated solid of given mass and specific heat directly into it.

Examples :—

1. 200 gm. of water at 98°C. are mixed with 200 c.c. of milk of density 1·03 at 30°C. contained in a brass vessel of thermal capacity equal to that of 8 gm.

of water; the temperature of the mixture is 64°C. Assuming that there is no loss of heat by radiation ,find the spacific heat of milk.

A. Let the required specific heat of milk $= s$

Mass of the milk taken $= 200 \times 1{\cdot}03$ gm.

Heat lost by water $= 200 \times (98 - 64)$ cal.

Heat gained by milk $= 200 \times 1{\cdot}03 \times (64 - 30) \times s$ cal.

Heat gained by the brass vessel $= 8 \times (64 - 30)$ cal.

$\therefore$ $200 \times (98 - 64) = (8 + 200 \times 1{\cdot}03s)\ (64 - 30)$

or $200 = 8 + 206s$ whence $s = 0{\cdot}93$.

2. The specific gravity of a certain liquid is 0·8, that of another liquid is 0·5. It is found that the heat capacity of 3 litres of the first is the same as that of 2 litres of the second. Compare their specific heats. (Pot. U.—1926)

A. Let the specific heats of the two liquids $= s_1$ and s_2 respectively.

Mass of 3 iltres of the first liquid $= 3000 \times 0{\cdot}8 = 2400$ gm.

Mass of 2 litres of the second liquid $= 2000 \times 0{\cdot}5 = 1000$ gm.

Heat capacity of the first liquid is given by $H_1 = 2400 \times s_1$.

Heat capacity of the second liquid is given by $H_2 = 1000, \times s_2$.

As $H_1 = H_2$ we have $2400s_1 = 1000s_2$, whence $s_1 : s_2 = 5 : 12$

48. **Determination of the Specific Heat of a Liquid.**—(*By the method of Cooling*)—If a given mass of liquid, heated to a known temperature, is kept in a certain vessel and allowed to cool in a enclosure at a lower temperature than the liquid, the ***rate of loss of heat*** is found to depend on (*i*) the temperature of the liquid (*ii*) the temperature of the enclosure, and (*iii*) the nature and extent of the surface of containing vessel.

The *rate of loss of heat* does not, therefore, depend on the nature of the liquide. Hence if allowed to cool in such a way that the above conditions do exactly hold good in each case, different liquids will *lose heat at the same rate*. This principle is applied in the *method of cooling*.

Take a copper calorimeter, blackened on the outside, with a stirrer of the same material and weigh the combination. Fill the calorimeter about two thirds with hot water at a temperature of 25° to 30° higher than the room temperature. Place the calorimeter on non-conducting supports inside a large copper vessel blackened on the inside. The outer vessel may be covered with a lid with small holes to introduce thermometer and the stirrer. Start a stop watch and stir the water gently and note the temperatures at intervals of one minute for about 20 minutes. Then weigh the calorimeter with its contents, when cool; the difference between this weight and that of the empty calorimeter with stirrer gives the mass of water taken.

Now throw away the water from the calorimeter. Heat a suitable quantity of the given liquid in another vessel to about the same temperature as the water previously taken and fill the calorimeter with the hot liquid upto the *same* level as before, the calorimeter being placed as before. Stir gently and note the temperatures at intervals of one minute for almost an equal range. Then weigh the calorimeter and its contents. The difference between this weight and that of empty calorimeter with stirrer gives the mass of the liquid taken. From these data draw on a graph paper time-temperature curves for the two liquids and thence find the times taken by the water and the liquid to cool through an equal range of temperature.

Let the mass of water taken $= m$ gm.

water equivalant of the calorimeter $= M_1$

time taken by water to cool from θ_1 to θ_2°C $= t_1$ sec.

mass of liquid taken $= M$, gm.

time taken by the liquid to cool from θ_1 to θ_2 $= t_2$ sec.

specific heat of the liquid $= s$

$\therefore$ Heat lost by water in cooling during t_1 sec $= m(\theta_1 - \theta_2)$

$\therefore$ Rate of loss of heat of water and calorimeter $= \dfrac{(m + M_1)(\theta_1 - \theta_2)}{t_1}$

Again heat lost by the liquid in time t_2 sec. $= Ms(\theta_1 - \theta_2)$

$\therefore$ Rate of loss of heat of liquid and calorimeter $= \dfrac{(Ms + M_1)(\theta_1 - \theta_2)}{t_2}$

Hence from the principle given above,

$$\frac{(m + M_1)(\theta_1 - \theta_2)}{t_1} = \frac{(Ms + M_1)(\theta_1 - \theta_2)}{t_1}$$

when $$s = \frac{(m + M_1)t_2}{Mt_1} - \frac{M_1}{M}$$

Example :—

1. 50 gm. of water and an equal volume of alcohol (relative density $= 0{\cdot}8$) are placed one after the other in the same calorimeter. They are found to cool from 60°C to 55°C in two minutes and one minute respectively. Find the specific heat of alcohol. The water equivalent of calorimeter is 2 gm. (U. P. B.—1951)

A. Heat lost by 50 gm. of water within a calorimeter of water equivalent 2 gm. in cooling from 60°C to 55°C in two minutes

$$= (50 + 2)\ (60 - 55) = 260 \text{ calories.}$$

$\therefore$ Rate of loss of heat from water and calorimeter $= \dfrac{260}{120} = \dfrac{13}{6}$ cal./sec.

Again 50 gm. of water occupies a volume of 50 c.c. Therefore volume of alcohol taken is 50 c.c. and its mass is $50 \times 0{\cdot}8 = 40$ gm. Let the sp. ht. of alcohol be s.

Heat lost by 40 gm. of alcohol and the same calorimeter in cooling through the same range in one minute

$$= (40s + 2)\ (60 - 55) = 200s + 10 \text{ calories.}$$

$\therefore$ Rate of loss of heat from alcohol and calorimeter $= \frac{200s + 10}{60}$ cal./sec.

Now, according to the law of cooling,

$$\frac{200s + 10}{60} = \frac{13}{6} \quad \text{whence} \quad s = 0{\cdot}6$$

The following Table supplies the specific heats of common substances.

Specific Heats of Common Substances

Substance	Sp.Ht.	Substance	Sp.Ht.	Substance	Sp.Ht.
Brass	·088	Gold	·03	Mercury	·04
Copper	·094	Glass	·16	Ice	·5
Iron	·119	Marble	·22	Mustard oil	·50
Lead	·031	Turpentine	·42	Castor oil	·44
Silver	·055	Olive Oil	·47	Bromine	·51

49. Specifie Heats of Gases.—The specific heat of a gas depends on whether the quantity of the gas is heated at constant volume or at constant pressure. Gases have, therefore two specific heats :—

The *specific heat of a gas at constant volume* (C_v) is the number of units of heat required to raise the temperatue of 1 gm. of the gas through 1°C, the volume being kept constant. The *specific heat of a gas at constant pressure* (C_p) is the number of units of heat required to raise 1 gm. of the gas through 1°C, the pressure being kept constant.

The specific heat of a gas at constant presure (C_p) *is always greater than* the specific heat of a gas at constant volume (C_v). For if unit mass, say 1 gm. of the gas is heated through 1°C, at a constant volume, a quantity of heat is required for the purpose. But if the mass is heated through 1°C, at constant pressure then in addition to the heat required to raise the temperature 1°C, some amount of heat must be necessary to supply the energy for the work done during expansion against the external pressure. The ratio of C_p to C_v in case of diatomic gases, *e g.*, nitrogen, hydrogen, *etc.*, is 1·41 and for monatomic gases it is about 1·6.

Determination of the Specific heat of a Gas at Constant Pressure (*Regnault's method*)—The specific heat of various gases at *constant pressure* was determined by Regnault. The gas is compressed in a reservoir R (Fig 40), kept in a constant temperature bath from which a steady stream of it is made to flow out through the pipe P. A valve V which regulates the amount of flow, keeps the pressure of the gas constant. The pressure is read by the manometer M. The gas then passes through a long spiral tube immersed in an oil bath H and is thus heated to known high temperature as found by the thermometer. It then passes through a copper coil immersed in water contained in a colorimeter C. The gas in circulating through the copper

spiral within the calorimeter gives up its heat to the water and it finally escapes into the air. The rise of temperature of the water in the calorimeter is noted and the mass of the gas which has come out is calculated from the change of pressure in the reservoir. On equating the heat lost by the heated gas to heat gained by the calorimeter and contents, the specific heat of the gas at constant pressure can be calculated.

If the temperature of the oil bath be T, and the initial and final temperatures of the calorimeter and contents be $t_1{}^\circ C$ and $t_2{}^\circ C$, then we

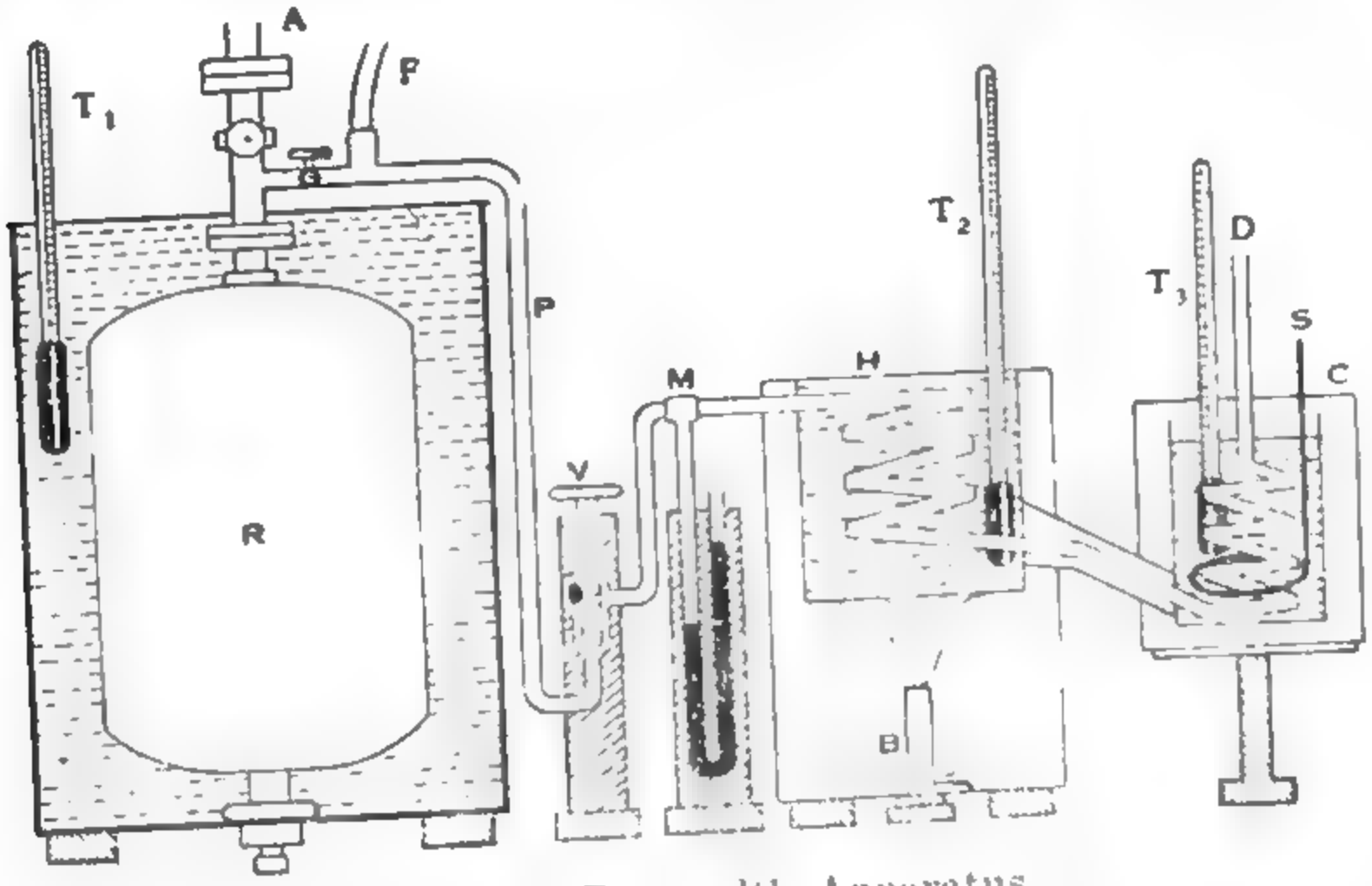


Fig. 40—Regnault's Apparatus

may suppose that the fall of temperature of the gas is $\{T_2 - (t_1 + t_2)\ 2\}$ If m be the mass of the gas which has left the reservoir and C_p its specific heat at constant presure : then the heat lost by the gas is

$mC_p\left(T_2 - \frac{t_1 + t_2}{2}\right)$. The heat gained by the calorimeter and its contents is $(M + m)(t_2 - t_1)$ where M and m are the water equivalent of the calorimeter and mass of water contained. Equating the two terms, C_p may be found.

Determination of the Specific heat of a Gas of Constant Volume.—The specific heat of a gas at *constant volume* has been determined by Joly with *steam calorimeter* (vide Art. 57). The gas is enclosed in a copper sphere which is suspended from the pan of a balance within an enclosure. By weighing the sphere empty and then filled with the gas, the mass of the enclosed gas is known. When steam is passed into the enclosure, it condenses on the sphere until the temperature of the latter with its contents comes to that of the steam. From the mass of steam condensed amount of heat taken by the sphere and the gas to rise through the observed range of temperature is known. From this the heat required by the mass of the enclosed gas for the rise of te-

perature is obtained and hence the specific heat of the gas at constant volume is found.

50. Observation on Specific Heat of Water.—It is found that of solids and liquids, *water has the highest specific heat* which means that a given mass of water absorbs more heat in warning and gives out more heat in cooling through a given range of temperature than an equal mass of any soil or any other liquid. It is for this reason that water is preferably used in hot water bottles and foot warmers and in hot-water pipes for heating rooms in cold countries. Again when kept in the sun for sometime, a quantity of water becomes less hot than an equal mass of any other material.

The use of *water as a calorimetric substance*, although almost universal, is found to be inconvenient for the following reasons :—

(1) Owing to the high specific heat of water, the variation of its temperature due to small quantities of heat cannot be appreciably measured.

(2) Owing to the variation of the specific heat with temperature, water is not suitable for very accurate work.

Again land is more easily warmed or cooled than water. The temperature of an island is therefore much influenced by that of the surrounding sea. In fact the temperature of the island is an average between that of the land and the sea. Thus, during summer, when the sun is above the horizon for a long period, the temperature of island is lower than that of the continent ; again during the winter when the sun is above the horizon for a shorter period, the temperature of the island is higher than that of the continent. Further, during day the sea is less heated than the continent ; but during night water cools more slowly than the land and so maintains the temperature of the island which does not therefore cool so much as the continent. It follows therefore that extremes of heat and cold are controlled by the sea, so that an island or a sea-side place has a more equitable climate than an inland place of the same latitude ; the sea may thus be called a *moderator of climate.*

***51. Measurement of High Temperature by the Calorimeter method.**—A *high* temperature, say that of a flame or a furnace, may be found by (1) Platinum Resistance thermometer, (2) Thermo-electric couple or (3) Calorimetric method.

The first two are dealt with in the Part on Voltaic Electricity.

Calorimetric method.—A suitable piece of a solid of known mass and specific heat is placed in contact with the source of the high temperature till the mass acquires this temperature A Calorimeter with sufficient water is taken, the heated solid is thrown into the water and the final temperature is carefully noted.

Let, water equivalent of calorimeter + stirrer = W gm.
mass of water taken = m gm.
mass of the solid taken = M gm.
specific heat of the solid = s
initial and final temperature of the water = t_1, t_2
unknown temperature of the source = t

$\therefore$ $Ms(t - t_2) = (W + m)(t_2 - t_1)$

whece t can be calculated

Example :—

1. A ball of platinum, whose mass is 80 gm. is kept for some time in a furnace and when it has acquired the temperature of the furnace it is immediately transferred to a vessel of water at 15°C. The temperature rises to 20°C. If the weight of water together with the water equivalent of the calorimeter be 400 gm., what was the temperature of the furnace? (Sp. ht. of platinum = ·0365).

(Gau. U.—1953)

A. Let T°C be the temperature of the furnace, which is the initial temperature of the platinum ball. The final temperature of the ball being 20°C, the heat lost by it is, $80 \times \cdot0365 \times (T - 20)$ calories.

Heat gained by calorimeter and water $= 400 \times (20 - 15) = 400 \times 5$

or $T = \frac{400 \times 5}{80 \times \cdot0365} + 20 = 705°C$ approximately.

52. Calorific value of a Fuel.—The *calorific value* of a fuel is the amount of heat given out when a definite mass, say 1 lb. of the fuel is completely burnt. Thus when we say that *the heating value of a sample of coal is 10,000 B Th. U. per pound*, we mean that the heat produced by the complete combustion of 1 lb. of coal of the particular sample is 10,000 B. Th. U. The calorific value of a fuel is usually determined by the *Bomb calorimeter.* A small quantity of the fuel, powdered and dried, is placed inside a strong steel bomb. The bomb is provided with gas-tight cover and is filled with oxygen at high pressure. The bomb is kept immersed in water contained in calorimeter ; the fuel is fired by heating a wire in contanct with it by electric current. The heat produced by combustion passes into the water, the initial and final temperatures of which are noted. Knowing the water equivalent of the calorimeter and of the bomb and the mass of water taken, the amount of heat produced by the combustion can be calculated and thus the calorific value is found.

53. Latent Heat.—When heat is continuously supplied to a mass of a solid, its temperature gradually rises until a certain temperature it begins to melt. On further heating more and more of the solid melts but the temperature remains *constant*, until the whole mass of the solid has completely changed into the liquid state. Conversely, if a mass of a liquid is continuously cooled, its temperatures gradually falls until at a certain temperature the liquid begins to solidify. On continuing the

process, more and more of the liquid solidifies but the temperature remains *the same* until the whole mass of the liquid has solidified.

Similarly, if a mass of a liquid is continuously heated, its temperature at first gradually rises till at a certain temperature depending on the superincumbent pressure, the liquid begins to boil and change into the vaporous state. On further heating, more and more of the liquid is vaporised but the temperature remains *constant* until the whole mass of the liquid has been evaporated. Conversely, if a mass of vapour is gradually cooled, its temperature falls until at a certain temperature the vapour begins to condense, which remains *constant* until the whole mass of the vapour has passed into the liquid state.

Hence when a piece of a solid melts (or a mass of a liquid is being vaporised), the heat supplied does not manifest itself in producing a rise in temperature but it solely utilised in bringing about the change of state. The heat absorbed during a change of state remains as if *latent* or hidden within the mass of the liquid (or the vapour) formed, reappearing only during the reverse process. The heat thus utilised in bringing about a change of state is called **latent heat,** as distinguished from *sensible heat* which produces a rise in temperature.

The **latent heat of fusion** *of a solid is the quantity of heat required to change unit mass of the substance at its melting point from the solid to the liquid state without change of temperature.* It is also equal to the quantity of heat given out by unit mass of the substance at its freezing point in changing from the liquid to the solid-state without any change of temperature.

"The latent heat of fusion of ice (or the latent heat of water) is 80 calories. This statement means that 80 calories of heat are required to change 1 gm. of ice at 0°C. into water at 0°C. Conversely, 80 calories of heat are given out when 1 gm. of water at 0°C. is changed into ice at 0°C. Similarly by the statement that the latent heat of fusion of lead is 5·7, we mean that the amount of heat required to convert 1 gm. of lead at its melting point into the liquid state is 5·7 calories.

In the pound-degree centigrade unit, the latent heat of fusion of ice is also 80 heat units (pound degree.) In the British Thermal Unit (pound degree Fahrenheit), the value becomes larger in the proportion of 9 : 5 and hence the latent heat of fusion of ice in B. Th. U. per pound = $80 \times 9 + 5 = 144$. If L be the latent heat of fusion of the soild, the amount of heat absorbed by m gm. of the solid in passing from the solid to the liquid state is equal to mL heat units.

The **Latent Heat of Vaporisation** *of a liquid is the quantity of heat required to change unit mass of the substance at its boiling point from the liquid to the vaporous state without change of temperature.* The same quantity of heat is also given out by unit mass of the vapour in condensing to the liquid state at the boiling point *without any change of temperature.*

It is experimentally found that the quantity of heat required to convert 1 gm. of water at 100°C. into steam at *the same temperature* is 537 calories. The same quantity of heat is also given out by 1 gm. of steam at 100°C. in condensing to water at 100°C ; in other words the latent heat of vaporisation of water (or the latent heat of steam) at 100°C. is 537 calories per gm. The value of the latent heat of steam in the pound-degree centigrade unit is also 537 heat units. Its value in the B. Th. U. (pound-degree Fahrenheit) per lb. $= 537 \times \frac{9}{5} = 966{\cdot}6$ B. Th. U.

Thus when ice melts, it first absorbs the latent heat and when steam condenses it first gives out the latent heat. Hence ice-cream appears so much colder to the teeth than iced water and more severe burns are caused by steam than by boiling water at the same temperature

54. Determination of the Latent Heat of Fusion of Ice.—Take a clean, dry calorimeter and weigh it together with a stirrer of the same material. A stirrer with a piece of wire-gauge fitted to the loop should preferably be taken. Fill the calorimeter about two-thirds with water and weigh again. The difference between the two weights gives the mass of water taken. Note the steady temperature of the water and calorimeter with a sensitive thermometer.

Put the calorimeter with the stirrer and the thermometer, inside a large vessel on non-conducting support. Take some pieces of ice of suitable size and after drying them crefully with blotting paper drop them one by one into the water in the calorimeter, taking care to hold the pieces with blotting paper while dropping them into water. Stir the water well all the time, keeping the pieces of ice under water with the wire-gauge stirrer. Note the lowest temperature reached. Then removing the thermometer, weight the calorimeter again with its contents. The difference between the last two weights gives the mass of ice added.

Let the mass of calorimeter and stirrer	=	W gm.
specific heat of material of the calorimeter	=	s
mass of water taken	=	m gm.
initial temperature of water and calorimeter	=	t°C
final temperature of the mixture	=	t'°C
mass of ice added	=	M gm.
required latent heat of fusion of ice	=	L

Heat lost by the calorimeter (with stirrer and the water) in cooling from t to $t' = Ws(t-t') + m(t-t') = (Ws+m)\ (t-t')$ cal. Heat gained (*i*) by M gm. of ice at 0°C. in melting to water at 0°C = ML calories. Heat gained (*ii*) by the M gm. of water formed in rising to $t' = Mt'$ calories.

Assuming that no heat is exchanged with any body outside.

$$\text{Heat lost} = \text{Heat gained}$$

$$\therefore \quad (Ws + m)(t - t') = ML + Mt'$$

$$\text{or} \quad ML = (Ws + m)(t - t') - Mt'$$

whence L can be calculated. The value of L is found to be about 80 calories per gramme.

Sources of Errors and Precautions :—

(1) If any water is carried with a piece of ice, the mass of ice is really less by the weight of this water and so the quantity of heat required for fusion will be less; this makes the observed value of L *too low*. Hence just before dropping into water, the pieces of ice should be carefully dried with blotting paper.

(2) A wire-gauge stirrer should be used, so that the piece of ice added are always kept under water and not allowed to float; otherwise the portion of ice above the surface of water while melting will take heat not only from the water but also from surrounding air.

(3) Some heat is gained by the colorimeter and its contents from outside by radiation. This will make the observed value of L *too low*. This is remedied by taking the initial mass of water a few degrees say 5°C. above the room temperature; ice is added till the final temperature of the 'mixture' goes below the room temperature to the same extent. This is known as the *method of compensation*.

(4) The quantity of ice added must be small as otherwise the lowering of temperature of the calorimeter and its contents may be so much that there may be actual condensation of moisture on the outer walls of the calorimeter. This *considerably* lowers the observed value of the latent heat of ice.

Examples :—

1. Find the result of placing 5 gm. of ice at 0°C., into 20 gm. of water at 45°C.

A. Heat lost by 20 gm. of water at 45°C. in cooling to 0°C = 900 cal.

Heat required to melt 5 gm. of ice at 0°C to water of 0°C = 400 cal.

∴ All the ice will melt.

Let the resulting temperature be t°C.

Heat lost by 20 gm. of water at 45°C, to come to t°C,

$$= 20 \times (45 - t) \text{ calories.}$$

Heat gained by 5 gm. of ice melting to water at 0°C = 400 calories.

Heat gained by 5 gm. of water (melted ice) to rise to t°C = $5t$ calories.

$$\therefore \quad 20 \times (45 - t) = 400 + 5t \quad \text{whence} \quad t = 20°\text{C}.$$

∴ There will be 25 gms. of water at 20°C.

2. 15 gm. of ice at − 20°C. are dissolved in 50 gm. of water at 40°C. and resultant temperature is 10°C. If the latent heat of fusion of ice is 80, find its specific heat.

A. Let the required specific heat of ice = s.

Heat lost by 50 gm. of water to cool from 40°C. to 10°C = 50 × 30 calories.

Heat gained by 15 gm. of the ice at − 20°C. to reach 0°C = 15 × s 20.

Heat gained by 15 gm. of the ice to melt to water at 0°C. = 15 × 80 cals.

Heat gained by 15 gm. of water to come to 10°C. $= 15 \times 10$

$\therefore \quad 50 \times 30 = 15 \times 20 \times s + 15 \times 80 + 15 \times 10$

or $\quad 300s = 150$ whence $s = 0{\cdot}5$.

3. Calculate the amount of heat necessary to convert 1 gm. of ice at $-10°C$. to steam at 100°C. (Sp. ht. of ice $= 0{\cdot}5$; latent heat of water $= 80$ cal/gm. and latent heat of steam $= 536$ cal./gm.). (Anna. U.—1950).

A. Heat necessary to raise 1 gm. of ice from $-10°C$. to 0°C. $= 1 \times {\cdot}5 \times 10$ cal.

Heat necessary to convert 1 gm. of ice into water at 0°C. $= 1 \times 80$ cal.

Heat necessary to raise 1 gm. of water from 0°C. to 100°C. $= 1 \times 100$ cal.

Heat necessary to convert 1 gm. of water at 100°C to steam $= 1 \times 536$ cal.

$\therefore$ Total amount of heat $= 5 + 80 + 100 + 536 = 721$ calories

3. In the experiment to determine the latent heat of fusion of ice, a carefully insulated calorimeter of water equivalent 20·1 gm. contains 300 gm. of water at 19°5°C Into this is dropped a lump of pure ice, *not previously dried*. The mass of the calorimeter with its contents is found to have increased by 17·9 gm. and the resultant temperature comes to 14·7°C. The correct value for latent heat of fusion of ice is 80 calories per gramme; what was the mass of water adhering to the ice?

Let mass of the adhering $= m$ gm.

$\therefore$ Mass of ice added $= (17{\cdot}9 - m)$ gm.

Heat lost by the calorimeter and water

$= 20{\cdot}1 \times (19{\cdot}5 - 14{\cdot}7) + 300 \times (19{\cdot}5 - 14{\cdot}7)$ cal. $= 320{\cdot}1 \times 4{\cdot}8$ calories.

Heat taken up by the ice to melt $= (17{\cdot}9 - m) \times 80$ cal.

Heat taken up by the water to rise to 14·7°C $= 17{\cdot}9 \times 14{\cdot}7$

$\therefore \quad 320{\cdot}1 \times 4{\cdot}8 = (17{\cdot}9 - m) \times 80 + 17{\cdot}9 \times 14{\cdot}7$ whence $m = 1{\cdot}98$ gm.

55. Ice Calorimeters.—The fact that certain quantity of ice in melting always absorbs a definite quantity of heat, has been utilised in the application of ice calorimeters for the determination of specific heat of a substance.

Black's Ice Calorimeter.—It consists merely of a large block of pure ice with a hole about an inch in diameter and an inch deep, scooped out into it (Fig 41). The mouth of the hole is covered by a slab of ice, which serves as a lid over the cavity. A mass of the solid, of which the specific heat is to be determined is taken and weighed. It is then heated to a definite temperature in a steam heater. The inside of the cavity is thoroughly dried with blotting paper or sponge The heated solid is then dropped into the cavity and the lid is replaced quickly. The body with the cavity comes to the temperature of 0°C, giving out heat which melts a certain quantity of ice. After a few minutes the water in the cavity as well as on the surface of the body is collected by means of a small pipette and a piece of sponge or blotting paper previously weighed (and

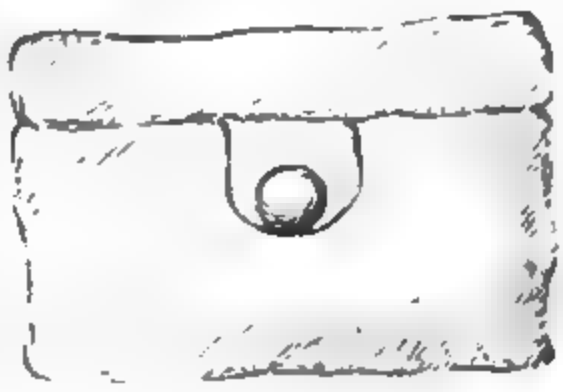


Fig. 41

cooled to 0°C). The mass of water formed is determined by direct weighing.

Let the mass of solid = M gm.
initial temperature of the solid = t°C.
mass of the melted ice = m gm.
sp. heat of the material of the solid = s
Heat lost by M gm. of the solid in cooling to 0°C = Mst cal.
Heat gained by m gm. of ice in melting = mL cal.
As no heat from outside can reach the solid

$$\therefore \quad Mst = mL \quad \text{where} \quad s = \frac{mL}{Mt}$$

The method is *not accurate*, since (*a*) the water formed by the melting of ice cannot be completely removed. (*b*) Some ice may melt by absorbing heat from the atmosphere during the period between the dropping of the solid inside and the closing of the lid.

The *advantages* of the method are that (1) it is a ready method, (2) there is no loss of heat by radiation and (3) no delicate thermometer is necessary for noting the final temperature.

Bunsen's Ice Calorimeter.—The action of this instrument depends on the fact that a given mass of ice on melting diminishes in volume to a definite extent, so that by accurately measuring the change in volume, the weight of ice melted and hence the quantity of heat required to melt in can be found.

Bunsen's ice calorimeter consists of a large thin-walled test tube A (Fig. 42) fused into a large glass vessel B, which extends at the bottom into a narrow tube CD, bent twice at right angles, as shown in the figure. At the other end of CD, there is a collar D closed by a cork through which passes a bent piece of capillary tubing T of uniform bore. To the horizontal part of tube T a finely graduated scale S is attached, so that the *volume* between any two divisions is accurately known. The lower part of B, the tube CD and a part of the capillary tube T are filled with pure dry mercury while the upper part of B is filled with pure distilled water, which has been boiled to free it from dissolved air.

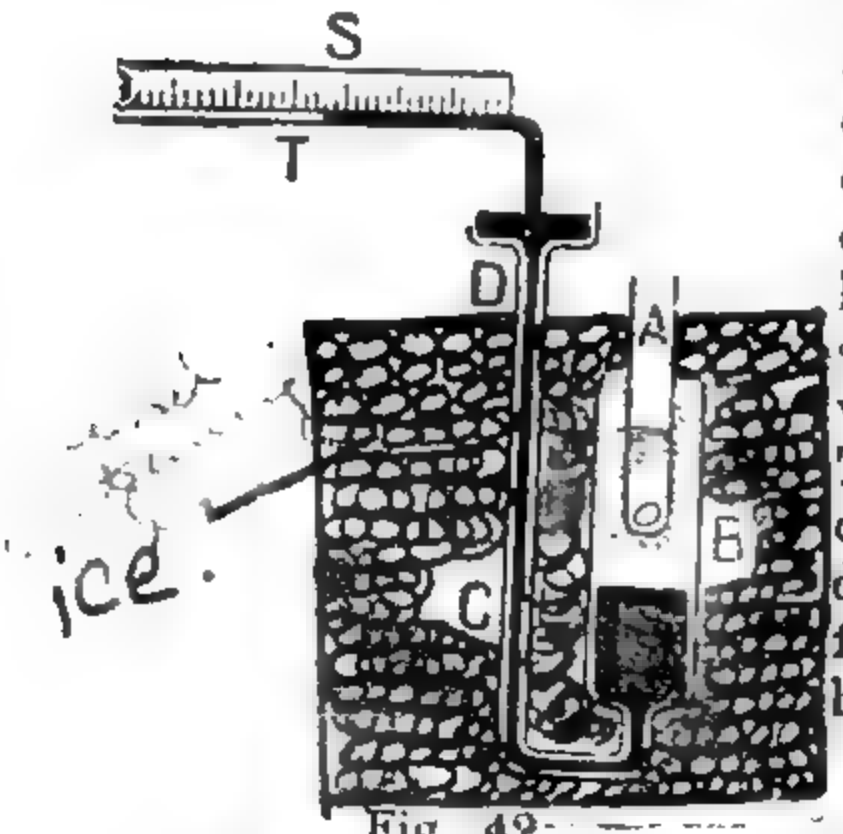


Fig. 42

Before using the instrument, it is kept surrounded as completely as possible with pure melting ice and a stream of alcohol or ether previously cooled by a freezing mixture is passed through A, till owing to the intense local cooling, a thin coating of ice is formed round the lower part of A. This gradually increases as the cooling is continued, while the mercury

meniscus in the capillary tube is found to move forward. The freezing mixture is then removed from A and a quantity of water already cooled at 0 C. is placed in it. After sufficient interval the whole of the instrument with the contents comes to 0°C and the mercury meniscus assumes a steady position as read from the scale.

To determine the specific heat of a metal, a small lump of it is taken and weighed. The lump is then heated to a constant high temperature and quickly dropped into the water in A which is at once corked. The heat given out by the solid is communicated by the water to the ice surrounding A and melts some of the ice. This continues until everything again is at 0°C. As the ice melts a contraction in the volume takes place, which causes the mercury meniscus to move inwards; when it has been steady, its position is read again. The cross section of the capillary tube having been previously known, the contraction in volume is easily found whence the quantity of ice melted and so the quantity of heat supplied can be calculated.

Let the mass of the solid = M gm.

sp. ht. of the material of the solid = s

initial high temperature of the solid = t°C.

contraction in volume of the ice = v c.c.

Now contraction of 1 gm. of ice at 0°C. on melting = 0·091 c.c.

∴ Ice melted in the present case = $v/0{\cdot}091$ gm.

Heat absorbed in the melting of ice = $vL/0{\cdot}091$ cal.

where L is the latent heat of fusion of ice,

Heat lost by the solid in cooling from t°C. to 0°C. = Mst cal.

As this heat is wholly utilised in melting the ice,

$$Mst = \frac{vL}{0{\cdot}091} \quad \text{or} \quad s = \frac{vL}{0{\cdot}091Mt} = \frac{10{\cdot}99Lv}{Mt}$$

whence s is calculated.

As v is very small, the cross-section should be accurately known and the contraction carefully noted. Sometimes the scale is previously calibrated directly in calories.

Advantages of Bunsen's Ice calorimeter :—

(*i*) It is very sensitive and accurate, (*ii*) It can be used to determine the specific heat of solids available in a very small quantity, (*iii*) There is no loss of heat due to radiation, (*iv*) The water equivalent of the calorimeter is not to be taken into account, (*v*) There is no necessity for a thermometer along with the calorimeter.

Example :—

1. A spherical iron ball is placed on a large block of dry ice at 0°C. into which it sinks until submerged. What was the temperature of the iron? (Density of iron = 7·7 gm. per c.c. and that of ice = 0·92 gm. per c.c.; sp. heat of iron = 0·12 and latent heat of fusion of ice = 80 calories per gm.

A. Evidently the loss of heat by radiation is not to be counted. Let the volume of the iron ball be V c.c. and its initial temperature be t°C. The ball melts a volume of ice equal to half its volume and falls to 0°C.

Now mass of the ball $= 7{\cdot}7$ V gm.; mass of ice melted $= 0{\cdot}92 \times \frac{V}{2}$ gm.

$\therefore$ Heat lost by the ball $= 7{\cdot}7 \times V \times 0{\cdot}12 \times t$ calories.

And heat gained by the ice $= 0{\cdot}92 \times \frac{V}{2} \times 80$ calories.

$\therefore$ $7{\cdot}7 \times V \times 0{\cdot}12 \times t = 0{\cdot}92 \times \frac{V}{2} \times 80$ whence $t = 39{\cdot}8$°C.

2. If a gram of ice at 0°C contracts ·091 c.c. in melting, calculate the specific heat of the substance when 40 gm. at 60°C dropped into an ice calorimeter cause a change in volume of ·273 c.c. Given latent heat of fusion of ice = 80 cals/gm.
(R. P. B.—1948)

A. The mass of melted ice $= \frac{\cdot 273}{\cdot 091}$ gm.

Heat gained by $\frac{\cdot 273}{\cdot 091}$ gm. of ice in melting $= \frac{\cdot 273}{\cdot 091} \times 80$ calories.

Again heat lost by 10 gm. of the substance at 60°C, when dropped into the ice calorimeter $= 40 \times 60 \times s$. where s = specific heat.

$\therefore$ $10 \times 60 \times s = \frac{\cdot 273 \times 80}{\cdot 091}$ whence $s = 0{\cdot}1$

3. A mass of 0·96 gm. of a substance heated to 100°C is dropped into a Bunsen's calorimeter. The thread of mercury recedes through a distance of 8·3 mm. in the capillary tube of 1 sq. mm. section. Calculate the sp. ht. of the substance given that 1 gm. of water on freezing expands by 0·0905 c.c. and evolves 80 calories.
(Nag. U.—1952)

A. The volume contraction effected by the movement of mercury thread by 8·3 mm. through a capillary tube of 1 sq. mm. cross-section $= 8{\cdot}3 \times 1 = 8{\cdot}3$ cu. mm. $= \cdot 0083$ c.c.

This volume contraction is due to some ice melting into water at 0°C.

Now, contraction is ·0905 c.c. when 1 gm. of ice melts into water at 0°C.

$\therefore$ " " 1 c.c. ... $\frac{1}{\cdot 0905}$ gm. " " "

or. " " ·0083 c.c. ... $\frac{\cdot 0083}{\cdot 0905}$ gm. " " "

Again 1 gm. of ice absorbs 80 calories in melting to water.

$\therefore$ $\frac{\cdot 0083}{\cdot 0905}$ ... $\frac{80 \times 83}{905}$ ...

This amount of heat is given out by 0·96 gm. of the substance in cooling through 100°C.

If s be its specific heat, then

$$0{\cdot}96 \times 100 \times s = \frac{80 \times 83}{905} \text{ whence } s = \frac{80 \times 83}{905 \times 96} = \cdot 075$$

56. Determination of Latent heat of Vaporisation of Water.—(*Latent heat of Steam*)—Take a clean, dry calorimeter and weigh it together with a stirrer of the same material. Fill the calorimeter about two-thirds with water and weigh again. The difference between the two weights gives the mass of water taken. Note the steady temperature of the water and the calorimeter with a sensitive thermometer.

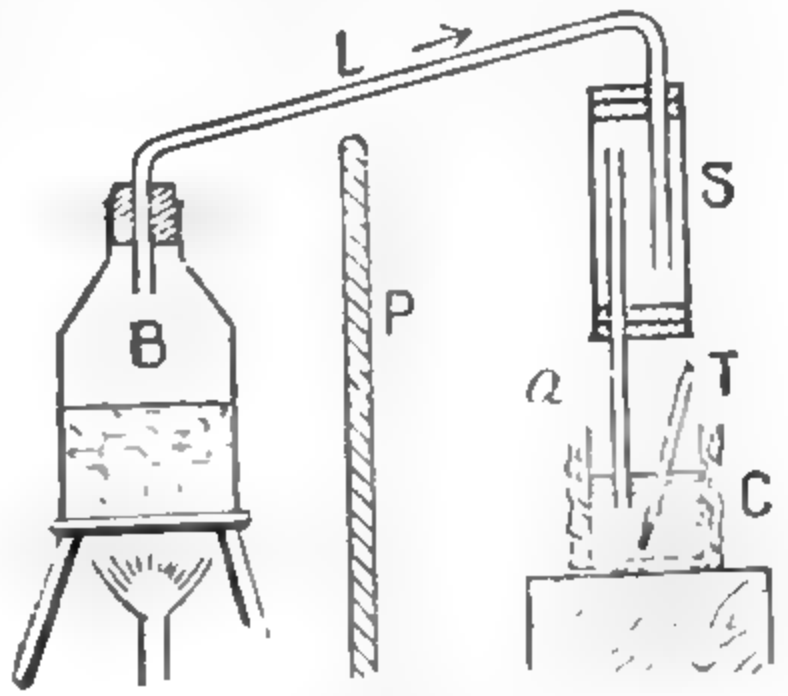


Fig. 43

Boil some water in a vessel B (Fig. 43), with its mouth closed by a cork through which a bent glass tube passes. The open end of the bent tube is connected to a *water-trap* or the so called **steam trap** S. This is a contrivance by which the steam is freed from any water accompanying it. It consists of a wide glass tube fitted with steam-tight corks through each of which passes a glass tube reaching nearly up to the opposite cork. The upper tube is connected with the delivery tube from the flask. Steam enters into the trap by the upper tube and the water formed by the condensation of any steam is deposited at the bottom of the wide tube, while the steam goes up and comes out by the exit tube *a* practically free from water

Put the calorimeter C, with its contents and the thermometer T, on non-conducting supports inside a large vessel Bring it under the steam trap, quickly put the nozzle of the exit tube well within the water and pass steam for some time, stirring the water all the while. A screen P is placed between the calorimeter and the boiler. Take away the nozzle quickly and note the highest temperature attained by the water. Allow the calorimeter to cool to the room temperature and remove the thermometer, taking care that no water adheres to it Weigh the calorimeter with the contents. The difference between the last two weights gives the mass of steam condensed.

Let the mass of the calorimeter and stirrer = W gm.
Sp. ht of material of the calorimeter = s
mass of water taken = m gm.
initial temperature of water and calorimeter = t°C
final temperature of the mixture = t_1°C.
temperature of the steam = 100°C.
mass of steam condensed = M gm.
required latent heat of steam = L

Heat lost by steam in condensing to water at 100°C = ML cal.
Heat lost by water in cooling from 100°C, to t_1 = $M(100 - t_1)$

Heat gained by calorimeter with stirrer and water in rising from t to t_1 = $Ws(t_1 - t) + m(t_1 - t)$ cal.

Assuming that no heat is exchanged with any body outside

$$\text{Heat lost} = \text{Heat gained}$$

$$\therefore \quad ML + M(100 - t_1) = (Ws + m)(t_1 - t)$$

$$\text{or} \quad ML = (Ws + m)(t_1 - t) - M(100 - t_1)$$

$$\therefore \quad L = \frac{Ws + m}{M}(t_1 - t) - (100 - t_1)$$

The value of L of steam at 100°C is found to be 537 calories per gramme.

Sources of Errors and Precautions :—

(1) If any water accompanies steam, the observed value of L would be *too low*. To avoid condensation of steam, the delivery tube should be covered with cotton wool along with the use of a steam-trap.

(2) Any loss of heat by radiation from the calorimeter and contents will make the observed value of L *too low*. To reduce the effect of radiation, the water in the calorimeter should be initially cooled a few degrees below the room temperature and steam should be passed till the temperature of the water rises as much above the room temperature as it was below.

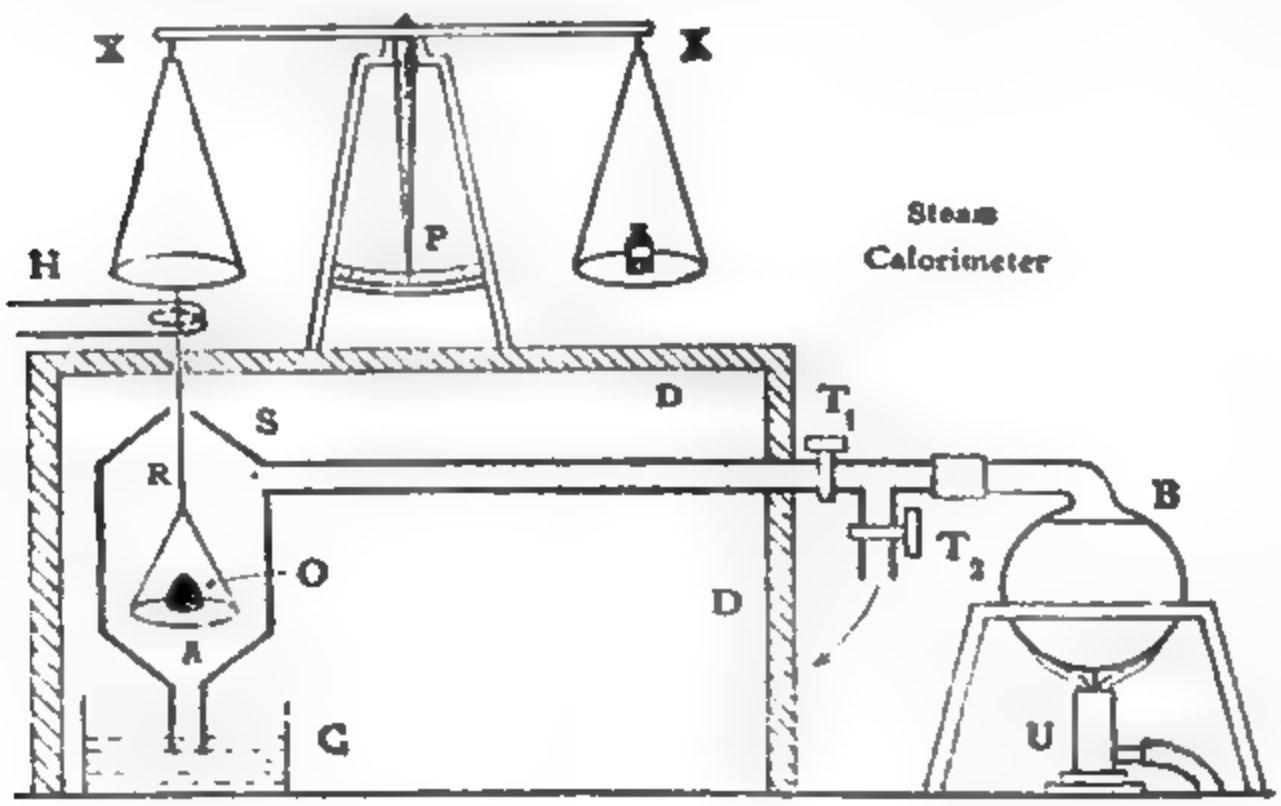
(3) The temperature of the steam may not be at 100°C. and so it should be determined in each case.

(4) To prevent the calorimeter from gaining any heat directly from the burner or the boiler by radiation, a screen is to be placed in between.

(5) The issue of steam should not be rapid, otherwise some water may be lost by splashing, which will make the observed value *too low*.

(6) The temperature of the calorimeter and its contents should not be allowed to rise much above 30°C. beyond the room temperature for in that case much vapour will be given off, causing considerable loss of heat.

57. Joly's Steam Calorimeter.—Joly, in 1886, devised a very accurate method of determining specific heat of a substance by the use of condensation of steam on it. The apparatus consists of a metal enclosure S, called the steam chamber, in which steam is supplied by a boiler B (Fig. 44). The steam chamber is enclosed within a box D on the top of which is placed a sensitive balance. From one arm of the balance a fine wire B is suspended, which passes down through a hole into the steam chamber. The wire carries a scale pan A at its other end.



Steam Calorimeter

Fig. 44—Steam Calorimeter

The body O whose specific heat is to be determined, is placed on the pan within the steam chamber *when steam is not passed through it* and its mass is determined. This gives the mass of the body in air at the room temperature t°C. Let the mass be m gm. Then steam is passed from the boiler into the chamber and when a *steady high temperature* of the chamber is recorded by the thermometer, 'weights' are placed on the other pan to counterpoise the body and the condensed steam. If this be M_1gm., then this gives the total mass of the body and the steam condensed on the body and the pan and suspension wire. Then the body is taken out and again steam is passed over the empty pan. The mass of steam condensed on the pan and its accessories is found by method of weighing. Let it be M_2gm. Then the mass of steam condensed on the body is $(M_1 - M_2)$ gm. Let the temperature of steam be T°C.

If the specific heat of the body be s, then the heat absorbed by the body when its temperature is raised from t°C to T°C is $m\,s\,(T-t)$. The heat given out by steam in condensation over the body is $(M_1 - M_2)L$, L being the latent heat of steam at T°C.

Thus $ms.\,(T-t) = (M_1 - M_2)L$ whence $s = \dfrac{(M_1 - M_2)L}{m\,(T-t)}$ ... (55,1)

In order that the steam might not condense on the upper left pan of the balance, an electric heater coil H is placed in front of the slit of the chamber D.

Example :—

1. Steam at 100°C is passed into a calorimeter of mass 55 gm. containing 150 gm. of mixture of ice and water, until all ice melts. The final mass of the calorimeter and its contents is 217·5 gm. If the latent heat of fusion of ice is 80 cal./gm. find how much ice was present in the mixture. (Latent heat of steam = 540 cal./gm.) (Del. H. S.—1953)

A. Let m gm. be the mass of ice originally present in the mixture. Then the mass of water in the mixture = $(150 - m)$ gm.

Mass of steam condensed into the mixture = $217·5 - (150 + 55) = 12·5$ gm.

Since *throughout the period* of melting of ice, temperature remains the same at 0°C. the calorimeter does neither grain nor lose heat. Therefore heat lost by steam in condensing to water at 100°C and that amount of water in coming down to 0°C must be equal to heat gained by m gm. of ice in the mixture at 0°C to be converted to water at the same temperature.

Now heat lost by steam in condensing to water at 100°C = $12·5 \times 540$ cal. and heat lost by 12·5 gm. of water at 100°C. to come to 0°C = $12·5 \times 100$ cal.

$\therefore$ Total heat lost = $12·5 \times 540 + 12·5 \times 100 = 8000$ calories

Again heat gained by m gm. of ice in melting = $m \times 80$ cal.

Thus from the principle, heat gained = heat lost,

$m \times 80 = 8000$ whence $m = 100$ gm.

2. A glass beaker of negligible thermal capacity contains 1 lb. of water and some ice. When 1 oz. of steam at 100°C. has been supplied to the vessel, the contents are at a temperature of 3°C; how much ice was there to begin with? Latent heat of ice = 79 cal./gm.; Latent heat of steam = 537 cal./gm.

(Utkal. U.——1951)

A. Let the mass of ice be x gm. Since 1 lb. = 453·6 gm. and 1 oz. = 28·35 gm.

Heat lost by 28·35 gm. of steam in condensing at 100°C = $28{\cdot}35 \times 537$ cals.

Heat lost by condensed steam to cool down to a temperature 3°C = $28{\cdot}35\ (100 - 3)$ cals.

Heat gained by ice in melting at 0°C. = $x \times 79$ cals.

Heat gained by melted ice in rising to 3°C = $x \times 3$ cals.

Equating Heat lost to Heat gained, we get,—

$$28{\cdot}35 \times 537 + 28{\cdot}35 \times 97 = 453{\cdot}6 \times 3 + x \times 82$$

whence $x = 202{\cdot}6$ gm. = 7·15 oz. nearly.

3. Steam at 100°C. is passed into a zinc vessel of mass 500 gm. containing 1 kgm. of ice and 1 kgm. of water at 0°C. and temperature of the 'mixture' rises to 30°C. If the mass of the calorimeter with its contents is found to increase by 232 gm. find the latent heat of steam. (Sp. ht. of zinc = 0·92; latent heat of water = 80).

A. Let the required latent heat be L calories per gm.

Heat lost by steam in cooling to water 100°C = 232 L calories

Heat lost by condensed steam in cooling to 30°C. = 232×70 „

Heat gained by the ice in melting = 1000×80 „

Heat gained by this water and the water in the calorimeter in rising from 0°C. to 30°C. = 2000×30 cal.

Heat gained by the vessel in rising from 0°C. to 30°C = $500 \times {\cdot}092 \times 30$ cal.

$\therefore$ $232L + 232 \times 70 = 80{,}000 + 60{,}000 + 1380$

or $232L = 125140$ whence L = 539 cals./gm.

4. Steam at 100°C. is passed into 100 gm. of ice at — 10°C. The temperature rise to 35°C. and the mixture is found to weigh 120 gm. Find the specific heat of ice. The latent heat of water and steam are 80 and 535 respectively.

A. Let the sp. ht. of ice = s

Mass of steam condensed = (120 — 100) gm. = 20 gm.

Heat lost by steam in condensing to water at 100°C. = 20×535 cal.

Heat lost by this water in cooling to 35°C. = 100×35 cal.

Heat gained by ice in rising from — 10°C. to 0°C = $100 \times s \times 10$ cal.

Heat gained by this water in rising to 35°C. = $20 \times (100 - 35)$ cal.

Heat gained by ice changing to water at 0°C = 100×80 cal.

$\therefore$ $20 \times 535 + 20\ (100 - 35) = 100 \times s \times 10 + 100 \times 80 + 100 \times 35$

or $1000s = 500$ whence $s = 0{\cdot}5$.

JOSEPH BLACK (1728-1799)

Black was born at Bordeaux in 1728. His childhood was spent in France but in 1740 he was sent to his home in Ireland to receive the education at the School. In 1746 he was admitted to the University of Glasgow. He studied medicine and chemistry at the University and at the same time investigated the physiological action of quicklime and caustic potash on the human system. A degree of Doctor of Medicine was conferred upon him as a reward of his labour.

Fig. 45— Joseph Black

In 1756 he was appointed a Professor of Anatomy and Analytical Chemistry in the Glasgow University. He was a great teacher, who could inspire students like James Watt, David Hume and Adam Smith. His finest research was on latent heat, which occupied a period from 1756 to 1761. He showed that when ice melts, each gramme of ice requires a certain amount of heat due to its melting without a change to temperature. The absorption of this heat is not revealed by a thermometer and hence he called it 'latent heat'. In 1766, he was appointed Professor of Chemistry at Edinburgh. He held on to this post till his death on December 6, 1799.

Summary

Unit of heat is the quantity of heat required to raise one gramme of pure water through 1°C. It is called the *calorie*.

The **British Thermal unit** (B. Th. U.) is the number of heat required to raise 1 lb. of water through 1°F.

The **specific heat** of a substance is equal to the number of units of heat required to raise 1 gm. of the substance through 1°C.

The **thermal capacity** of a body is the total quantity of heat required to raise the temperature of the body by 1°C.

The **water equivalent** of a body is the quantity of water in grammes which will be raised through 1°C. by the amount of heat required to raise the body through 1°C.

Methods for the determination of the specific heat.—(1) Method of mixture: (2) method of cooling (in case of liquids): (3) Ice Calorimeter.

The **Latent heat of fusion** of a solid (L) is the quantity of heat required to change unit mass of the substance at its melting point from the solid to the liquid state without change of temperature.

The **Latent heat of vaporisation** of liquid is the quantity of heat required to change unit mass of the substance at its boiling point from the liquid to the vapour state without change of temperature.

Exercises on Chapter V

Reference

Arts. 40 & 44 — 1. What is the unit quantity of heat? Define thermal capacity of a body. How would you find the thermal capacity of a body practically? (C. U.—1952)

Arts. 41 & 44 — 2. What is meant by unit quantity of heat? Define 'B. Th. U.' and 'calorie' and indicate the numerical relation between them. Explain the term 'water equivalent' of a calorimeter and describe how it is determined. (Gau. U.—1952)

Arts. 43 & 44 — 3. Explain the terms 'specific heat', 'thermal capacity' and 'water equivalent' of a body, and state the units used in expressing them. (Del. U. S.—1952; C. U.—1953; Utkal. U.—1949)

Art. 45 — 4. A copper calorimeter weighs 180 gm. and the sp. ht. of the material is 0·9. Find the quantity of heat required to raise its temperature from 25°C. to 65°C. (C. U.—1943)

A. 648 cal.

Art. 45 — 5. How would you find the thermal capacity of a given calorimeter by experiment? (Mad. U.—1947)

Art. 46 — 6. Describe an experiment to determine the specific heat of a solid. Mention the sources of error and how they are avoided. (Utkal. U.—1953; Mad. U.—1947)

Arts. 45 & 46 — 7. Define specific heat. Describe an experiment to determine the water equivalent of a copper calorimeter and the specific heat of copper. (C. U.—1943)

Art. 45 — 8. A calorimete rcontains 129·4 gm. of water at 12·2°C., 65·7 gm. of water at 38·6°C. are added to this and after stirring the temperature of the mixture is found to be 21°C. Find the water equivalent of the calorimeter.

A. 2 gm.

Art. 45 — 9. Supposing you were given a thermometer reading only from 50°C to 100°C and some water of which the temperature was below 20°C, describe an experiment to determine roughly the temperature of water, without using any other thermometer. (C. U.—1953)

Reference

HINTS :—Heat sufficient quantity of water in a vessel upto a *known* higher temperature T°C to be measured with the given thermometer Pour a quantity of this water into the calorimeter containing the cold water, so that after stirring the mixture, the temperatue is above 50°C. the exact value being known from the thermometer. Thus knowing the water equivalent of the calorimeter and masses of hot and cold water, the initial temperature of cold water may be found. This temperature is approximate because there is heat loss from the mixture during the process of heat exchange.

10. 96 gm. of lead shots (sp. ht.= 0·03) at 99°C. are poured into 148 gm. of a liquid at 29°C contained in a lead vessel weighing 300 gm. If the final temperature of the mixture is 33°C. find the specific heat of the liquid. (Mad. U.—1947) Art. 46

A. 0·26

11. A copper calorimeter of 10 gm. contains 12 gm. of water at 98·1°C. A copper coin at 15°C. is added to this and the temperature of the calorimeter and its contents decreases to 92·8°C. Find the mass of the coin. (Sp. ht. of copper = ·093). Art. 46

A. 9·5 gm.

12. Describe an experiment to determine the specific heat of a solid which is soluble in water explaining carefully how you would calculate your result. (Gau. U.—1953) Arts. 46 & 47

13. A calorimeter whose water equivalent is 10 gm. is filled with 50 gm. of water at 80°C. and the time for the temperature to fall to 75°C is 4 minutes. When filled with another liquid, the weight being 40 gm., the time taken for the same fall is 130 seconds. Find the specific heat of the liquid. (U. P. B.—1947; R. P. B.—1940) Art. 48

A. 0·56

14. A copper calorimeter of mass 50 gm. contains 20 gm. of an oil of sp. ht. ·47, at 20·5°C. Into this is poured another oil at 98·5°C. The temperature of the mixture is found to be 38·5°C. and the mass of the calorimeter and oil is found to be 75·4 gm. Find the sp. ht. of the added oil. (Sp. ht. of copper = ·09). Art. 47

A. ·77

15. A calorimeter, whose water equivalent is 5·0 grams, is filled with 25·0 grams of water. It takes 4 minutes to cool from 25°C to 17°C. When the same calorimeter is filled with 30·0 grams of a liquid it takes 180 seconds to cool through the same range. Calculate the specific heat of the liquid. (Raj. U.—1953) Art. 48

A. 0·58

16 Describe how the sp. ht. of a liquid is measured by the method of cooling. State the principle of the method and write down the formula used for calculating the result. Art. 48
(Raj. U.—1953; Del. U.—1942; R. P. B.—1940; Mad. U.—1949)

17. How do you account for the difference between the two specific heats of a gas? (R. P. B.—1941) Art. 49

18. Define the term specific heat. Explain why sp. ht. of a gas depends on the conditions under which this quantity is measured. Art. 49
(U. P. B.—1944) Art. 49

Reference

Art. 49 19. Explain why a gas has two specific heats. Describe how the specific heat of a gas at constant pressure has been measured. Give a neat diagram. (Del. H. S.—1951; R. P. B.—1942)

Art. 49 20. Why is the specific heat of a gas at constant pressure greater than that at constant volume? How will you use the knowledge of these specific heats to find the volume? Deduce any formula you use. (U. P. B.—1950)

Art. 51 21. A brass weight of 100 gm. is heated so that a particle of solder placed upon it just melts. It is then put into 100 c.c. of water at 15°C. contained in a calorimeter of water equivalent 12 gm. If the final temperature of water is 35°C. what is the melting point of the solder? (sp. ht. of brass = ·088). (Pat. U.—1935)

A. 289·5°C.

Art. 51 22. A lump of platinum weighing 100 gm. is heated in a flame until its temperature has reached that of the flame. It is then removed and dropped quickly into a calorimeter which has a water equivalent of 5 gm. and contains 495 gm. of water. If the temperature of the water rises from 22°C. to 30°C. find the temperature of the flame. (Sp. ht. of platinum = ·036).

A. 1141°C approximately.

Art. 51 23. How will you proceed to measure the temperature of a furnace by a calorimetric method? Develop the working formula and discuss the source of error in the method. (Utkal. U.—1951)

Art. 51 24. What is meant by the statement that 'the latent heat' of fusion of ice is 80? (C. U.—1950)

25. Define specific heat and latent heat of fusion of a solid.

Arts. 43 & 53 How many units of heat are required to melt 100 gm. of tin originally at 20°C [melting point of tin = 232°C., latent heat of fusion of tin = 14 cal./gm.; sp. ht. of tin = ·05].

Does the value of these constants depend on the kind of thermometer used, Fahrenheit or Centigrade?

A. 2460 cal., yes. (C. U.—1944)

Art. 51 26. How would you determine the latent heat of fusion of ice mentioning the precautions needed.
(C. U.—1927; Mad. U.—1949, '50; Pat. U.—1938; *cf.* And. U.—1951)

Art. 52 27. What would be the final temperature of the mixture when 5 grammes of ice at — 10°C. are mixed up with 20 grammes of water at 30°C.? The specific heat of ice is ·5.

A. 7°C.

Art. 54 28. A ball of copper of mass 30 gm. was heated to 100°C. and dropped in an ice calorimeter. In cooling down it evolved sufficient heat to melt 5·45 gm. of ice. If the latent heat of fusion of ice is 80, what is the specific heat of copper? (Dac. U.—1933)

A. ·092.

Art. 54 29. A copper calorimeter of mass 100 gm. and specific heat 0·1 contains 250 gm. of a liquid of sp. ht. 0·4 at a temperature of 30°C.

Reference

10 gm. of pure ice at 0°C are dropped into the liquid. Find the resulting temperature. Latent heat of fusion of ice = 80 cals/gm.

A. 16·03°C. (Mad. U.—1946)

30. Describe and indicate the use of Black's Ice calorimeter. Art. 55
(C. U.—1940)

31. A piece of iron weighing 15 gm. is dropped at a temperature of 113·6°C. into a cavity in a block of ice of which it melts 2·5 gm. If the latent heat of ice is 80 calories per gram, find the specific heat of iron. Art. 55
(C. U.—1948)

A. ·11.

32. Describe Bunsen's ice calorimeter and its working. How will you use it to find the density of ice? Show the calculations clearly. Art. 55
(P. U.—1952; R. P. B.—1943, '48)

33. The density of ice at 0°C. is ·916 and that of water is 1 gm. per c.c. A piece of metal weighing 600 gm. is raised to a temp. of 80°C. and placed in a mixture of ice and water melting some of the former. The volume of mixture is found to decrease by 8·4 c.c. How much ice is melted and what is the sp. ht. of metal? Art. 55

A. 91·6 gm; ·152.

34. 25 gms. of wax are heated to 60°C. and poured into 100 gms. of water at 5°C in a calorimeter of water equivalent of 10 gms. Find the final temperature of the mixture. (Melting point of wax = 45°C; sp. ht. of solid wax = 0·3; sp. ht. of liquid wax = 0·4; latent heat of fusion of wax = 40 cal/gm.) Arts. 46 & 53
(Utkal. U.—1948)

A. 19·85°C. approx.

35. Describe the Bunsen's ice calorimeter and explain how it is used to determine sp. ht. of a substance. Art. 55
(Nag. U.—1952; E. P. U.—1951, '52 Del. H. S.—1952)

36. The open end of the capillary tube of a Bunsen's ice calorimeter is placed under the surface of mercury. When 25 gm. of water at 15°C. are placed in the inner tube of the calorimeter, it is found that 6·8 gm. of mercury are drawn in. Calculate the density of ice. Latent heat of ice = 79 cals/gm. and density of mercury = 13·6 gm./c.c.). Art. 55
(R. P. B.—1940)

A. 0·904 gm./c.c.

37. 20 gm of water at 15°C are put in the tube of a Bunsen's ice calorimeter and it is observed that the mercury thread moves through 29 cm. 12 gm. of a metal at 100°C are then placed in water and mercury thread moves through 12 cm. Find the specific heat of metal. Art. 55

A. 0·1.

38. Explain the construction and use of Bunsen's ice calorimeter indicating the special advantage of the method over the other method of measuring specific heat. Art. 55
(P. U.—1953; R. P. B.—1952; Utkal. U.—1948)

39. Steam at 100°C is passed into a calorimeter of mass 55 gms. containing 150 gms. of a mixture of ice and water, until all the ice is just melted. The mass of the calorimeter and its contents is then found to be 217 gms. How much ice was originally present in the mixture. Arts. 54 & 53

Reference

(Latent heat of fusion of ice = 80 cal/gm.; Latent heat of steam = 540 cal./gm.). (Utkal. U.—1947)

A. 96 gm.

Arts. 54 & 56

40. Define latent heat of evaporation and describe an experiment to measure the latent heat of steam. (Utk. U.—1953; And. U.—1952; Del. H. S.—1950; Pat. U.—1951, '53)

Arts. 54 & 56

41. Into a calorimeter, containing 175 gm. of water and some ice, steam of mass 10 gm. and temperature 100°C. is passed. The temperature of the contents rises to 10°C. If the water equivalent of the calorimeter is 5 gm. calculate the mass of ice initially present. Given latent heat of water = 80 cal../gm.; latent heat of steam = 540 cal./gm. (And. U.—1952)

A. 50 gm.

Art. 56

42. In an experiment to find the latent heat of steam, the following observations were recorded. Mass of calorimeter and stirrer = 132 gm. (sp. ht. = 0·095); Mass of calorimeter with water at 28·5°C = 229 gm; Mass of steam at 98°C condensed = 0·72 gm.; Final corrected temperature = 32·5°C. Find the latent heat. Point out the chief sources of error and suggest the necessary precautions or corrections. (Mad. U.—1947)

A. 543 cal./gm.

Arts. 54 & 56

43. A copper calorimeter, weighing 100 gm. contains a mixture of 10 gm. of ice and 100 gm. of water at 0°C. Steam at 100°C is passed into the mixture until the final temperature of the calorimeter and contents is 10°C. Determine the mass of steam which has passed into the calorimeter, given that:—Latent heat of steam = 540 cal./gm.; Latent heat of ice = 80 cal./gm.; sp. ht. of copper = 0·1. (Pat. U.—1953)

A. 3·17 gm.

44. The diameter of a capillary tube of a Bunsen's ice calorimeter is 1·4 mm. When a piece of metal weighing 11·09 gm. at 100°C is dropped into the calorimeter, the mercury thread moves by 10 cm. Calculate the specific heat of the metal. The latent heat of ice is 80 cal./gm. and density of ice is 0·9 gm./c.c. (Del. H. S.—1952)

A. 0·1.

Art. 56

45. A copper vessel of water equivalent 60 gms. contains 600 gms. of water at 30°C. A bunsen burner, adjusted to supply 100 calories per second, is used to heat the vessel. Neglecting all heat losses calculate (*a*) the time required to raise the water to the boiling point and (*b*) The time required to boil away 50 gms. of water. (Latent heat of steam = 540 calories). (C. U.—1952)

A. (*a*) 7 min. 42 sec.; (*b*) 12 min. 12 sec. (from starting).

Art. 55

46. Describe Joly's Steam Calorimeter and explain how it may be used to determine the specific heat of a gas at constant volume. (Nag. U.—1950; U. P. B.—1952; R. P. B.—1945, '49)

CHAPTER VI

CHANGE OF STATE

58. Fusion and Solidification.—Fusion or *melting* is the change of a solid substance from the solid to the liquid state. Solidification or *freezing* is the change of a liquid substance from the liquid to the solid state. If a solid body be heated continuously, its temperature gradually rises until at a certain temperature it begins to melt. If once the process of melting has started, the temperature does not change until the whole of the substance is converted into liquid state. This temperature which is different for different solids, is called the **melting point** of the substance. If the heating is continued even after the whole of the solid has melted, the temperature of the liquefied mass rises in the usual way.

Take a mass of melting ice in a suitable vessel and put a thermometer in it ; the temperature of the ice is found to be 0°C. Apply heat to the vessel ; ice begins to melt, stir well. The thermometer however, does not show any change of temperature until the *whole* of the ice is melted. If a quantity of paraffin wax is heated in a similar way, it is found that the paraffin begins to melt at nearly 55°C, and the temperature remains practically constant at this point until the whole mass is melted.

On the other hand, if a mass of liquid is continuously cooled, the temperature gradually falls until a certain temperature the liquid begins to solidify. The temperature, which is different for different liquids, remains constant throughout the process of solidification and is called the **freezing point** or the *solidification temperature* of the liquid. If the cooling is continued even after the whole of liquid is solidified, the temperature of the resulting solid decreases in the usual way.

Super-cooling.—Many liquids, if cooled slowly and without any disturbance, may be cooled below their solidifying point without solidification taking place ; *e.g.*, pure water freed from dissolved air, has been cooled to −20°C. by Despretz without being frozen. This phenomenon is known as *super-cooling*, *surfusion* or *superfusion* and the liquid in this condition is said to be *super-cooled*. This condition is unstable ; for if the liquid is slightly shaken or if a piece of the substance in a solid form is dropped into the liquid, solidification at once starts and the temperature quickly rises to the true solidification point. For super cooling, the liquid taken must be *pure* and free from any suspended or dissolved foreign substances.

Sublimation.—There are certain solids which under ordinary circumstances pass directly into the vaporous state without passing through the intermediate liquid state and *vice versa*. The process is

known as *sublimation*. Iodine crystals, camphor, sulphur, arsenic, benzoic acid, napthalene are examples of these ; ice and snow also sub lime slowly, even when below the freezing point.

59. **Determination of Melting Point.**—The melting point of a solid can be determined by any one of the following methods.—

(a) Capillary Tube Method.—A small quantity of the substance in the liquid state is sucked into a thin walled capillary tube and then the lower end of the tube is sealed. The tube C (Fig. 46) is fastened to the thermometer T, the sealed end being close to the bulb. The combination is now supported in a suitable water or oil bath in a glass vessel, the open end of the capillary tube being of course above the liquid. The bath is slowly heated and kept well-stirred. After some time a temperature is reached when the solid in the capillary tube just begins to liquefy ; the temperature of the bath is noted. The flame is then removed, the bath is kept well-stirred and is allowed to cool. The temperature is again noted just when the substance in the tube begins to solidify. If carefully observed, the two readings should not differ more than one degree and their mean is taken as the melting point of the solid.

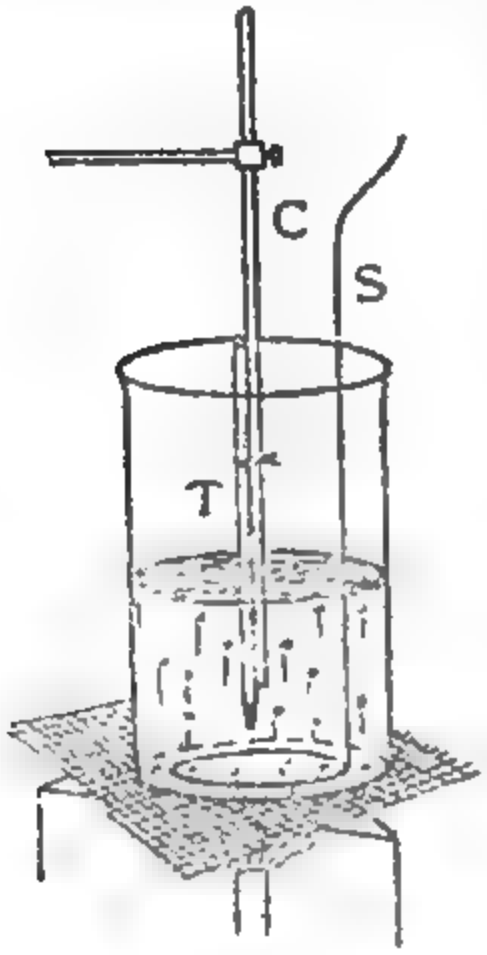


Fig. 46

This method is specially suitable when a very small quantity of the solid is available which has not a high melting point. (Vide. J. Chatterjee's Intermediate Practical Physics).

(b) Cooling Curve Method.—A mass of the solid is taken in a small calorimeter and is heated in a water or oil bath, as the case may be, until the whole is melted and the temperature rises a few degrees above the melting point. The bulb of a thermometer is kept immersed in the liquid. The calorimeter is placed on a piece of cork inside a larger vessel and is allowed to cool undisturbed. The temperature of the substance in the calorimeter is noted at regular intervals, say half a minute, as the cooling proceeds. At a certain temperature, solidification sets in and the thermometer records a constant temperature until the whole mass is solidified. After this the temperature decreases again and the observation is continued for some time after the whole mass is solidified. A *curve* is now plotted with time as abscissa and temperature as ordinate. The general form of the cooling curve is as shown in fig. 47. It would be noticed that during the process of solidification the portion of the curve is practically parallel to the time axis, which indicates a *constant* value for the temperature during the period. If the solid is heated again, it would be found that at a certain temperature the solid beings to melt and the temperature remains constant until the whole mass of the solid has melted. The temperature which remains constant during the process of solidification as well as of fusion is found to be

the same. This is termed the *melting point* of the particular solid or the *freezing point* of the corresponding liquid.

There are certain substances, such as glass, pitch, wax, fat, etc which are solid at ordinary temperatures but on being heated they pass before melting through a *plastic* or viscous state, intermediate between solid and liquid. This state may extend over a considerable range of temperature, depending on the particular substance. Again there are certain liquids such as glycerine, acetic acid and some other organic acids and oils, which pass through an intermediate *viscous* state before changing into solid state. These liquids have, therefore, no definite solidification point. In case of a substance which is a mixture of different substances *e g.*, paraffin wax, we do not get a single level step in the cooling curve, as shown in fig. 47, but a series of level steps.

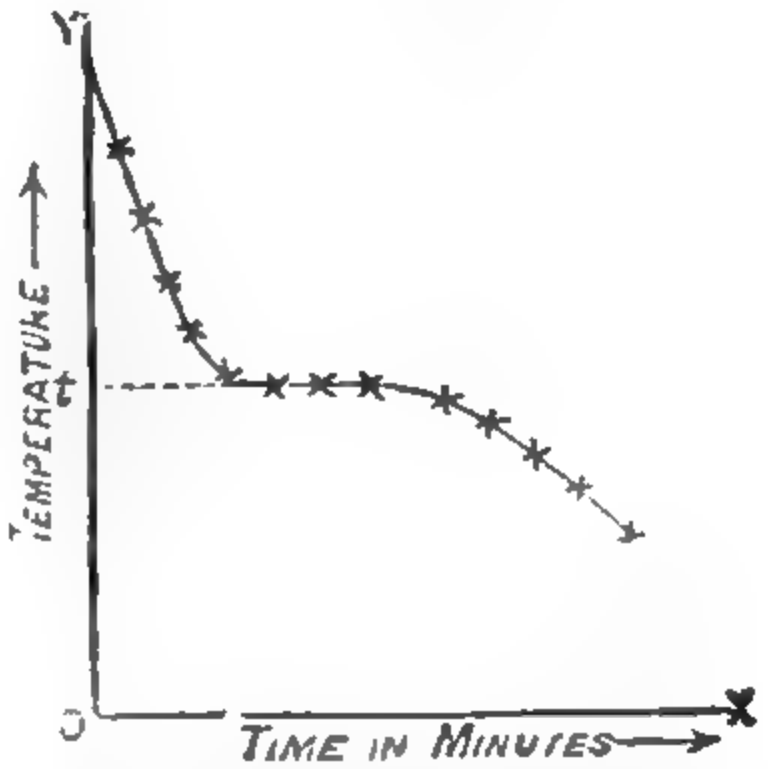


Fig. 47—Cooling Curve

60. Change of Volume on Fusion and Solidification.—A substance always changes in volume on melting. In general, a substance expands on melting and contracts on solidification. There are, however, a few solids, *e g.*, ice, cast iron, bismuth, antimony, which contract on melting. In the former case, the solid sinks in the resulting liquid and in the latter the solid floats on the corresponding liquid.

The *expansion of water on freezing* is very nearly 9% so that 11 parts by volume of water at 0°C. become 12 parts by volume of ice at the same temperature, which means that, 1 c.c. of water at 0°C. forms 1·09 c c. of ice at 0°C. Ice therefore, floats on pure water. A tumbler of water with a lump of ice floating on it is taken. The level of the water is noted. Ice gradually melts away. The volume of water formed by the melting of ice is less by $\frac{1}{11}$th of the volume of ice, also the volume of ice above water is $\frac{1}{11}$th of the total volume of ice taken. Hence the volume of the water formed by the melting of ice exactly occupies the space formerly occupied by the immersed portion of ice ; *so there is no change in the level of the water in the vessel when the ice melts.*

If water is contained in a closed space, its expansion on freezing takes place with a great force A hollow ball of stout cast iron is completely filled with water, closed with a screw stopper and is placed in a freezing mixture of ice and salt. After a time the water inside the ball freezes when the ball bursts asunder with a crackling sound. If a bottle, completely filled with water and tightly corked, be kept in open air on a frosty night, it would burst. In cold countries water pipes may burst during a frost owing to the freezing of water inside.

If water would contract on freezing, the ice formed would go to the bottom and a fresh layer of water would freeze ; so during a frost in cold countries rivers, lakes, etc. would be converted into blocks of solid ice and all aquatic animals would perish. But ice being lighter floats on the surface of water averting such a calamity. Also ice, being a bad conductor of heat, does not allow all the heat of water below ice to pass out and so the water does not freeze to a very great depth however low might be the temperature above. Cast iron expands on solidification : a lump of cast iron therefore floats on the molten metal. In an iron casting, the mass of molten iron expands on solidification and is forced into intimate contact with every part of the mould, so the castings are very sharp and well-defined in outline.

61. Effect of Pressure on Melting Point.—The melting point of a solid is found to depend on the pressure to which it is subjected. The nature of the effect is as follows : substances which *contract on melting e.g.*, ice, have their melting points *lowered* by increase of pressure ; while substances which *expand on melting e g.*, paraffin wax, have their melting points *raised* by increase of pressure.

The above facts might be expected from a simple consideration. Ice contracts in volume on melting. External pressure tends to decrease the volume and therefore to help the process of melting. Hence under increased pressure ice melts at a lower temperature. It has been calculated that the melting point of ice at 0°C is lowered by about 0·0073°C. for an increase of pressure of one atmosphere, that is to say that the melting point of ice is lowered to —1°C. by a pressure of about 137 atmospheres.

Again paraffin wax expands on melting ; external pressure tends to decrease the volume and therefore to oppose the process of melting. Hence under increased pressure its temperature is raised to bring about fusion. This is true for all substances which expand on melting.

Regelation.—If two pieces of ice are pressed together for a few seconds, then on releasing the pressure, the pieces are found to be frozen together at the region of contact. The pressure lowers the melting point of ice and so the ice melts at the surface of contact. On releasing the pressure, the melting point rises again and the water formed freezes, binding the two pieces. The phenomenon of the melting of ice due to pressure and the subsequent re-solidification when the pressure is removed, is known as **regelation** (*re*, again : *gelare*, to freeze). This was first noticed by Faraday. The *lowering of the melting point of ice by increased pressure* can be demonstrated by the following experiments.

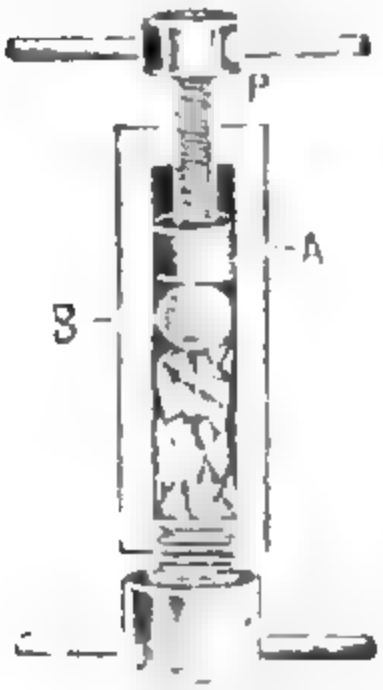


Fig. 48

Mousson's Aapparatus.—This consists of a stout iron cylinder A (Fig 48), closed at one end with a screw plug and fitted at the other with a screw plunger P. The cylinder is partly filled with water which is frozen by placing the

cylinder inside a mixture of ice and salt. A small metal ball B is then placed on the top of the ice within the cylinder which is now closed by the screw plunger. The whole is then surrounded by melting ice and pressure is gradually applied by working the plunger in. When the screw at the bottom is taken out, the metal ball is found to come out first. This shows that during the application of pressure ice melted and the ball went down ; hence the melting point of ice is lowered by the increased pressure. When the pressure is released, the water freezes again, leaving the ball at the bottom.

Bottomley's Experiment.—A large block of ice is placed on a suitable support (Fig. 49). A loop of wire with a heavy weight attached underneath is passed over the block near about its middle. It is found that the wire cuts its way slowly through the block and ultimately passes out. The block, however, remains unbroken, the track of the wire being marked by a section of ice.

It can be explained thus The ice just under the wire being subjected to great pressure has its melting point lowered and therefore it melts. The latent heat necessary for the melting of ice is taken practically from the surrounding ice. Hence the temperature of ice near the wire gets below 0°C. The wire passes down and the water produced flows above. The water is now free from pressure and since its temperature is 0°C. in contact with ice below 0°C., it freezes again and thus joins the two surfaces of ice together.

As the heat necessary for fusion is to be partly conducted through the wire from the upper to the lower side, the wire passes heat more rapidly if it be made of a better conductor ; thus, a copper wire cuts through more quickly than iron wire of the same diameter and a piece of thread will cut through very slowly. Snow balls can be formed by pressing pieces of snow. The markings of the slate or of the wheels of a heavy cart on ice can be similarly explained. The motion of *glaciers* may be attributed to some extent to this cause. The layer of ice at the bottom of the glacier being subjected to a very great pressure melts and the water oozes out but re-solidifies. Thus the bottom being soft, the glacier begins to move down the incline.

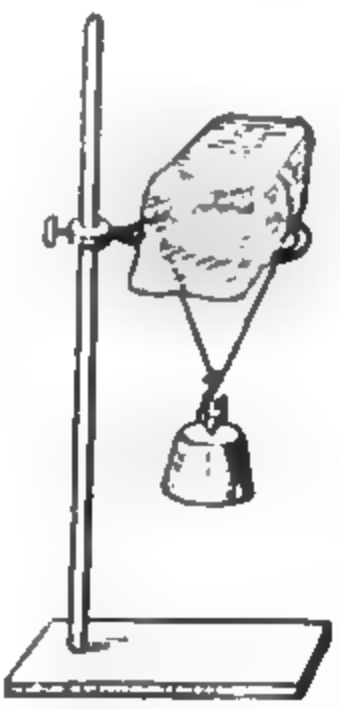


Fig. 49

62. Freezing Mixture.—The process of the solution of a solid in a liquid is akin to a change of state. Thus, if a quantity of sugar or any salt is dissolved in water, the necessary heat for solution is taken from the water and the solution, therefore, cools down. When common salt is mixed with ice, a quantity of salt goes into solution absorbing some heat from ice. Thus the mixture would have a temperature below 0°C. This is the principle upon which *freezing mixture* acts. Thus by dissolving crystals of sodium sulphate or ammonium nitrate in cold water, a temperature of −15°C can be obtained.

When the temperature of the freezing mixture falls below 0°C and some salt goes into solution, there is an internal pressure due to dissolved salt, acting within the solution. This is called the solution pressure. This solution pressure helps to liquefy more ice and then an additional quantity of salt goes into the solution absorbing more heat from the mixture. This explains the lowering of temperature of a freezing mixture. Ultimately an equilibrium of temperature is attained when the rate of supply of heat from the surrounding atmosphere is equal to the rate of absorption of heat due to the dissolved substance. This is the minimum temperature attained by the mixture.

Some Freezing Mixtures

Ingredients	Parts by weight	Temp. of the mixture
Ammonium Nitrate	10	— 17·4°C
Ice	10	
Powdered ice	10	— 22°C
Common salt	1	
Calcium chloride	10	— 35°C
Ice	2	
Carbon dioxide (solid)	4	— 77°C
Ether	1	

When accumulated ice melts, it absorbs the latent heat from air whereby the temperature of the air falls and fog is formed. Again when a liquid passes into the solid state it gives out the latent heat. Thus during a snow-fall, the moisture in the atmosphere changes into snow and gives out the latent heat, whereby the air feels warmer.

63. Laws of Fusion—(1) A solid under constant pressure begins to melt at a certain temperature which is constant for the same solid and which is called the *melting point* of the solid at the given pressure. The melting point of a solid is the same as the solidification point of the corresponding liquid.

(2) The rate of fusion is proportional to the supply of heat but the temperature of the solid remains constant at the melting point until fusion is complete.

(3) If a substance expands on melting such as paraffin wax, its melting point is raised by pressure. If the substance contracts on melting, such as ice, its melting point is lowered by pressure.

(4) Until mass of each substance absorbs during fusion a definite quantity of heat, known as its *latent heat of fusion*, which is constant for the particular substance melting under the same conditions.

64. Vaporisation and Liquefaction.—The change of a substance from the liquid to the vapour or gaseous state is known as **vaporisation** ; while the change of a substance from vapour to liquid is known as **liquefaction** or *condensation*.

According to the circumstances under which it takes place, the

process of *evaporation* is distinguished as (*i*) **evaporation** and (*ii*) **ebullition** or **boiling.**

Evaporation —If some water contained in a shallow dish be left at the ordinary temperature in a fairly dry room or in the open air, the quantity of water is found to decrease gradually and to disappear ultimately. Methylated spirit, turpentine, ether, etc., disappear much more rapidly. In all these cases the liquid gradually passes into the gaseous state and as the mass of the liquid is not disturbed, the change must be taking place at the surface of the liquid. The process is known as *evaporation*. Evaporation, then, is a slow change from the liquid to the gaseous state which takes place at the surface of the liquid and at all temperatures. It is found from above that in an *unlimited space* such as open atmosphere, the process of evaporation goes on continuously until the whole of the liquid is changed into vapour. But in a closed or *limited space*, it would be found that after some time the liquid does not further decrease in volume. This shows that under definite conditions a limited space can contain only a limited quantity of vapour.

Ebullition or Boiling.—Let us take some water in a glass flask closed with a stopper through which an exit tube is fitted and a thermometer is inserted into the water. As the flask is heated over a Bunsen's flame, the mass of the liquid gradually rises in temperature and the rate of evaporation is also found to increase. On the inside of the walls of the flask some bubbles appear which become larger with rise of temperature of the liquid and finally detach from the walls of the flask and rise towards the surface. These are bubbles of air dissolved in the liquid. As temperature further rises, bubbles of steam form at the bottom of the flask and while rising up they collapse and condense with a sharp click which causes the water to 'simmer'. As the temperature of the liquid increases, bubbles reach the surface of the liquid when simmering cease. Bubbles of steam are then found to escape vigorously throughout the mass of the liquid and temperature of the water is found to remain steady. The water is now said to be *boiling* or in a state of ebullition.

Thus *ebullition* or *boiling* is the *rapid* change of a substance from the liquid to the gaseous state which takes place *throughout the mass* of the liquid and at a *definite* temperature, under given conditions, for a particular liquid. The temperature remains constant throughout the process provided the pressure remains unchanged. The **boiling point** of a liquid is a particular temperature, at which it changes from liquid to vapour under definite conditions and which remains constant so long boiling goes on. Different liquids have different boiling points

If a quantity of water which has been previously boiled to drive away dissolved air be allowed to boil, it may sometimes be found that there is no continuous bubbling. For suddenly a large bubble forms and bursts forth so as to throw the water violently upwards. The temperature of the liquid fluctuates slightly. The liquid again remains

quiescent and similar boiling takes place after an interval. This is known as **boiling by bumping.** If a small piece of cotton or a fragment of glass or porcelain be introduced into the liquid, bumping ceases and continuous boiling takes place ; then bubbles of steam are given off from the surface of the substance.

Distinction between Evaporation and Ebullition.—The following are the distinguishing features between evaporation and ebullition.—

(*i*) Evaporation is a *slow* process but ebullition is a *rapid* one.

(*ii*) Evaporation takes place at the *surface* of a liquid while ebullition occurs *throughout its mass.*

(*iii*) Evaporation takes place at *all temperatures* while ebullition occurs at a *definite* and *constant* temperature, called the boiling point, depending on the nature of the liquid and the general conditions.

Similarity between Fusion and Ebullition.—The following are the points of similarity between fusion and ebullition.—

(*i*) The temperature remains constant throughout each process.

(*ii*) The latent heat is absorbed during each change.

(*iii*) There is usually an increase in volume during each change.

(*iv*) Most pure liquids may be cooled under suitable conditions below their normal freezing point without solidification taking place, so that they may be heated above their boiling points without boiling.

(*v*) The freezing point as well as the boiling point of a liquid change with pressure.

(*vi*) The freezing point of a solution is *lower* than that of the pure solvent and the boiling point of solution is *higher* than that of the pure solvent, the difference in each case being approximately proportional to the concentration of the solution, *e.g.*, a saturated solution of common salt freezes at about – 22°C. and boils at 108°C.

65. Factors influencing Evaporation.—The rate of *evaporation* of a liquid is experimentally found to depend upon the following factors :—

(*a*) *The nature of the liquid.*—The lower is the boiling point of a liquid, the greater is the rate of evaporation.

(*b*) *The temperature of the liquid.*—The higher is the temperature of the liquid, more rapid is the evaporation. This liquid medicines containing alcohol as an ingredient are labelled 'to be kept in a cool place', to prevent rapid evaporation.

(*c*) *The exposed surface.*—As a general rule, the larger is the surface of the liquid exposed to air, the quicker is the evaporation : *e.g.*, a given quantity of a volatile liquid will evaporate more rapidly when contained in a plate than in a tumbler.

(*d*) *The pressure on the surface*—The less is the pressure on the surface, the higher is the rate of evaporation. Thus in vacuum the evaporation is exteremely rapid. In pharmaceutical works, liquid

extracts are rapidly concentrated and solid extracts are manufactured by the process of *evaporation in vacuo*.

(*e*) *The pressure of the vapour in air close to the liquid surface.*—The less is the quantity of the vapour already present in a given volume of the air, the greater is the rate of evaporation. The water evaporates more rapidly in dry than in damp air. For this reason wet linen and muddy roads dry more quickly in winter than in wet seasons.

(*f*) *The renewal of the air in contact with the surface of the liquid.*—When the layer of air above the liquid surface, charged with the vapour, is removed either by wind or mechanically, say by a fan, a fresh layer of air comes in the place and helps evaporation which thus proceeds rapidly and continuously. Hence wet linen dry more rapidly on a windy than on a calm day.

Factors influencing Boiling Point.—The boiling point of a liquid is influenced more or less by—

(*a*) *the vapour pressure on the surface of the liquid.*

(*b*) *the presence of any dissolved impurity,*—solid, liquid or gaseous. It is for this reason that in determination of the boiling point of a liquid, the bulb of the thermometer should be placed *in the vapour* of the liquid and not in the liquid itself. Again the boiling point of a solution is always higher than that of the pure solvent.

(*c*) The boiling pont depends to a small extent on the material of the vessel and degree of cleanness of its inner surface.

66. Vapour Pressure.—The vapour of a liquid formed at any temperature exerts pressure like a gas, which is called the *vapour pressure of* the liquid at the corresponding temperature.

Take two barometer tubes A and B (Fig. 50), each about a metre long completely filled with pure dry mercury. Close the open end and invert both of them over a trough containing pure dry mercury. Clamp both the tubes vertically side by side. The mercury in both of them stands at the same level. The height of the mercury column from the surface of mercury in the trough indicates the atmospheric pressure at the time. The space above mercury in each tube is Torricellian vacuum and contains only mercury vapour, having negligible pressure.

Into one of the tubes, say B, introduce one drop of water by means of a small bent pipette. The water being lighter rises to the top of the mercury column and is immediately evaporated. The level of the mercury column in B comes down to some extent. This indicates that the vapour formed, exerts pressure within the closed space of B and the difference of mercury levels in A and B measures the pressure of the water vapour.

Again introduce one or two drops of water into it. The water evaporates and the increased quantity of vapour exerts greater pressure, as is shown by a further depression of the mercury column. Continue adding water till a thin layer of water at C appears on the surface of the mercury column, showing that the liquid does no longer evaporate.

From the difference of the heights of the mercury columns, the pressure exerted by the vapour at the time is obtained. Introduce a few more drops of water; the mercury column is no longer depressed which indicates that the pressure exerted by the vapour has reached its *maximum value*.

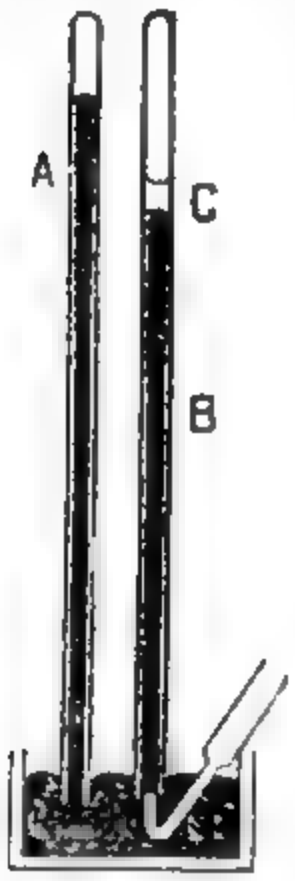


Fig. 50

This shows that a closed space can hold only a limited quantity of vapour under given conditions. In this condition the space is said to be **saturated** with the vapour. Hence, if in a closed space the vapour remains in contact with its liquid, the space is definitely saturated with the vapour. The pressure exerted by saturated vapour is consequently *maximum* and is referred to as the maximum vapour tension or *saturated vapour pressure* at the given temperature.

When a space contains less than the maximum amount of vapour that it can possibly contain at the given temperature, it is said to be **unsaturated** and the pressure now exerted is known as the *unsaturated vapour pressure*. It is evident that at a given temperature the pressure of unsaturated vapour is less than that of the saturated vapour. If the above experiment be repeated with different liquids, it will be found that *at any given temperature, the saturated vapour pressure of different liquid is different.*

67. Effect of Pressure on Saturated and Unsaturated Vapours.—Any vapour in contact with its liquid in a closed space is always saturated. It is found that *saturated vapour does not obey Boyle's law but unsaturated vapour does.*

(1) *Effect of Change of Volume of Saturated vapour at Constant Temperature,*—Take a clean barometer tube and filling it completely with pure dry mercury invert the tube over a deep cylindrical trough D. (Fig. 51) also containing pure mercury. Introduce sufficient water into the tube so that the mercury column is depressed to the maximum extent by the water vapour and a layer of water remains on the mercury column. The volume of the enclosed vapour is proportional to the length of the space above the mercury head and the difference between the heights of the mercury column and the barometric height gives the pressure of the vapour.

Now push the tube gradually into the trough and observe the height of the mercury column. Note that as the space above the mercury column occupied by the vapour is *decreased*, more and more of the vapour is condensed into water, but the height of the mercury column from the surface of the mercury in the trough remains *unaltered*, showing that the pressure of saturated vapour does not change with volume.

Again gradually raise the tube, taking care that its open end is always under mercury and that some water is left on the surface of

mercury. As the inside volume is *increased*, some of the water quickly evaporates to saturate the space but the height of the mercury column from the mercury in *D* remains the same. This indicates that the saturated vapour pressure is *unaltered*. This takes place until the volume of the space above mercury is increased to such an extent that the whole amount of water gets evaporated. Any further increase in the volume would make the space unsaturated and the pressure of the vapour would be proportional inversely as the volume following Boyle's law.

Fig. 51

The above experiment indicates that at constant temperature the amount of vapour necessary to saturate a space is proportional to the volume of the space but the pressure exerted by the saturated vapour remains the same whatever be the volume of the space. Hence at constant temperature the pressure exerted by saturated vapour is independent of its volume which means that **saturated vapour does not obey Boyle's law.**

(*i*) *Effect of Change of volume of Unsaturated Vapour at Constant Temperature.*—Take the same apparatus as in fig. 51. Introduce a drop or two of water into the tube and observe the depression of the mercury column as the water evaporates. Raise the tube until the water is completely evaporated ; now raise the tube a little further to be sure that the space above the mercury becomes completely *unsaturated*. Now gradually raise the tube, taking care that the open end remains always under mercury. Note that as the volume of the vapour is *increased*, the height of the mercury column increases, which indicates that the pressure of the enclosed vapour *decreases*. Find the pressure of vapour in each case and note that the product of the volume and the corresponding pressure of the vapour is practically constant.

Again push the tube gradually into the trough Observe that as the volume of the vapour is *decreased*, the height of the mercury column decreases, indicating an *increase* in the pressure of the vapour. Note that in this case the product of the volume and the corresponding pressure of the vapour is practically constant. This goes on until the diminished space becomes saturated with the vapour contained when liquid would appear and the pressure would be the maximum for that temperature. Any further decrease in volume will *not* change the pressure, only more and more of the vapour will be condensed. Hence at a constant temperature the product of the volume and the pressure of unsaturated vapour is approximately constant *i e*, **unsaturated vapour approximately obeys Boyle's law.** The above facts can be verified with vapour of any other liquid.

68. Effect of Temperature on Saturated and Unsaturated Vapours.—It would be found that saturated vapour does not obey Charles' law but unsaturated vapour does.

(1) *Change of Temperature of Saturated Vapour.*—Take two barometer tubes *A* and *B* filled with pure dry mercury and invert them over a mercury through *C* (Fig. 52). Jacket the tubes by a water glass tube J, reaching almost upto the mercury surface and having its lower end closed by a cork through which the tube passes. The jacket contains water which can be heated by introducing steam through a glass tube reaching almost down to the bottom and the temperature is noted by a thermometer T, suspended within the jacket. Fill the jacket with water at room temperature. The height of the column of mercury from the mercury surface in the trough gives the barometric height at the time. Introduce sufficient water into one of the tubes, say B, by means of a bent pipette, so that the mercury colomn is depressed to the maximum extent and a layer of water remains on the mercury column. The difference in height of the two columns gives directly the saturated vapour pressure at the temperature of the bath. By putting the tubes within the same bath both the mercury columns are equally affected by a change of temperature.

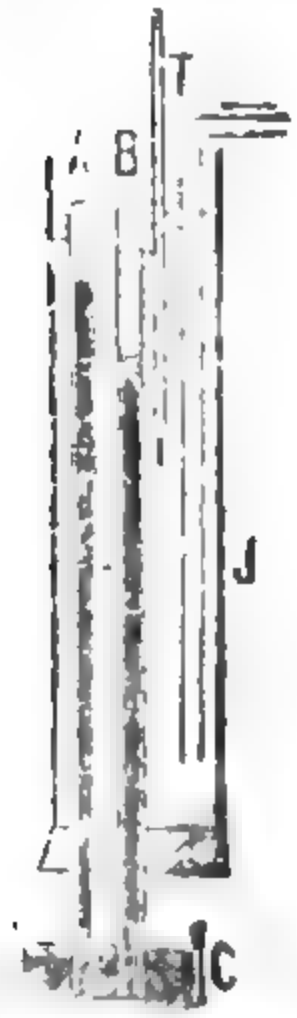


Fig. 52

Now lower the temperature of the bath by putting ice into it. Note that as the temperature falls, more and more of the vapour is condensed and the height of the mercury column increases which indicates that the pressure of the vapour, which is always saturated, decreases as the temperature falls. The difference in the heights of the mercury columns in the two tubes gives directly the pressure of the vapour at the corresponding temperature. Again raise the temperature of the bath by passing steam into it Note that as the temperature rises, more and more of the water is evaporated and the mercury column gradually goes down. This indicates that the pressure of the saturated vapour increases with a rise of temperature. The experiment can be continued upto a temperature till the water is just completely evaporated, after which the vapour becomes unsaturated.

It is to be remembered that the pressure of the *saturated* vapour is not affected by a change of volume so long as the temperature remains constant. Therefore the pressure of *saturated vapour* changes with change of temperature, but not according to Charles' law, being always equal to saturated vapour pressure at the particular temperature, which means that **saturated vapour does not obey Charles' Law.**

(2) *Change of Temperature of Unsaturated Vapour.*—The apparatus which can be conveniently used for the purpose consists of a wide graduated tube AB (Fig. 53), having at one end a funnel tube E provided with stopcocks S_1 and S_2. The lower end of AB is connected by a thick walled flexible rubber tubing to another glass tube DE which is open at the top. The lower parts of AB and DF and the whole of the rubber tubing are filled with pure dry mercury. The tube AB is jacketted by a wide glass tube J closed at the lower end by a cork through which the lower end of AB passes. The jacket contains water which can be

heated by passing steam into it by a glass tube I, reaching almost upto the bottom of the bath ; the temperature is given by the thermometer T. The jacket tube is held vertically by the clamp C fixed to a suitable stand. The tube DF can be moved up and down along a vertical stand against a metre scale and can be clamped at any position.

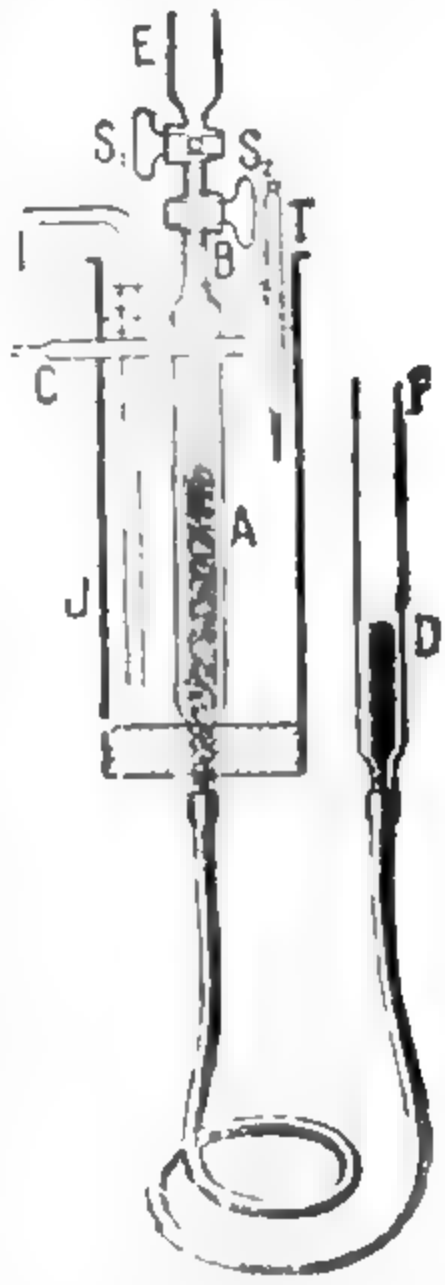


Fig. 53

Open S_1, S_2 and raise DF until mercury in AB comes out of S_1, so that the air in AB is completely driven out. Close the stop-cocks and lower FD until the mercury level in AB goes down sufficiently to leave a vacuous space above the mercury surface. Evidently the difference in the mercury levels in AB and DF is the barometric height at the time. Put some liquid, say water in the funnel ; open S_1 when some of the liquid occupies the space between S_1 and S_2. Close S_1 and then open S_2 so that a very *small* quantity of water is discharged into the vacuous space in AB and gets at once evaporated. Observe that the level of mercury in AB sinks down owing to the pressure of the unsaturated vapour. Increase the space above mercury a little more by lowering FD to be sure that the space becomes unsaturated. Note down the position of the mercury level in AB. The difference in the mercury levels in AB and DF subtracted from the initial difference *i.e.*, the barometric height, gives the pressure of the unsaturated vapour at the temperature.

Now gradually raise the temperature of the bath by passing steam into it and at intervals maintain the temperature constant for some time by regulating the flow of steam. Observe that as the temperature rises, the mercury column in AB is depressed more and more. Raise DF each time till the mercury lever in AB comes to the original position so that the volume of the vapour is kept constant. Note that the difference in the mercury levels in AB and DF gradually decreases which indicates an increase in the pressure of the vapour. It is found that the increase of pressure with temperature takes place approximately according to Charles' law. If the temperature of the jacket is decreased and if the volume of vapour is kept constant by lowering DF, the pressure of the vapour is found to decrease with a fall in temperature approximately according to Charles' law. This goes on, till for a certain temperature the space become saturated with the vapour contained. If the temperature is further decreased, the pressure falls at a quicker rate, being always the saturated pressure for the temperature in question. It is found therefore, that with change of temperature, the pressure of unsaturated vapour changes according to Charles' law, which means that the *unsaturated vapour approximately obeys Charles' law.* The above facts may be verified with vapour of any other liquid.

69. Distinction between Saturated and Unsaturated Vapours. —When a space contains the maximum quantity of any vapour which

can be possibly contained in the volume as the given temperature, the space is said to be *saturated* with the vapour. The pressure exerted by the saturated vapour is the maximum for that temperature. The presence of some of the liquid in a closed space ensures that space is saturated. If the space does not contain the maximum possible amount of vapour at the given temperature, the space is then *unsaturated* with the vapour.

At constant temperature, the *pressure of saturated* vapour does not depend on a change of volume. If the volume is *decreased*, some of the vapour is condensed, but the pressure remains the same. Again if the volume is *increased*, some of the liquid at once evaporates to saturate the space, the pressure remaining the same as before. In case of *unsaturated* vapour, a change in volume at constant temperature produces a change in pressure approximately according to Boyle's law. If the pressure is increased, the volume decreases; this goes on until the volume become saturated with the vapour, after which the pressure can not be changed any further but any attempt to compress the volume would condense more and more of the vapour. *Saturated vapour, therefore, does not obey Boyle's law but unsaturated vapour does approximately.*

If the *temperature of saturated* vapour in contact with its liquid is increased, more and more of the liquid evaporates, the pressure increases with temperature, being always the saturated pressure at the particular temperature. If the temperature is *decreased*, some of the vapour is condensed, the pressure falling to the saturation pressure for that temperature. The change of pressure in either case does not, however, take place according to Charles' law. But in case of *unsaturated* vapour, the pressure changes with temperature approximately in accordance with Charles' law. If the temperature decreases, the pressure decreases till at a certain temperature the space becomes saturated with the vapour present. On further lowering the temperature, more and more of the vapour is condensed, the pressure each time coming to the saturated pressure at the particular temperature. *Saturated vapour, therefore does not obey Charles' law but unsaturated vapour does approximately.*

The following Table supplies the saturated vapour pressure of water with corresponding temperature.—

Saturated Vapour Pressure of water with Temperature

Temp. °C	Press. mm.	Temp. °C	Press. mm.	Temp. °C	Press. mm.
4	6·54	16	14·52	28	29·94
6	7·51	18	16·46	30	33·57
8	8·61	20	18·62	32	37·59
10	9·84	22	21·02	34	42·02
12	11·23	24	23·69	36	46·90
14	12·73	26	26·65	38	52·26

70. Pressure by a Mixture of Gases and Vapours.—Take the same apparatus as in Fig. 53. Keep the jacket filled with water at the room temperature. Open S_1 and S_2 and draw a quantity of dry air into AB. Adjust DF, so that the mercury in AB and DF is at the same level. The air in AB is now at atmospheric presure. Mark the position of the mercury in AB. Put some ether in the funnel and introduce through the stop cocks sufficient liquid into AB, so as to leave some of it unevaporated. Observe that the mercury column in AB is depressed by the pressur of the vapour. Now raise DF till the level of mercury in AB is brought back to its former position. As the volume of the air is the same as before, its pressure is equal to the atmospheric pressure. The difference of mercury levels in DF and AB is therefore, equal to the pressure of saturated vapour at the room temperature.

Again open S_1 and S_2 and raise DF till the air, the ether vapour and a little of the ether are driven out of AB, some only being left on the mercury surface in AB. Close S_1 and S_2 and lower DF sufficiently to get the ether evaporated. The difference of mercury levels in DF and AB remains the same even though DF is lowered still further, provided some ether is left in AB. This difference gives the pressure of the saturated ether vapour at the room temperature, when formed in vacuum. It is found that the *pressure of the saturated ether vapour at a particular temperature, whether formed in vacuum or in presence of air, is the same.*

If in the above experiment the volume of the space over mercury in AB be increased or decreased by lowering or raising DF, the total pressure due to air and vapour will be found to change. But if the alteration in pressure due to a change of volume of the air, as given by Boyle's law, be taken into account, the pressure due to the vapour alone is found to remain the same, provided there is enough of the liquid to keep the space saturated. But if the space is unsaturated, then for any change in volume, the total pressure is found to change according to Boyle's law. If the experiment be repeated with water or any other liquid, similar result will be obtained. Again if the saturated pressures of two liquids be *separately* determined in a closed space, the saturated pressure, when both the liquids are *simultaneously* present in a given volume, would be found to be equal to the sum of the pressures exerted by each, provided there is no chemical action between the two. The results, summarised in *Dalton's Law*, are given below.

Dalton's Laws of Vapours.—The laws *of Partial Vapour Pressure*, known as Dalton's Law, may be stated thus :—

(1) The saturated pressure of a particular vapour in a closed space at a given temperature depends only on this temperature and is independent of its volume or of the pressure of any other vapour or gas, provided they do not chemically react with each other.

(2) The total pressure exerted in a closed space by a mixture of gases and vapours having no chemical action upon one another is the sum of pressures which each would separately exert if it alone occupied the whole space at the existing temperature.

The first law applies only when the space is saturated while the second law holds good whether the space is saturated or not.

Examples :—

1. Two bulbs of volumes 1 litre and 2 litres are connected by a tube of narrow bore. Initially the air inside the two bulbs is at 0°C. and 76 cm. pressure. The smaller bulb is alone raised to the temperature of 100°C. What is the pressure inside the system? (And. U.—1952)

A. Suppose that the smaller bulb is isolated and then raised to 100°C. Then at this temperature, the pressure inside the bulb is given by $76\ (1 + 100 \times \cdot00366)$ cm. Now let the bulb be connected to 2 litres bulb supposed to be empty. If P_1 be the pressure of the bulbs of total capacity 3 litres, then from Boyles law.

$$76\ (1 + 100 \times \cdot00366) \times 1 = P_1 \times 3 \qquad \text{whence } P_1 = \frac{76 \times 1\cdot366}{3} \text{ cm.}$$

Again suppose that bigger bulb is isolated, the pressure inside being 76 cm. It is connected to smaller one supposed to be empty. Then from Boyle's law, if P_2 be the pressure of the system.

$$76 \times 2 = P_2 \times 3 \qquad \text{whence} \qquad P_2 = \frac{76 \times 2}{3} \text{ cm.}$$

From Dalton's law of Partial Pressures, the total pressure of the system then

$$P_1 + P_2 = \frac{76 \times 1\cdot366}{3} + \frac{76 \times 2}{3} = 76 \times 1\cdot122 = 85\cdot272 \text{ cm.}$$

71. Vapour and Gas : Critical Temperature.—There is not a hard-and-fast distinction in the use of the two terms. Generally the *vapour* is used to mean the gaseous state of a substance which is liquid or solid at ordinary temperature and pressure : for example, water vapour, ether vapour, alcohol vapour, vapour of iodine or of camphor. The word *gas* generally denotes a substance which under ordinary temperature and pressure remains, in the gaseous state *e g.*, oxygen, nitrogen, etc. Again a *vapour* can be easily liquefied at the ordinary temperature by increase of pressure alone, while a *gas* under similar conditions would be more and more compressed.

It is found that a vapour cannot be liquefied by pressure if it be above a particular temperature, depending on the nature of the vapour, however great the pressure may be. Hence above this temperature the vapour behaves like a gas, obeying Boyle's law. On the other hand, a so-called permanent gas, *e.g.*, hydrogen or oxygen, if cooled below a certain very low temperature, can be liquefied by suitable pressure, so that below this temperature the gas behaves as the vapour of its liquid.

There is then for every substance in the gaseous state a certain temperature such that the substance can be liquefied by suitable pressure alone if it be below this temperature ; but it can not be liquefied if it be above the temperature, however great the pressure may be. This temperature is known as the **critical temperature** of the gaseous substance. Hence a substance in the gaseous state *below* its critical temperature behaves like a *vapour* and *above* this it behaves as a *gas*. In other words, *a gas is simply a vapour above its critical temperature.*

The critical temperatures of oxygen, nitrogen and hydrogen are very *low* being respectively—118·8°C.,—147·1°C.,—239·9 C. Each gas must therefore, be cooled below its critical temperature before it can be liquefied suitable pressure.

Critical Pressure.—The pressure required to liquefy a gas at its critical temperature is called its *critical pressure.*

Critical Volume.—The volume occupied by one gramme of a gaseous substance at the critical temperature and critical pressure is called its *critical volume.*

Examples :—

1. A barometer tube dipping into mercury contains some air. A quantity of water is introduced, a part of which remains unevaporated and the height of the mercury column is now found to be 70 cm. What is the height of the column when the tube is depressed so as to reduce the volume occupied by the air to half the original volume? (Atmospheric pressure and temperature of the time = 76 cm. and 20°C. respectively; saturated vapour pressure of water at 20°C = 17·5 mm).

A. Pressure of air and saturated vapour inside = 76 − 70 = 6 cm.

∴ Initial pressure only of air = 6 − 1·75 = 4·25 cm.

Whence the volume of the air reduced to one-half, the pressure of the saturated vapour remains the same but new pressure of air becomes equal to 8·5 cm.

Let the new length of the mercury column = h cm.

Thus, $h + 8·5 + 1·75 = 76$ Whence $h = 65·75$ cm.

2. 500 c.c. hydrogen is collected over water at 25°C when the barometric height is 762 mm. Calculate the weight of hydrogen when dried. (Density of hydrogen N. T. P. = 0·09 gm. per litre; saturation water pressure at 25°C = 23·55 mm). (Mad. U.—1950)

A. Since hydrogen is collected over water, the gas is saturated with water vapour, and the pressure exerted by the mixture is the sum of the individual pressure. Let the pressure exerted by hydrogen alone at 25°C and having a volume of 500 c.c. be p.

Then according to the condition of experiment, $p + 23·55 = 762$ mm.: whence $p = 738·45$ mm.

Then from Boyle's law, $$\frac{738·45 \times 500}{273 + 25} = \frac{760 \times v}{273}$$

whence $v = 445·1$ c.c. at N. T. P.

The weight (mass) of hydrogen of volume 445·1 c.c.

$= 445·1 \times ·00009 = 0·04$ gm. approximately.

3. The uniform tube 80 cm. long and 1 sq. cm. in cross-section is filled with mercury and is inverted over a mercury trough with the open end 2 cm. below the mercury. The height of the mercury column is found to be 74·4 cm., the true atmospheric pressure being 76 cm. If the error is due to the pressure of water vapour (unsaturated), what will be the volume of the latter at the atmospheric pressure?

A. Length of space above mercury $= 80 - (2 + 74{\cdot}4) = 3{\cdot}6$ cm.

$\therefore$ Volume of the water vapour $= 3{\cdot}6$ c.c.

And its pressure $= 76 - 74{\cdot}4 = 1{\cdot}6$ cm. of mercury.

Let the volume of the vapour at atmospheric pressure be v c.c.

When $76 \times v = 1{\cdot}6 \times 3{\cdot}6$ whence $v = {\cdot}07$ c.c.

4. A mass of air is saturated with water vapour at a temperature of 100°C.; on raising the temperature to 200°C. without change of volume the pressure is raised to two atmospheres. Find the pressure at 0°C. of this volume of the dry air alone. (Pat. U.—1926)

A. Let the pressures of dry air at 0°C. and 100°C, be P_o cm. and P cm. of mercury respectively.

Now the pressure of saturated water vapour at 100°C. = 76 cm.

$\therefore$ Total pressure of the moist air (saturated) at 100°C. $= (P + 76)$ cm.

As the vapour is unsaturated at 200°C.

From $\frac{P}{T} = \frac{P'}{T'}$, we get $\frac{P \times 76}{273 + 100} = \frac{2 \times 76}{272 + 200}$ whence $P = 43{\cdot}784$ cm.

Again $\frac{P_o}{273} = \frac{43{\cdot}784}{273 + 100}$ or, $P_o = \frac{43{\cdot}784 \times 273}{373} = 32{\cdot}01$ cm.

5. A capillary tube of uniform bore is sealed at one end and a column of air is enclosed in the tube by a pellet of water. The enclosed air column measures 10 cm. at a temperature of 17°C. and 16 cm. at a temperature of 67°C. Assuming that the vapour pressure of water at 17°C. is 15 mm. find the vapour pressure at 67°C. (Barometric height at the time = 760 mm.).

A. Let the vapour pressure of water at 67°C. be x mm.

Owing to the pellet of water, the column of air always saturated with water vapour; it is known that the saturated vapour pressure is not affected by change of volume and the total pressure inside is therefore always equal to the atmospheric pressure.

Initial pressure due to dry air only $= 760 - 15 = 745$ mm.

Final pressure due to dry air only $= (760 - x)$ mm.

From $\frac{PV}{T} = \frac{P'V'}{T'}$, we get $\frac{745 \times 10}{278 + 17} = \frac{(760 - x) \times 16}{273 + 67}$ whence $x = 214{\cdot}1$ mm.

72. Cold caused by Evaporation.—Whenever a liquid evaporates at any temperature, some heat is necessary to be supplied as latent heat for such change of state. If the heat is not supplied from external source, it is absorbed from the liquid itself, the surrounding air and other objects with which the liquid and the containing vessel may be in contact. These are, therefore, cooled during the process and the more rapid is the evaporation, the greater is the cooling produced.

The following experiments and observations verify the above statements;—A few drops of ether or alcohol are poured on the palm. As the liquid evaporates, it absorbs heat from the palm which cools

thereby. In some minor surgical operations local anæsthesia is produced by an ether spray. The bulb of a thermometer is wrapped with a piece of muslin and a few drops of ether are poured over the muslin ; as ether evaporates, the thermometer shows a fall in temperature. Tea is sometimes poured out in a saucer before sipping ; the tea in the saucer, being exposed over a large surface, evaporates more quickly and gets cooled. If a person remains for some time with wet clothes on, he is likely to catch cold, for while water evaporates it takes heat from his body.

In hot weather water kept in unglazed earthen pitchers becomes cooler than the water kept in glass or metal vessels. The walls of earthen pitchers being porous, water always oozes out and evaporation takes place all over the vessel while in a metal vessel it may take place directly from the water surface only through the mouth. The quicker evaporation, therefore, produces greater cooling of water in the porous vessel. But when the atmosphere is saturated as in rainy seasons, very little evaporation occurs and we do not find any appreciable difference in the temperatures of the water in the two vessels. When streets are watered, the water evaporates, taking the latent heat from the surrounding air which is thus cooled down.

The perspiration coming out of the pores of our skin gets evaporated *slowly*. The vapour remains as a sort of enveloping layer over the skin. Due to the wind of a fan, this layer of vapour is removed and fresh layer quickly takes its place, which produces *rapid* evaporation. This causes removal of heat from the skin producing cold. When the forehead of a person is sponged with eau-de-cologne, he feels refreshed ; for the liquid which contains a large proportion of alcohol evaporates quickly and takes away much of the local heat. The same purpose is partially effected by supplying piece of wet linen on the forehead and fanning briskly.

A wet *khus khus* screen or a moist coarse canvas screen hung near the doors and windows of a room keeps it cool. As the water evaporates, the latent heat is taken from the air in the room, which is thereby cooled. Some water is placed on a block of wood ; a beaker containing some ether is placed in the water. If the ether is quickly evaporated by blowing air through it, rapid cooling is produced, whereby the water on the wood may ultimately freeze.

Leslie's Experiment.—A small shallow metallic dish containing some water and a vessel containing strong sulphuric acid are placed under the receiver of an air pump. As the pump works, the pressure inside falls ; also some of the vapour formed is removed by the pump and the rest is absorbed by sulphuric acid. This produces a very rapid evaporation, whereby the temperature of the water falls down quickly, so that a thin layer of ice appears on the surface of the water. Liquid carbon dioxide contained under pressure in a vessel is allowed to escape through a fine jet inside a closed wooden box ; the rapid evaporation of the liquid coming out cools it so much that it solidifies.

Wollaston's Cryophorus.—It consists of a long bent glass tube with a bulb at each end (Fig. 54) and containing a little water and water vapour, but no air. All the water is transferred into one of the bulbs, say B and the bulb A is kept inside a freezing mixture. The vapour in A condenses, the pressure inside falls and *rapid* evaporation of the water in B takes place; the water in B is thus gradually cooled.

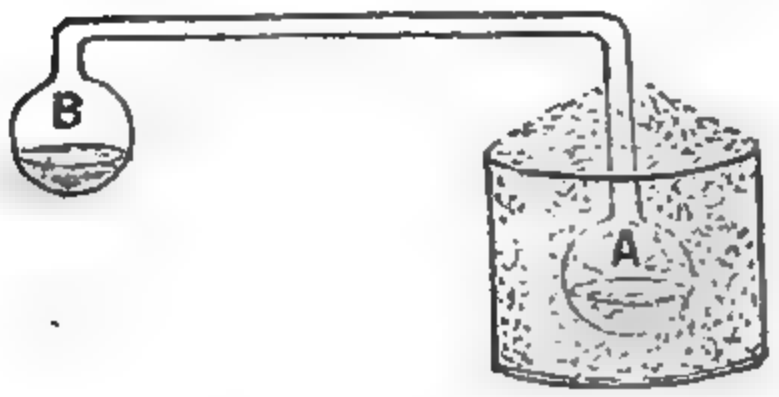


Fig. 54

On holding any bulb with palm it is slightly warmed and the liquid shows boiling under diminished pressure even at the room temperature. The lowering of temperature caused by rapid evaporation of ammonia is utilised in *Cold Storage* where meat and other perishable foodstuff are stored in warm climates. The same principle is also utilised in refrigerating machines.

73. Ice Machine –This is used for the manufacture of ice on a large scale. The essential **parts** of the machine are :—(Fig. 55)

(1) A *tank T.*—This is filled with a strong solution of brine or of calcium chloride.

(2) *Coils.*—Liquefied ammonia or carbon dioxide is contained in *coils* immersed horizontally within the solution. The circles in this tank indicate transverse section of the coils. These coils are called the *evaporator coils.*

(3) A *Ice-cans*, C.C.—These contain water to be frozen.

(4) A *pump*, P —This is a combined exhaust and compression pump and is worked electrically. It is connected to one side *directly* with the evaporator coils.

(5) A *condensing coil*, A.—The pump P is connected to the other side with the condensing coil A and *regulating valve* V.

(6) *Pressure gauges*, L and H.—The former is for low and the latter is for high pressure.

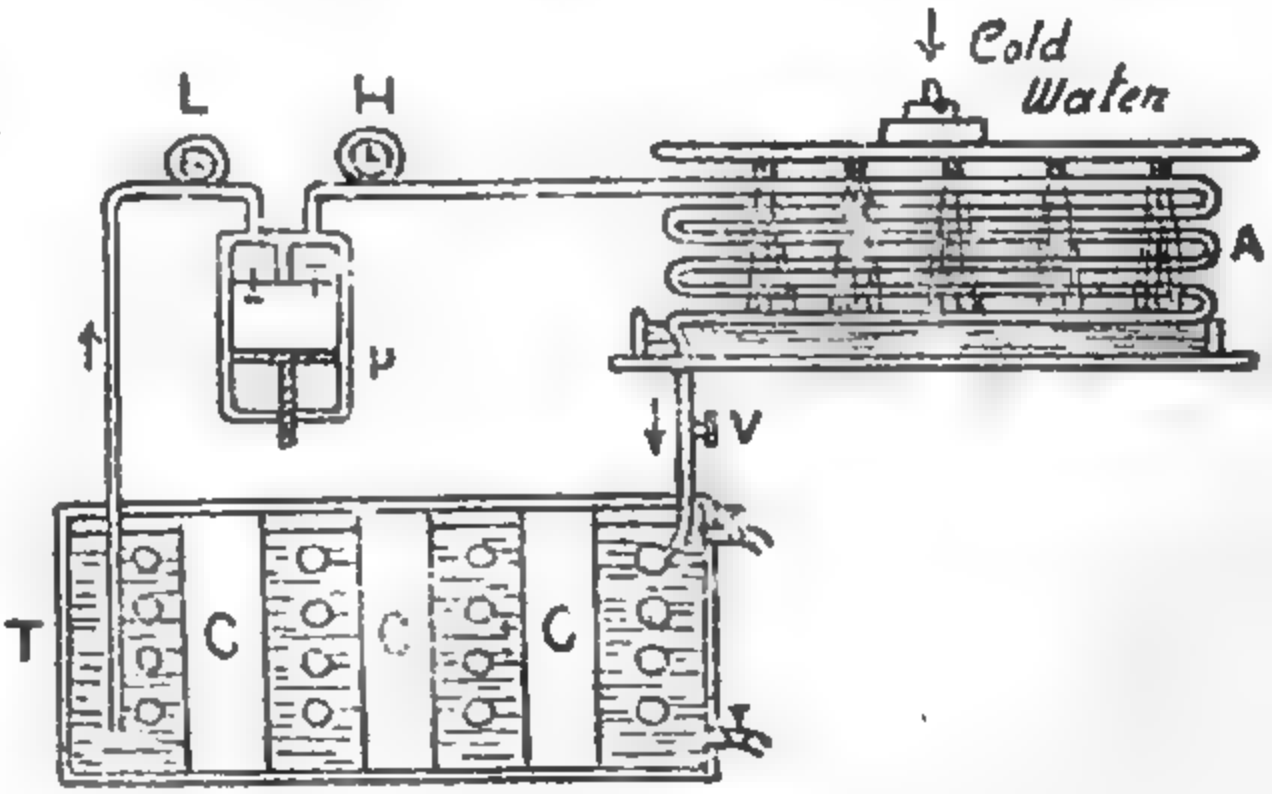


Fig. 55—A common ice machine

Action. — When the pump P works as a compression pump, it forces gaseous ammonia under a high pressure of about 155 pounds per sq inch in the condenser coil A, which is kept cool by a constant flow of water over its surface. The ammonia is thus

liquefied and is allowed to pass slowly through the valve V into the evaporator coils. The pump P now works as an exhaust pump and the liquid ammonia in the evaporator coils evaporates very rapidly absorbing the latent heat from the brine. The evaporated gas is then compressed again into the condenser coil and is again liquefied under the influence of the cold bath The liquid ammonia pass into the evaporator coils and is treated as before The evaporated ammonia is pumped out so quickly that the pressure inside the coil does not exceed 34 pounds per sq. inch. The brine is thus continuously cooled and is brought down to the temperature of 16°F, so that the water in the cans, C, C, is frozen into blocks of ice.

74. Cold caused by Expansion of a Gas.—When a compressed gas is allowed to expand through a narrow orifice, the gas has to do work against the molecular attraction The energy necessary for this is taken from the gas which is thereby cooled This is called the Joule-Thomson effect If carbon dioxide, stored under great pressure be allowed to escape through a narrow opening into a vessel open to the atmosphere, intense cooling is produced whereby the carbonic acid may be solidified. The principle of the cooling by expansion is utilised in the modern method of liquefying air and other gases.

Liquefaction of Air (*Linde's Method*).—The essential parts of the machine are :- (Fig. 56).

(1) A powerful *pump*, P acting as a compressor.

(2) The *cooler*, R is a cylindrical vessel containing cold water in which pipes conveying the compressed air are placed.

(3) *Double-walled Coil*, S—This has an inner tube s and is placed round with a non-conducting material within a box It has at its lower end a small *needle valve* V.

Action.—The air is at first freed completely from CO_2 by being slowly drawn through a series of lime towers (not shown). This air is then pumped to a pressure of nearly 200 atmospheres by the pump P. As the air is thus again heated it is cooled by being passed through the cooler R. The gas under the high pressure may be cooled to −4°C. by the help of ammonia refrigerating machine. The cold compressed air then passes down the inner tube s of the double-walled coil and escapes through the valve V. As the air suddenly expands, its pressure falls to about 16 atmospheres and its temperature falls considerably. This cooled air returns through the space between the two tubes, thus cooling the air moving through

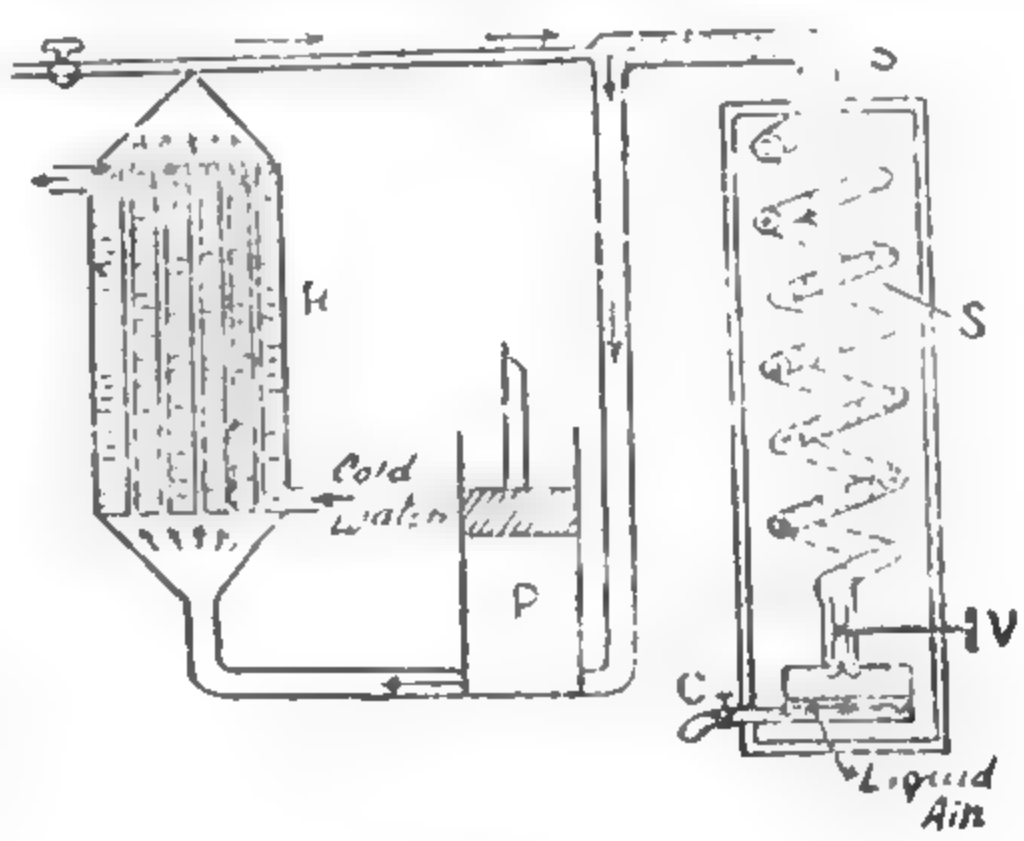


Fig. 56

s towards the valve V. Hence as the pump is worked, the temperature of liquefaction is reached. The critical temperature for oxygen is $-118{\cdot}82^{\circ}C$. The liquid air collects to a vessel attached below the valve and can be drawn off through the stopcock C. The liquid air is stored in *Dewar Vessel* which is double-walled glass vessel with the intervening space made vacuum. The boiling point of liquid air is $-183^{\circ}C$, under a pressure of one atmosphere. By making it boil under reduced pressure, a temperature of $-217^{\circ}C$. ($56^{\circ}K$) can be attained.

***75. Production of Low Temperature.**—By using a modified apparatus working on the same principle, hydrogen was liquefied by Dewar in 1891 and then by Travers. The gas was previously cooled to $-200^{\circ}C$., and was subjected to a pressure of 290 atmospheres. The critical temperature of hydrogen is $-239{\cdot}9^{\circ}C$. and liquid hydrogen boils at $-262{\cdot}7^{\circ}C$. under a pressure of 1 atmosphere. By making it boil under a reduced pressure, the liquid can be frozen to a white solid.

Helium was liquefied in 1908 by Kamerlingh Onnes by cooling the gas upto $-269^{\circ}C$. ($4^{\circ}K$), the critical temperature of the gas being $5{\cdot}26^{\circ}K$ and its boiling point under a pressure of one atmosphere being $4{\cdot}26^{\circ}K$. Onnes could not solidify helium, although he reduced the temparature to $0{\cdot}99^{\circ}K$ under a pressure of 0·013 mm. in 1931 In 1926 Dr. Keesom succeeded in partially solidifying helium by subjecting it to an enormous pressure of 130 atmospheres and later in 1932, the temperature was lowered to $0{\cdot}17^{\circ}K$. It was pointed out by Langevin in 1905 and by Giauque in 1927 that a lowering of temperature can be effected by demagnetising magnetised oxygen and the magnitude of the fall was independently calculated by Debye in 1926 in the case of gadolinium sulphate. Giauque reached a temperature of $\cdot 325^{\circ}K$ by using this substance Afterwards De Hass, Krammers and others have reached a temperature of $0{\cdot}05^{\circ}K$.

76. Boiling Point of a Liquid.—A barometer tube about a metre long is completely filled with pure dry mercury and is inverted over a mercury trough (Fig 57). Some quantity of water is introduced into the tube by means of a bent pipette. The tube is then jacketted completely with a wider glass tube with its ends closed by corks, as shown in the figure. Steam is now introduced into the jacket tube by a small tube at the upper end and is allowed to escape through a tube below. With rise of temperature, the pressure of the water vapour in the tube increases and the mercury column is gradually depressed. The difference between the initial height and the height of the mercury column at any instant gives the pressure of the water vapour at the corresponding temperature. Ultimately a steady state is reached ; if there be sufficient water present, the level of mercury inside the tube is found to be the same as in the trough. At this stage the water vapour inside is at the temperature of boiling water and *the pressure of the vapour is equal to the atmospheric pressure.* So we find that the vapour pressure of water at the boiling point is equal to the atmospheric pressure. Hence the **boiling point of a liquid** *may be*

defined as the temperature at which the saturated vapour pressure of the liquid is equal to the pressure of the atmosphere above its surface.

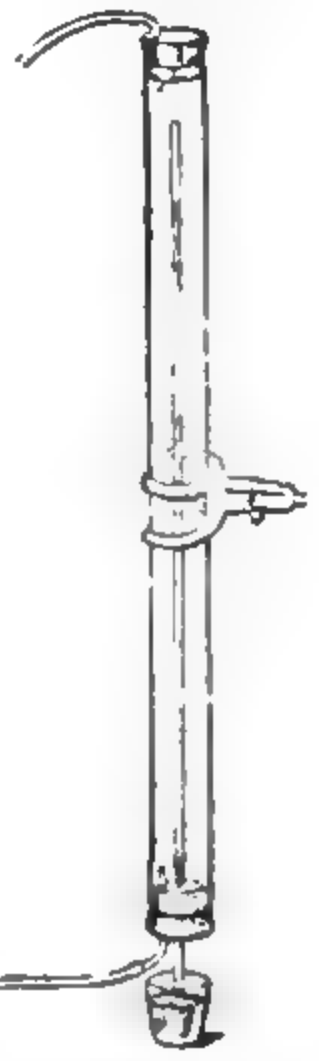


Fig. 57

Hence so long as the vapour pressure is less than the pressure on the surface, the liquid would evaporate and when the two pressures are equal, the liquid would boil. The *standard* boiling point of a liquid is referred to under *normal* atmospheric pressure. Thus, the boiling point of water is 100°C. means that at 100°C. the pressure of water vapour is equal to the normal atmospheric prossure *i.e.*, a pressure of 76 cm. of mercury.

77. Effect of Pressure on the Boiling Point.—The definition of the boiling point indicates that a liquid would boil at a lower temperature if the pressure on its surface is decreased and at a higher temperature if this pressure is increased.

(a) Boiling under Diminished Pressure.—This may be demonstrated by the following experiments : Boil some water in a beaker and place it quickly under the receiver of an air pump, wait till the boiling ceases and then work the pump vigorously. The water is found to be boiling again. The vapour pressure at the decreased temperature is less ; so when by working the pump, the pressure on the surface of the liquid is reduced to a value equal to the vapour pressure at the time, boiling recommences.

Franklin's Experiment.—Take a strong round-bottomed flask and fill it about half with water. Boil water for some time to drive off all the air by the steam. Remove the burner, immediately cork up the flask tightly and support it inverted on the ring of a retort stand (Fig. 58). Wait till the boiling ceases. The space above the water is now filled with saturated vapour at a fairly high temperature, which produces the pressure on the water surface. Now pour a stream of cold water on the flask ; the water within the flask is seen to boil again, although temperature has fallen below the normal boiling point. The addition of fresh water starts the boiling again, so long as the water in the flask remains sufficiently warm The process can be repeated several times, the boiling taking place each time at a lower temperature.

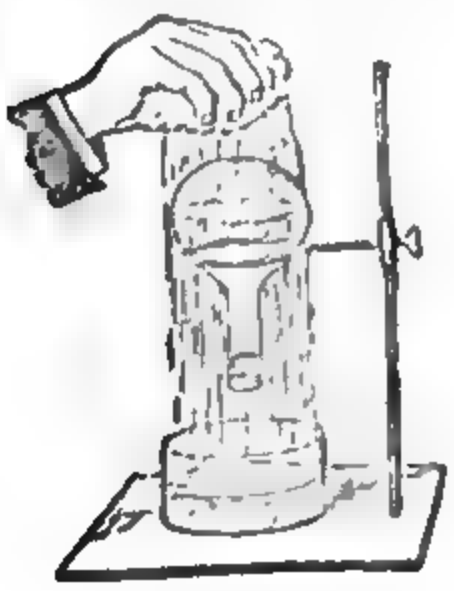


Fig. 58

Explanation.—On cooling, some of the vapour contained within flask is condonsed. The pressure on the surface of the water is thereby reduced. The temperature of the water although much below the normal boiling point is about the boiling point corresponding to this reduced pressure and so boiling appears and continues until the space above is saturated with vapour and the vapour pressure becomes equal to the saturated pressure at this lower

temperature. The pouring of more water causes further condensation. which starts the boiling again.

(b) Boiling under Increased Pressure.—This may be demonstrated by :—

Papin's Digester.—It consists of a strong metal vessel, suitably closed by a lid and fitted with a pressure gauge and a safety valve. The pressure gauge indicates the pressure of the steam which is formed when water within it is heated while the latter controls the pressure of the steam to a particular limit. When the steam pressure exceed this limit, the valve automatically opens and allows the excess steam to escape. The valve can be adjusted to different pressure limit. A thermometer is inserted through the lid with its bulb in the steam.

When the vessel is heated, the vapour accumulates inside and the pressure due to this vapour on the surface of water increases. Water, therefore, does not boil until a temperature is reached at which the maximum vapour pressure is equal to the pressure on the surface. It is found that water boils at about 120°C when the pressure on its surface is increased to 2 atmospheres and at 200°C under a pressure of 15 atmospheres.

78. **Regnaults' Experiments.**—*The variation of the boiling point of a liquid with pressure*, greater as well as less than one atmosphere, can be determined by the apparatus devised by Regnault to find the *saturated vapour pressure of water at various temperatures* (from 50°C. to about 250 C) It is known that a liquid boils when its saturated vapour pressure is equal to the superincumbent pressure. Hence by knowing the variation of the saturated vapour pressure with temperature, the variation of the boiling point of liquid with the pressure on its surface can be found.

The liquid is contained in the air-tight copper boiler B (Fig. 59), the upper part of which is connected through a Liebig's condenser C with a large round bottomed flask E. The flask is kept immersed in a bath of constant temperature. The pressure of the air within it can be increased or decreased by a pump and can be measured by an open U-tube mercury manometer M. Through the lid of the boiler B a sensitive thermometer is introduced to read the temperature at which the liquid boils.

The pressure of the enclosed air is at first reduced to a certain value by an exhaust pump Then the liquid within the boiler is heated, the vapour formed in trying to escape through the condenser comes in contact with cold walls, condenses to liquid and comes back into B. After sometime the liquid is found to boil and the temperature is noted. The pressure of the vapour at this temperature, which is equal to that of the air E, is obtained from the manometer.

The pressure inside is then increased to a certain value above one atmosphere by means of a condensing pump. The temperature of the liquid, is found to rise and become steady when the liquid boils at some

higher temperature. The pressure is *increased* in successive steps ; the corresponding boiling point is noted in each case and is found to *increase*. As the saturated vapour pressure of a liquid at its boiling point is equal to the superincumbent pressure, this method gives the boiling point of the liquid under various pressures. This method also gives the saturated vapour pressure of the liquid, as given by the manometer at various temperatures. By this method Regnault was able to find the saturated vapour pressure of water from 50 C. to 230 C., the value at the latter temperature being 27·5 atmospheres. Thus by knowing the boiling point of water, the corresponding vapour pressure is known from Regnault's table.

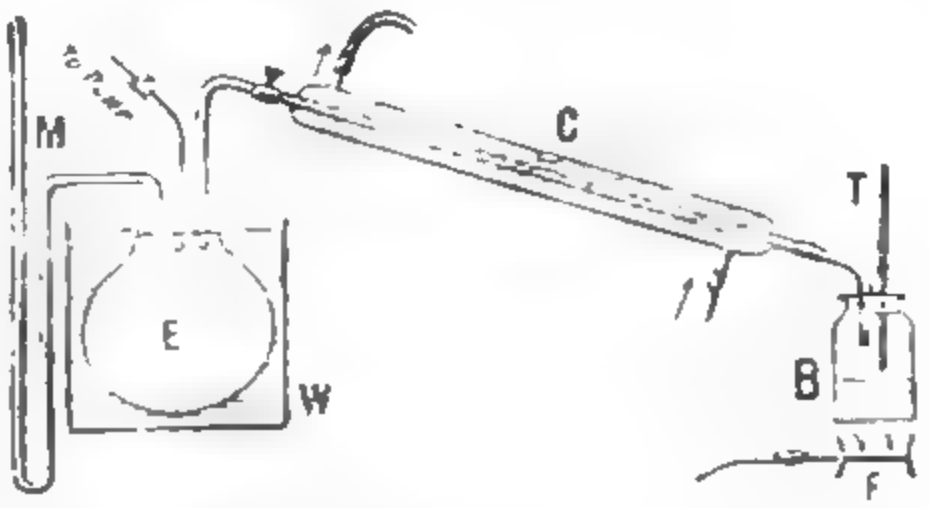


Fig. 59

It is evident that any variation in the atmospheric pressure changes the boiling point of the liquid. Thus at sea level, the atmospheric pressure being maximum, the boiling point is also the highest. At higher altitudes, the atmospheric pressure decreases and so the boiling point comes down. The boiling point of water at the top of a mountain is, therefore, less than 100 C. Thus at Darjeeling (6800 ft. above the sea level) water boils at about 91°C At Quito, which is the highest city in the world (9520 ft above the sea level) water boils at 90°C. Hence at very high altitudes it would be impossible to cook food properly in open vessels. This difficulty can be avoided by taking closed vessels in which the vapour formed will automatically increases the pressure on the surface of the water and hence the boiling point rises. The boiling point of water is found to decrease by 1 C. as the pressure falls by about 26·1 mm. Hence in determining the upper fixed point of a thermometer, the atmospheric pressure at the time is to be carefully noted and a correction, if necessary, is to be made for the variation in the boiling point of water.

The following Table suppiles the variation of Boiling Point of water with Pressure.—

Boiling Point of water with Pressure

Pressure mm.	Boiling Point° C	Pressure mm.	Boiling Point C
750	99·68	764	100·15
752	99·70	766	100·22
756	99·85	768	100·29
758	99·93	770	100·87
760	100·00	772	100·44
762	100·07	774	100·51

79. Measurement of Saturated Vapour Pressure of Water.— The maximum vapour pressure of water at various temperatures can be determined in the following way :—

(*a*) *For ordinary Temperatures* from (0°C. to 100°C) : *Regnault's method*—Two barometer tubes are completly filled with pure mercury and are inverted over a mercury trough. They are jacketted (as in Fig. 52) by a wider glass tube, reaching almost upto the mercury surface in the trough. Into one of the tubes sufficient water is introduced for a thin layer of it to remain on the surface of the mercury, while the other serves as the barometer. The water in the jacket is heated by passing steam into it. The temperature is noted by a thermometer while the pressure of the saturated vapour is given directly by the difference of the mercury levels in the two tubes.

(*b*) *For low temperature* (below 0°C) : *Gay Lussac's apparatus.*— The tube in which water is to be introduced is bent over at the upper end. The bent protion which generally ends in a bulb contains the liquid and is surrounded by a freezing mixture to lower the temperature to the required extent. The other tube gives the atmospheric pressure and the difference of mercury levels gives the vapour pressure directly.

(*c*) *For higher temperature* (above 100°C.)—For this purpose Regnault's apparatus as in fig. 59 is used and the usual procedure is followed.

80. Laws of Ebullition.—The following are the observed laws under which a liquid boils :—

(*a*) A liquid boils at the temperature at which its maximum vapour pressure is equal to the pressure on the surface.

(*b*) Under a particular pressure, every liquid has a definite boiling point. The boiling point rises or falls according as the superincumbent pressure increases or diminishes.

(*c*) If the pressure remains unchanged, the temperature at which a liquid begins to boil remains stationary until the whole of the liquid is converted into vapour.

(*d*) To convert unit mass of a liquid into vapour without any change of temperature, a definite quantity of heat, known as the latent heat of vaporisation of the liquid, is required.

81. Determination of Height by Boiling points (Hypsometry).—When a liquid boils, its vapour pressure is equal to the superincumbent pressure. The boiling point of water is conveniently determined at any place by means of a *hypsometer*. The corresponding saturated vapour pressure is obtained from Ragnault's table and hence the atmospheric pressure at the place is found. The difference between the atmospheric pressure at two stations can be thus determined ; but this difference is equal to the weight of a vertical column of air of unit cross-section and of height equal to the difference of altitudes between the two stations. The pressure and the temperature of the air column are not uniform throughout but may be taken to be the mean of the

pressure and the temperatures at the two stations, as they do not differ much.

Let difference of altitudes between the two stations = H cm.

Atmospheric pressure at lower station = P_1 cm. of mercury

Atmospheric pressure at upper station = P_2 cm. of mercury

Atmospheric temps. at the two stations = t_1, t_2 respectively

Then the difference of pressure between the two stations = $(P_1 - P_2)$ cm. of mercury

$\therefore$ The mean pressure = $\frac{P_1 - P_2}{2} = P$ say,

Also the mean temperature = $\frac{t_1 + t_2}{2} = t$ say,

The density of air at N.T.P. = 0·001293 gm. If ρ be the mean density at pressure P and temperature t,

$$\frac{76}{0{\cdot}001293 \times 273} = \frac{P}{\rho \times (273 + t)} \text{ whence } \rho = \frac{P \times 0{\cdot}001293 \times 273}{76 \times (273 + t)}$$

$\therefore$ the weight of H c.c. of air of mean density ρ

$$= \frac{H \times P \times 0{\cdot}001293 \times 273 \times 981}{76 \times (273 + t)}$$

As this is equal to the weight of a column of mercury of unit cross-section and the height $(P_1 - P_2)$ cm.

$$(P_1 - P_2) \times 13{\cdot}6 \times 981 = \frac{H \times P \times 0{\cdot}001293 \times 273 \times 981}{76 \times (273 + t)}$$

whence $$H = \frac{(P_1 - P_2) \times 13{\cdot}6 \times 76 \times (273 + t)}{P \times 273 \times 0{\cdot}001293} \text{ cm.}$$

*82. **Molecular States of Matter.** – According to the *molecular theory of matter*, a vapour is considered to contain groups of small aggregation of molecules. These groups move about like single molecule but with relatively low velocities. As temperature is lowered, the number of these groups per unit volume increases and ultimately they may coalesce into a liquid drop.

In *liquid*, there are groups which are composed of larger number of molecules. A molecule in a liquid state, therefore, moves with a smaller velocity than that of a molecule of its vapour at the same temperature. In a *solid*, the heavy groups lose even the small linear velocity they had in the liquid state and consequently have fixed positions relative to other groups of molecules. They can only oscillate over a small distance about their normal positions of rest. When the groups are arranged in a regular way, the form of the solid is *crystalline* and where irregular, it is *amorphous*.

Effect of Heat.—When a solid is heated, the molecular groups take up the heat energy and vibrate more violently about their mean

positions. The heat supplies the consequent increase in the kinetic energy of the molecules.

As the temperature rises, the average energy of vibration of the groups goes on increasing till at a certain temperature some of the groups begin to detach themselves from the adjacent groups, break up and form into liquid type molecular groups. Fusion now sets in and any further supply of heat energy is utilised in breaking the groups. Hence the temperature remains constant during fusion. If the body be crystalline, the groups begin to break up at a *definite* temperature which give a *definite* **melting point** of such a solid ; but in an amorphous state the loosely held molecules are detached at a lower temperature than the more regular or more closely packed ones and hence in such a substance the melting begins at one temperature and is completed at some higher temperature.

The energy of motion of some of the groups may occasionally increases so much that they may overcome the attraction of the adjacent molecules and escape from the liquid surface ; this explains **evaporation.** As the molecules with the highest kinetic energy leave the liquid, the mean kinetic energy of the remaining molecules decreases to some extent and so the temperature of the liquid somewhat falls. This explains the *cooling produced by evaporation.*

With a rise of temperature, evaporation increases. With further supply of heat the gas molecules increase in energy so much that they escape as bubbles from the liquid when **boiling** begins. The temperature at which this occurs depends on the **nature and aggregation** of the gas molecules. Hence for a particular liquid **under given conditions** boiling occurs at a *definite* temperature, called the **boiling point.**

Any further supply of heat energy is used to break up the liquid groups into similar gas molecules and so the temperature remains constant at the boiling point so long boiling continues. The kinetic energy of the gaseous molecules is far greater than the kinetic energy of the molecules in the liquid groups at the boiling point ; this excess of energy supplied by the heat absorbed while boiling is going on forms the **latent heat of vaporisation.**

If evaporation takes place in a *closed space,* there will be constant interchange of molecules between the vapour and the liquid. At any time a certain number of gas molecules leave the liquid surface move about in the space above and while moving towards the liquid some may be caught back into it. The rate of escape of molecules depends on the temperature while the number of molecules per unit volume in the space of the liquid depends on pressure

If at first the space contains little or no vapour, the number of molecules leaving the liquid must exceed the number of molecules re-entering the liquid per unit time. But the latter increases relatively as the quantity of vapour in the space increases until finally as many molecules re-enter the liquid per second as leave the liquid. At

this stage the number of vapour molecules per unit volume of the space remains the same. The space is then said to be *saturated* with the vapour; the pressure of the vapour is now the **saturated vapour pressure** at the temperature, which remains constant so long the temperature remains the same. A change of volume causes a corresponding change in the number of the molecules but the pressure remains the same. An increase in the superincumbent pressure opposes the escape of the gas molecules from the liquid, the kinetic energy of escape being supplied in the from of heat, *more* heat energy is now necessary. This explains the *rise of the boiling point under increased pressure*.

A liquid then contains a small portion of the free gas molecules, and gas, a small number of liquid type group molecules. The **critical temperature** may be taken to be the temperature at which a liquid and its vapour can simultaneously exist in all proportions.

HENRI VICTOR REGNAULT (1810—1878)

Regnault was born of poor parents on July 9, 1810 at Aix-la chapelle in France. In his boyhood he could not get proper school education because of poverty. He was compelled to take a job at a Druggist's Pharmacy as a clerk. Here he used to work during day and read elements of Chemistry and medicine at night. In 1832 he went to Paris and after much hard work was able to be admitted to the Ecole Polytechnic. He passed out of the institution with distinction in 1836 and became a professor at Lyons. His earlier works were concentrated on Organic Chemistry.

By 1840 he became famous as a scientist and that year he was appointed a professor of Natural Philosophy at Ecole Polytechnic. From this time on almost all his contributions were in the domain of Physics. He designed a sensitive type of calorimeter and made a systematic measurements of the specific heats of various substances. His other subjects of research were density of water vapour, deviations from Boyle's law, expansions of liquids and gases, specific heats of gases, vapour pressure and velocity of sound. He died in 1878.

Summary

Fusion and Solidification.—*Fusion or melting* is change of a substance from the solid to the liquid state. *Solidification* or *freezing* is the change of a substance from the liquid to the solid state.

The **melting point of a solid** is the temperature at which melting begins under given conditions and which remains constant until melting is complete.

Effect of Pressure on melting point.—By increase of pressure, the melting point of a substance which *expands* on melting is *raised* while that of a substance which *contracts* on melting is *lowered*.

Evaporation and Ebullition.—*Evaporation* is a *slow* change from the liquid to the gaseous state, which takes place at the *surface* of the liquid at *all* tempera-

tures while *ebullition* is a *rapid* change which takes place *throughout* the mass of the liquid at a *definite* temperature.

Vapour Pressure.—The vapour of a liquid exerts pressure like a gas, which is called the *vapour pressure* of the liquid at the particular temperature.

Saturated and Unsaturated Vapours.—If a space contains as much vapour as it is possible for it to contain under the given circumstances, the space is said to be *saturated* with the vapour; otherwise it is *unsaturated*. The pressure exerted by saturated vapour at any temperature is *maximum*.

Vapour and Gas.—A substance in the gaseous state behaves like a vapour below its *critical temperature* and as a gas, above this temperature.

Cold can be produced by (*a*) *rapid evaporation* and (*b*) sudden expansion.

The **boiling point** of a liquid is the temperature at which the saturated vapour pressure of the liquid is equal to the superincumbent pressure.

EXERCISES ON CHAPTER VI

Reference	
Art. 61	1. Two bits of ice being pressed together form one mass. Explain. (C. U.—1933, '50)
Art. 61	2. Write a short note on regelation. (E. P. U.—1953)
Art. 61	3. How can you demonstrate that pressure changes the melting point of a body and evaporation produces cold? (Utkal. U.—1952)
Arts. 62 & 73	4. Write short notes on freezing mixtures and ice making machine. (Mad. U.—1950)
Art. 68	5. State the laws of fusion of a substance. (Mad. U.—1949, '51)
Art. 64	6. Distinguish between boiling and evaporation. What condition determines whether a liquid will boil or evaporate? (C. U.—1950; P. U.—1921; Dac. U.—1937; Pat. U.—1941, '44, '52)
Arts. 64 & 79	7. Define boiling point of a liquid. Describe carefully how the maximum tension of aqueous vapour is determined for temperatures below and above the normal boiling point. (Del. U.—1938)
Art. 64 67 & 68	8. What are the points of distinction between evaporation and ebullition? Define vapour tension of a liquid. A glass cylinder is provided with an air tight piston and contains inside some saturated water vapour at 20°C. State and explain what will happen when (*a*) temperature is raised to 30°C, the piston remaining in position; (*b*) temperature is lowered to 10°C, the volume remaining unaltered; (*c*) the piston is raised up, temperature being kept constant; (*d*) the piston is lowered down, the temperature being unaltered. (Utkal. U.—1949)
Art. 65	9. State and explain the various factors that influence the rate of evaporation of water. (Mad. U.—1950)
Art. 66	10. Explain saturation vapour pressure of a liquid. (Gau. U.—1952; Mad. U.—1949)
Arts. 67 & 68	11. Two barometers stand side by side. A few drops of water are introduced into the vacuum of one and a little air into the other. What would be the effects on the errors of the barometer reading thus

Reference

produced of (*a*) change in the atmospheric pressure, (*b*) a change in the temperature? (C. U.—1929)

12. A quantity of air saturated with water vapour at 30°C exerts a pressure of 74 cm of mercury when its volume is 200 c.c and a pressure of 116·6 cm. when its volume is 125 c.c. temperature remaining constant. Calculate the pressure exerted by vapour — Arts. 67 & 68

A. 30 mm.

13. A gas is collected over water in 100 c.c tube and measures 72·8 c.c.; the temperature and the pressure of the atmosphere at the time are 25°C. and 74·39 cm. respectively Calculate the volume of the dry gas at N. T. P. The vapour pressure of water at 25°C. is 23·45 mm. and the density of mercury is 13·6 gm. per c.c. — Arts. 66 & 69

A. 61·51 c.c.

14. Distingmsh between saturated and unsaturated vapours (Del. H. S.—1954, Pat. U.—1941; C. U.—1941, '47; Gau. U.—1952; *Cf.* E. P. U.—1950, '52; P. U.—1952) — Art 69

15. Enunciate and explain Dalton's laws of partial pressure. (Pat. U.—1940) — Art. 70

16. Why is it unwise to sit in a draught with wet clothes on? (C. U.—1947, *Cf.* '50) — Art. 72

17. Explain why the reading of a thermometer is altered by wrapping a wet rag round its bulb. (C. U.—1948) — Art. 72

18. If you dip your hand in lukewarm water and then expose it to the air, the hand feels cold If you make the same experiment with ether, the hand feels much colder on exposure. Explain these facts. (C. U.—1938) — Art. 72

19. Explain the principle, construction and working of an ice-plant. (Raj. U.—1953; E. P. U.—1952) — Art. 73

20. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperatures greater or less than 100°C. At Darjeeling the barometric height is about 23 inches. If there is a change of ·01°C. in the boiling point of water for a change of 1 mm. of Hg. at what temperature will water boil there? (Due. U.—1942) — Arts. 76 & 78

A. 92·97°C.

21. Write a short note on Papin's digester. (E. P. U.—1953) — Art. 77

22. Describe an experiment to show that the vapour pressure of a liquid exposed to air at its boiling point is equal to the atmospheric pressure. (C. U.—1925; P. U.—1931) — Art 76

23. Define boiling point of a liquid. Describe suitable experiments to show that water can be made to boil at temperature greater or less than 100°C. (P. U.—1932; C. U.—1941; *Cf.* Pat. U.—1952) — Arts. 76 & 78

24. A flask is half filled with water and heated till the water boils After a short time when the upper part of the flask is filled with steam, it is well-corked and inverted. Cold water is now poured over the flask. What will happen? Why? (C. U.—1938) — Art. 77

25. Explain how the maximum tension of aqueous vapour is determined at temperatures below and above normal boiling point. (C. U.—1947) — Art. 77

Reference

Arts. 78 & 80 — 26. State laws of boiling. How is it that things cannot be cooked properly on high mountains? How can water be made to boil at any temperature above 100°C? (Del. U.—1943)

Art. 79 — 27. Describe an accurate method for determining the pressure of saturated vapour at different temperatures.

The pressure of the saturated vapour of water at 50°C is 9·2 cm. of mercury and it is found that under a pressure of 9·2 cm. of mercury water boils at 50°C. Explain fully the relation expressed in the statement. (Utkal. U.—1952)

Art. 79 — 28. How would you determine the maximum pressure of aqueous vapour between 0°C. and 100°C.?

(C. U.—1947; Dac. U.—1931; Gau. U.—1952; *Cf.* Mad. U.—1949)

Art. 79 — 29. Describe how you will measure the saturated vapour pressure of water at the room temperature. (C. U.—1949)

Art. 81 — 30. Explain how you are able to determine (approximately) the height of a mountain by finding the boiling points of water at its top and bottom. (C. U.—1945; Pat. U.—1943)

CHAPTER VII

HYGROMETRY

83. Water Vapour in the Atmosphere.—As evaporation continually goes on from all water surfaces such as seas, rivers, lakes etc, there is always some aqueous vapour present in the atmosphere. The outside of a tumbler containing iced water gets after some time moist. This is due to the condensation of water vapour of the atmosphere on the tumbler. The formation of dew, mist, cloud, rain, etc., also indicates the presence of water vapour in the atmosphere The study and measurement of the quantity of aqueous vapour in the atmosphere is called **hygrometry.**

84. Dew Point.—The air is not ordinarily saturated with the water vapour. If moist air in an open space is gradually cooled, the pressure of the moist air at the region falls below that of neighbouring atmosphere which is at the ordinary temperature. Hence moist air from surrounding places comes to the space to equalise the pressure. As the cooling proceeds, a temperature is ultimately reached, at which the moisture accumulated at the region is *just sufficient* to saturate the space, pressure remaining unchanged. This temperature is called the *dew point.* If the temperature is slightly lowered below the dew point, some of the vapour is condensed and is deposited as *dew.* Hence the dew point *is the temperature at which a given mass of air becomes just saturated with the quantity of aqueous vapour actually present in it.* Also it follows that the pressure of the aqueous vapour actually present in air at the ordinary temperature is equal to the saturated vapour pressure at the *dew point* at that time. The statement, "The dew point of the atmosphere is 23°C at some time," means that the amount of water vapour present in a given volume of air at the time would just saturate the air at 23°C.

85. Relative Humidity or Hygrometric State of the Air.—Humidity gives us information about the amount of aqueous vapour present in the atmosphere at some particular hour.

Absolute Humidity is defined as the quantity of aqueous vapour actually present in a given volume of the atmospheric air. It is usually expressed in grammes per cubic metre of air.

Relative Humidity is defined as the ratio of the mass of aqueous vapour actually present in a given volume of air to the mass of water vapour required to saturate the volume at the observed temperature. So relative humidity gives the degree of saturation of the atmosphere as regards moisture. It is, therefore, termed the **hygrometric state** or fraction of saturation of the air.

Hence ***Relative Humidity***

$$= \frac{\text{Mass of aqueous vapour actually present in a volume of air at } t^\circ C.}{\text{Mass of aqueous vapour required to saturate the same volume at } t\ C.}$$

But since unsaturated vapour present in a given volume of air obeys Boyle's law, the pressure exerted by the vapour is nearly proportional to the mass of the vapour present. So the ratio between two masses of vapour may be taken to be equal to the ratio of the corresponding vapour pressures. Hence relative humidity may also be defined as the ratio of the pressure of the aqueous vapour actually present in the atmosphere to the saturated vapour pressure at the temperature of the air.

$$\therefore \text{ Relative Humidity} = \frac{\text{Pressure of aqueous vapour actually present in the air at } t^\circ C.}{\text{Pressure of aqueous vapour required to saturated the air at } t^\circ C.}$$

$$= \frac{\text{Saturated vapour pressure at the dew point at the time}}{\text{Saturated vapour pressure at the temperature of the air}}$$

The *relative humidity* is generally expressed as the ***percentage saturation*** of air In this case it is expressed as a ratio

$$= \frac{\text{Saturated vapour pressure at dew point} \times 100}{\text{Saturated vapour pressure at air temperature}} \text{ per cent.}$$

Denoting the saturated vapour pressure at the dew point by f and that at the temperature of air by F, we have

$$\text{Relative Humidity} = \frac{f \times 100}{F} \text{ per cent.}$$

The feeling of dampness or dryness of air depends on the relative humidity. It, therefore, does not depend only on the absolute quantity of water vapour *actually present* or on the quantity necessary to saturate the air under the given conditions but it depends on the *ratio* of the former to the latter quantity. Thus, on a misty winter day, the vapour in a given volume of air may be less than that on a hot summer day when the air appears to be dry ; that is because the amount of aqueous vapour present on the latter occasion is a *smaller fraction* of the amount necessary to saturate the space.

Again the lower is the relative humidity, the more rapid is the evaporation ; hence the dryness or dampness of the air is *judged by the rate of evaporation at the time*. A very moist day in summer feels much more uncomfortable than a dry day ; for in the former case the atmosphere is nearly saturated with water vapour producing a stuffy sensation. When the relative humidity is low, the atmosphere takes up the water vapour more rapidly. For this reason in our country a damp cloth dries more rapidly in winter than in summer.

Relative humidity is determined from day to day at meteorological stations to forecast weather When the relative humidity is high, we may expect rain. A knowledge about the actual atmospheric condition and a forecast for immediate future are essential now-a days to pilots of aeroplanes for a safe navigation. Information about relative humidity is necessary for Public Health Department, as germs of certain disease multiply well in moist atmosphere. In hot houses supply of moisture is to be regulated for the growth of plant. The

knowledge is also of importance for textile industries ; elaborate arrangements are made to maintain artificially a uniform degree of dampness in spinning rooms. A successful control of humidity is needed at the cold storage of meat and fruit in the holds of ship and also in artificial seasoning of timbers for aeroplanes.

Hygrometers (Gr. *hygros*, wet ; *metron*, a measure) are instruments for determining the humidity of the air at any particular time and place. According to the principle upon which they work, hygrometers are divided into three classes :—

(1) *Dew-point Hygrometer.* (2) *Chemical Hygrometer.*
(3) *Wet and dry Bulb Hygrometer.* (4) *Hair Hygrometer.*

86. Dew point Hygrometers (*Condensation Hygrometers*)—The dew point is directly determined by any one of these instruments. The ratio of the saturated vapour pressure at the dew point to that at the atmospheric temperature gives the relative humidity.

(1) Daniell's Hygrometer.—It consists of two glass bulbs A and B (Fig. 60) connected by a glass tube bent twice at right angles and mounted on a vertical stand, so that the bulbs project downwards. The instrument contains only ether and ether vapour, all the air having been removed before the apparatus is finally sealed. In order to observe clearly the deposition of any dew on the surface of the bulb A, it is blackened from inside or a gilt band is fastened over the surface near the middle. There is a sensitive thermometer T mounted inside A. The thermometer T_1 mounted on the stem of the instrument gives the temperature of the outside.

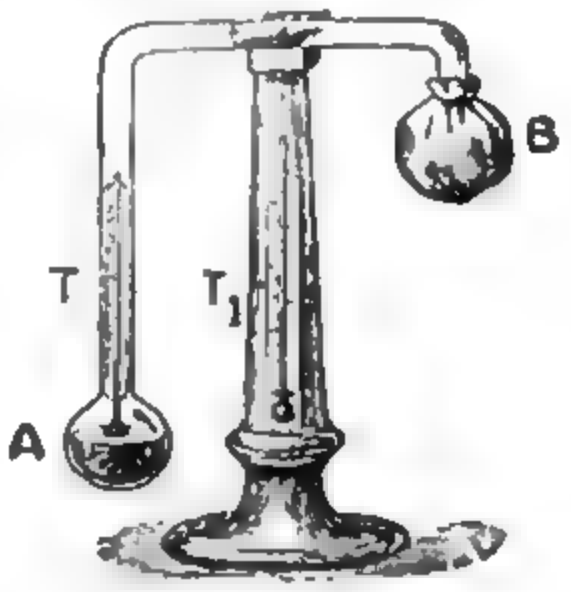


Fig. 60—Daniell's Hygrometer

In using this instrument, all the ether is at first transferred into the bulb A by tilting the apparatus and a piece of muslin or linen is wrapped round B. Ether is then poured drop by drop on the linen. As the ether evaporates, it takes the necessary heat from the bulb B and cools it rapidly. Consequently the ether vapour in B is condensed, which causes rapid evaporation of the ether in A. The temperature of this ether and hence that of the bulb A gradually decreases so long as the linen is kept moist with ether. The mass of the air surrounding the bulb A also falls in temperature and ultimately a thin film of dew forms on the bulb, making the surface of A appear dull. The temperature at which the dew just appears is read from T. The dropping of ether is discontinued and the linen is allowed to dry up. The bulb A warms up and the temperature at which the dew just disappears is noted. The mean of these two temperatures is taken as the dew point at the time. Observation should be made either by a telescope or through an intervening plate glass, so that the breath or heat of the body of the observer might not disturb the deposition of dew on the instrument.

Sources of Error :—

(1) As evaporation takes place from the upper surface of the ether in A and as there is no stirring of the mass, the temperature of the ether is not uniform throughout.

(2) Glass being a bad conductor, the temperature of the outside surface of A can not take up that of the ether in the bulb simultaneously.

(3) The appearance or disappearance of dew cannot be exactly ascertained.

(4) The rate of evaporation of the ether in A and hence the cooling of the bulb cannot be controlled at will.

(5) Owing to the proximity of the observer, the temperature of the surface of A also the hygrometric state of the air at the place may be altered by the breath and the heat from the body of the observer.

(6) The presence of a large quantity of ether vapour in the surrounding air is also objectionable being likely to affect the Hygrometric state.

(7) When the air is hot and dry, it is difficult to get indication with the instrument and a large quantity of ether is wasted for the purpose.

87. Regnault's Hygrometer.—It essentially consists of a wide test tube G (Fig. 61), the lower part S of which is a thimble of a thin and polished silver sheet and the upper part is glass. The mouth of the tube is closed by a cork through which a sensitive thermometer T_1 and a piece of glass tubing *a* are inserted reaching almost to the bottom of G. A short side tube is sealed near about the top of G and is connected to a vertical brass tube *b* ; *b* is again connected through a rubber tube *c* to an aspirator A filled with water. A second tube B similar to G is attached to the same vertical tube *b* ; its surface is used for the sake of comparison while the thermometer T_2 placed in it merely gives the temperature of the air in the room.

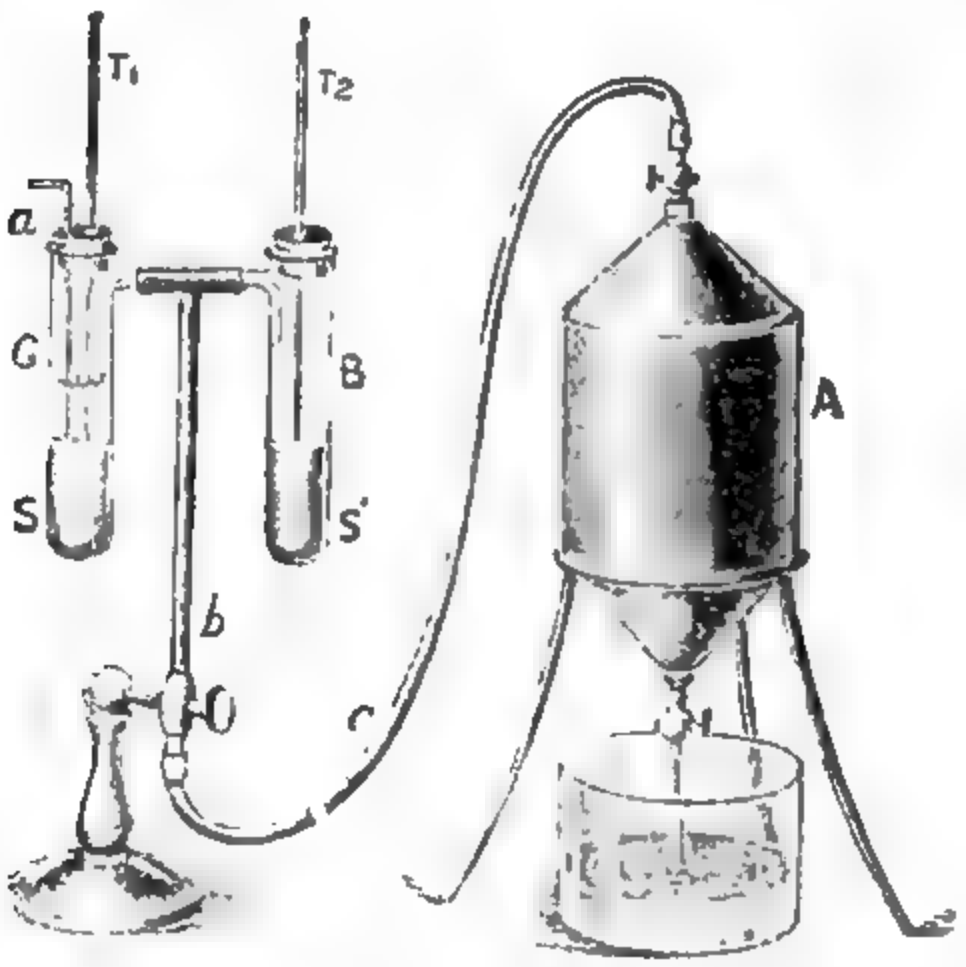


Fig. 61—Regnault's Hygrometer

In using the instrument, G is partly filled with ether, so that the lower end of *a* and the bulb of the thermometer T_1 are well within the ether. The aspirator is opened ; as water goes out, a current of air is drawn in, bubbling through the earth and thus causing it to evaporate rapidly. The liquid therefore gradually falls in temperature and as it is automatically stirred by the air bubbles, it attains uniform temperature throughout the mass ; further, as silver is a very good conductor and the sheet is very thin, the temperature attained by the silver surface, both inside and outside, is practically the same. The liquid, the silver surface and the layer of air surrounding it gradually cool down and ultimately the moisture in the air is condensed on the

bright silver suface. The appearance of dew as indicated by loss of brilliancy of the surface of S in contrast to that of S′ is observed from a distance by a telescope and the temperature at which the dew just appears is noted from T_1. Then aspirator is stopped where upon the apparatus warms up slowly and the temperature at which the dew just disappears is noted again. The mean of these two temperatures is taken as the *dew point* at the time.

To find the relative humidity (R.H.) the saturated vapour pressure f, at the dew point and F, that at the room temperature are obtained from Regnault's table ; then R. H. $= \frac{f}{F} \times 100$ per cent.

Advantages of Regnault's Hygrometer.

(1) As the liquid is continuously agitated by the air bubbles, the temperature throughout the mass is uniform.

(2) As silver is a very good conducter, the temperature of the silver surface is practically the same as that of ether.

(3) The change of the brightness of the two tubes greatly facilitates the observation of the appearance and disappearance of dew.

(4) As the rate of evaporation can be altered by regulating the aspirator the rate of cooling can be controlled and hence the successive observations can be made at desired intervals.

(5) As the observation is made from a distance, the breath or heat from the body of the observer does not affect the hygrometric state of the air in the neighbourhood of the tubes.

(6) As the ether vapour is carried into the aspirator, it cannot affect the relative humidity.

***88. Chemical Hygrometer** (*Absorption Hygrometer*).—The mass of water vapour present in a given volume of air can be directly determined by this hygrometer and so the absolute humidity can be found by this process. The pressure of the vapour is calculated and the *relative humidity* is also determined.

The apparatus consists of a large aspirator (Fig 62) filled with water and connected with a system of U-tubes through the 'trap' bottle. The U-tubes contain either lumps of freshly fused calcium chloride or pieces of pumice stone soaked in strong sulphuric acid while the trap bottle contains strong sulphuric acid. All the connections are made air tight by short pieces of India rubber tubing. The drying U-tubes with their contents are carefully weighed before the experiment is started.

The aspirator is opened when water escapes and air is sucked through the system of U-tubes. The moisture of the air is absorbed by the drying tubes while any moisture rising from the water in the aspirator is absorbed in the 'trap' bottle. After continuing the experiment for some time, the aspirator is closed, the drying tubes are disconnected and weighed again. The increase in the weight gives the mass of aqueous vapour with the volume of air drawn in, which is equal to the volume of water that has flown out ; but the air becomes

saturated with moisture when it comes in contact with water within the aspirator. The temperature of the air outside and inside the aspirator are noted.

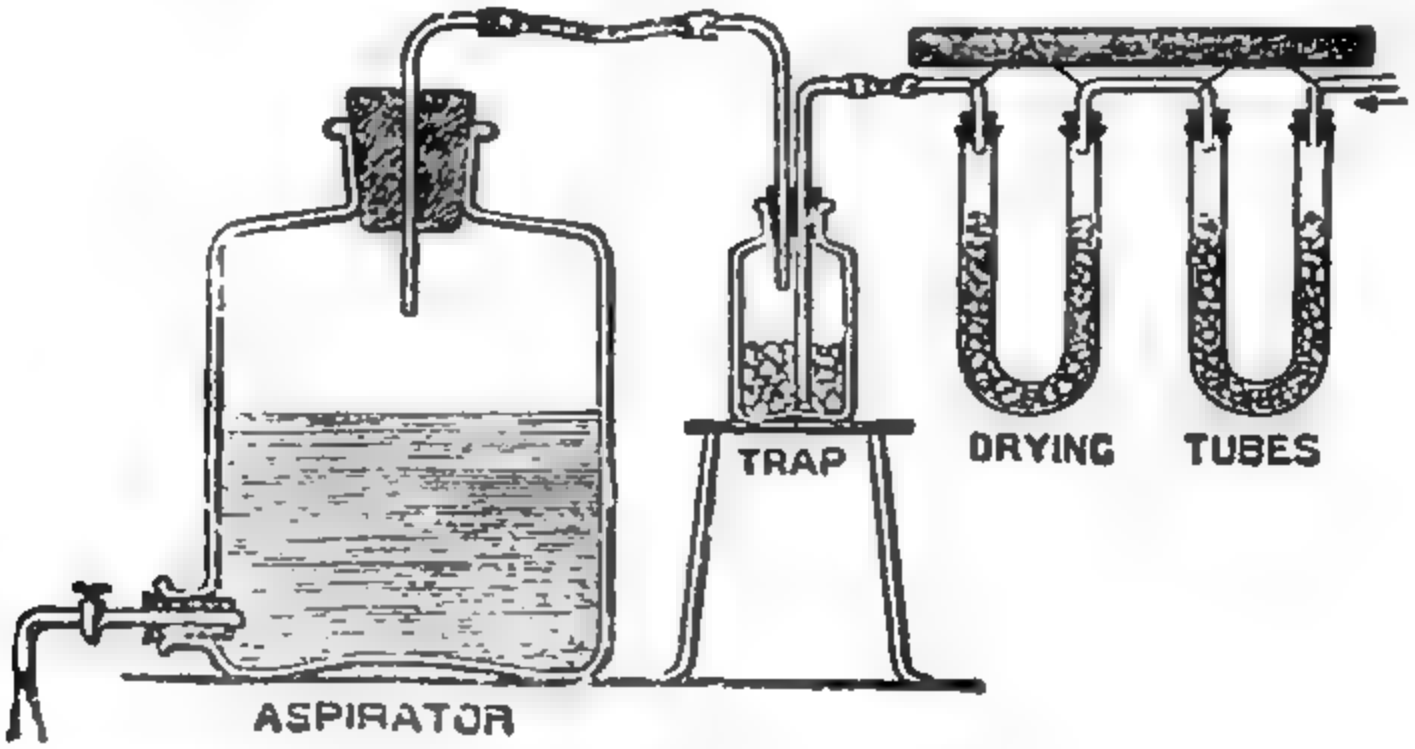


Fig. 62—Chemical Hygrometer

Let mass of the aqueous vapour absorbed = m gm.

volume of the air drawn in = V c. c.

temperature of air inside = t°C

pressure of aqueous vapour = f mm. of mercury

atmospheric pressure at the time = H „ „ „

Since the relative density of aqueous vapour to air is 0·622 and the density of dry air at N. T. P. is 0 001293 gm. per c.c.

∴ Density of aq. vapour at N. T. P. = 0·622 × 0·001293 gm. per c.c.

Let the density of aqueous vapour at pressure f and temperature t = d gm. per c.c.

From Boyle's Law, $$\frac{f}{d(273+t)} = \frac{760}{0{\cdot}001293 \times 0\,622 \times 273}$$

whence $$d = \frac{f \times 0{\cdot}001293 \times 0\,622 \times 273}{760 \times (273+t)} \quad \ldots \quad (88,1)$$

As $d = m/V$, it can be obtained from experiment. Hence f, the pressure of the vapour actually present is known from eqn. (88,1) and is given by

$$f = \frac{d \times 760 \times (273+t)}{0\,001293 \times 0{\cdot}622 \times 273} \text{ mm.} = 3460 \times (273+t) \times d \text{ mm.}$$

If the saturated vapour pressure at t°C., as obtained from Regnault's Table, be F then

$$\text{Relative humidity} = \frac{f}{F} = \frac{m \times 760 \times (273+t)}{V \times F \times 0\,001293 \times 0\,622 \times 273}$$

$$= \frac{3460 \times m \times (273+t)}{V \times F} \quad \ldots \quad (88,2)$$

To obtain the relative humidity as *percentage*, the above expression is to be multiplied by 100. To obtain absolute humidity, take the ratio of m and V/1000000. Thus the ratio gives the amount of water in gm. present in a cubic metre of air.

It is to be noted that the temperature of the air outside may be different from that of air inside the aspirator : also the atmospheric air is generally unsturated while that within the aspirator gets saturated and so the pressure of the vapour and hence that of the dry air, inside and outside, would not be the same. But as the differences mentioned are very small, they are not generally taken into account.

89. The Wet and Dry Bulb Hygrometer (*Psychrometer or Mason's Hygrometer*).—It consists of two exactly similar mercury thermometers (Fig. 63), mounted vertically side by side on the same board with their bulbs freely exposed to the air. One thermometer gives the temperature of the air ; the bulb of the other is covered with muslin, which is kept continuously moistened with water by means of a cotton wick dipping into water, kept in a small attached vessel.

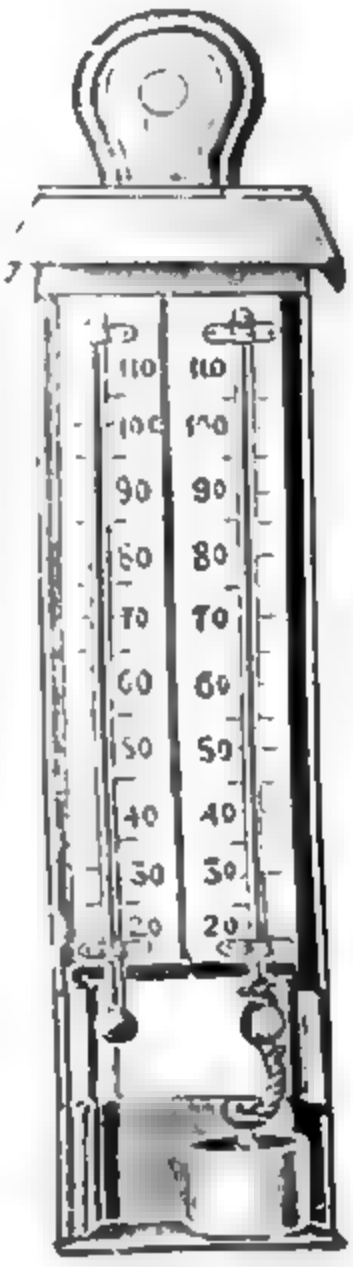


Fig. 63

As water evaporates from the muslin, the temperature indicated by the wet bulb thermometer is lower than that of the dry bulb one. The difference in the readings depends on the hygrometric state of the air ; for example, on a very dry day evaporation would be very rapid and so the difference in the readings increases and on a damp day evaporation would be slow and the difference decreases. If the atmosphere is already saturated, no evaporation takes place and the two thermometers give the same reading.

The relative humidity can be determined by obtaining the dew point with the help of Glaisher's factor.

Let the temperatures given by the dry and the wet bulb thermometers respectively $= t_1$ and t_2
temperature of the dew point at the time $= t$
Then from Glaisher's formula, $t_1 - t = F(t_1 - t_2)$ ... (89,1)
where F is the Glaisher's factor.

The dew point is thus found ; obtaining the saturated pressure at t and t_1 from Regnault's table, the relative humidity is calculated.

90. Hair Hygrometer.—The apparatus consists of a human hair fixed at a point A of the instrument and stretched over three small pulleys B, C and D (Fig. 64). Then the hair is wound a few turns over a revolving small drum E and is finally fastened to the end of a light spring S at F. A pointer P moving over a graduated dial is fixed to the drum E.

The hair has a property of increasing continuously in length with a

rise in atmospheric humidity. Consequently when the hair elongates, the spring draws it further downwards. As a result, the drum with its attached pointer rotates over the scale graduated in units of relative humidity. In a practical case, a strand of 20 to 30 fibres of hair is put together to give the required tension. The following informations are available regarding the behaviour of a human hair in humidity according to Griffith's investigations.

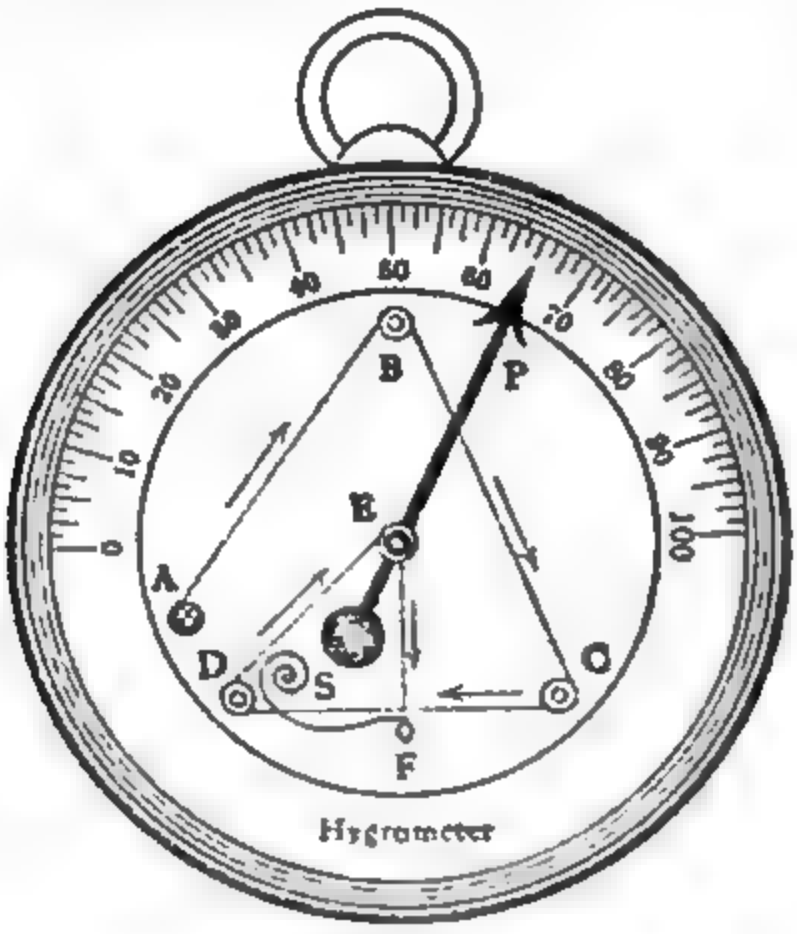


Fig. 64

The readings of a hair hygrometer tend to give higher readings with lapse of time. This is probably due to the fact that the hair being always in tension due to the spring gets a permanent set. The hair should be guarded against any undue strain. The calibration of the instrument should be checked from time to time. A hair hygrometer has the advantage of being a direct reading instrument. It can be used at a temperature below the freezing point of water where a dry and wet bulb thermometer cannot work.

The following Table supplies the temperature of the Dry bulb with the corresponding value of Glaisher's factor.—

Dry and Wet Bulb Thermometers

Dry Bulb Temp. °C.	Glaisher's Factor	Dry bulb Temp. °C.	Glaisher's Factor	Dry Bulb Temp. °C.	Glaisher's Factor
[illegible]	7·82	11	2·02	24	1·72
5	7·28	12	1·99	26	1·69
6	6·62	14	1·92	28	1·67
7	5·77	16	1·87	30	1·65
8	4·92	18	1·38	32	1·63
9	4·04	20	1·79	34	1·61
10	2·06	22	1·75	[illegible]	1·59

Example.—At some hour the readings of a Wet and Dry bulb hygrometer are 52°F and 58°F. Find the humidity.

A. Dry bulb temperature, $t_1 = 58°F = 14·4°C$ and Wet bulb temperature $t_2 = 52°F = 11·1°C$. Hence from equ. (89.1) $14·4 - t = F \times 3·3$. The value of F for $14°C = 1·92$ and for $16°C = 1·87$. Hence from a proportionate value, F at $13·4°C = 1·91$. Thus $14·4 - t = 1·91 \times 3·3 = 6·3$. Hence dew point $t = 8·1°C$. Now from Regnault's table of Saturated Vapour, maximum vapour pressure at $8·1°C = 8·67$ mm. and that at $14·4°C = 13·1$ mm. Thus the relative humidity

$$= \frac{8·67}{13·1} \times 100 = 66·1\%.$$

91. Mass of the Moist Air.—To find the mass of a given volume of *moist* air, find (1) the mass of the *dry* air of the same volume at the given temperature and at pressure due to the air only and (2) the mass of an equal volume of aqueous vapour at the temperature and at pressure due to the vapour only; the sum of the two gives required mass.

Let temperature of the atmosphere at the time = t°C

pressure " " " " = P mm. of mercury

pressure of aqueous vapour actually present = f " " "

From Dalton's law, pressure due to air only = $(P-f)$ " "

Now density of dry air at N. T. P. per c.c. = 0·001293 gm.

Let the density of dry air at temperature t and pressure $(P-f)$ = ρ gm. per c.c.

From Boyle's law, $$\frac{P-f}{\rho(273+t)} = \frac{760}{0{\cdot}001293 \times 273}$$

$$\therefore \quad \rho = 0{\cdot}001293 \times \frac{273}{273+t} \times \frac{P-f}{760}$$

∴ Mass of 1 litre of dry air at pressure $(P-f)$ and temperature t°C is given by

$$m_1 = 1000 \times \rho = 1{\cdot}293 \times \frac{273}{273+t} \times \frac{\rho-f}{760} \text{ gm.}$$

Again density of aqueous vapour with respect to air = 0·622

And the pressure due to aqueous vapour alone = f

∴ Mass of 1 litre of aq. vapour at pressure f and temp. t is

$$m_2 = 1{\cdot}293 \times 0{\cdot}622 \times \frac{273}{273+t} \times \frac{f}{760} \text{ gm.}$$

∴ mass of one litre of moist air at the time $= m_1 + m_2$

$$= 1{\cdot}293 \times \frac{273}{273+t} + \frac{P-f}{760} + 1{\cdot}293 \times 0{\cdot}622 \times \frac{273}{273+t} \times \frac{f}{760}$$

$$= 1{\cdot}293 \times \frac{273}{273+t} \times \frac{1}{760} \times \{P-f+0{\cdot}622f\}$$

$$= 1{\cdot}293 \times \frac{273}{273+t} \times \frac{P-0{\cdot}378f}{760} \text{gm.}$$

Examples :—

1. On a certain day the dew point is 8·5°C. and the temperature of air is 18·4°C. Find the relative humidity; the maximum vapour pressure for 8°C., 9°C., 18°C. and 19°C. is respectively 8·04, 8·16, 15·46 and 16·16 mm. of mercury. (P. U.—1928)

A. Increase of vapour pressure for an increase of 1°C. from 8°C. =
8·61 − 8·04 = 0·57 mm.

∴ " " " " " 0·5°C. from 8°C. = 0·285 mm.

Again increase of pressure for an increase of 1°C. from 18°C.

$16{\cdot}46 - 15{\cdot}46 = 1{\cdot}0$ mm.

$\therefore$ " " " " " 0·4°C. from 18°C. $= 0{\cdot}4$ mm.

Hence pressure at 8·50°C. $= 8{\cdot}04 + 0{\cdot}285 = 8{\cdot}325$ mm. of mercury

And pressure at 18·4°C. $= 15{\cdot}46 + 0{\cdot}4 = 15{\cdot}86$ mm. of mercury

$\therefore$ Relative humidity $= 8{\cdot}325 \div 15{\cdot}86 = 0{\cdot}525 = 52{\cdot}5\%$ approx.

2. The dew point on a certain day was 20·2°C. and the temperature of the air 28·6°C. Calculate the relative humidity taking the necessary data from the Book of Tables. (Mad. U.—1949)

A. From the table, saturation pressure at 20°C = 18·66 mm. and that at 21°C = 19·76 mm. So for 1°C, the variation of pressure = 1·14 mm. Hence for a variation of 0·2°C, by applying the law of proportion, the variation of pressure = 0·23 mm. Thus saturation pressure at 20·2°C = 18·62 + 0·23 = 18·85 mm.

Again, saturation pressure at 28°C = 29·94 mm. and at 29°C = 31·54 mm.

Hence for a similar reasoning, saturation pressure at 28·6°C = 30·90 mm.

$\therefore$ Relative Humidity $= \dfrac{18{\cdot}85}{30{\cdot}90} \times 100 = 61\%$.

3. The temperature of air on a certain day is 23°C., and the relative humidity is 60%. What fraction of the mass of the water vapour in the air would condense, if the temperature decreases to 10°C. ? (Saturated pressure of water vapour at 23°C. = 21·1 mm.; at 10°C. = 9·2 mm.).

A. Let the mass of water vapour present in a given volume = m gm.

Now relative humidity $= \dfrac{\text{mass of water vapour present } (m)}{\text{mass of water vapour reqd. to saturate}} = \dfrac{60}{100}$

$\therefore$ Mass of aq. vapour required to saturate at 23°C. $= m \times \frac{5}{3}$

Again $\dfrac{\text{mass of water vapour required to saturate at 10°C.}}{\text{mass of water vapour required to saturate at 23°C.}}$

$= \dfrac{\text{Saturated pressure at 10°C.}}{\text{Saturated pressure at 23°C.}} = \dfrac{9{\cdot}2}{21{\cdot}1}$

Mass of aq. vapour reqd. to saturate at 10°C. $= \dfrac{9{\cdot}2}{21{\cdot}1} \times \dfrac{5}{3} m = 0{\cdot}73m$

$\therefore$ The required *fraction* that would condense $= 1 - 0{\cdot}73 = 0{\cdot}27$.

4. Find the mass of a litre of moist air at 30°C. when the barometer reads 758·6 mm. and the dew point is 16·1°C. (Max. pressure of water vapour at 16·1°C. = 13·6 mm. of mercury).

A. Pressure due to water vapour only = 13·6 mm. of mercury

$\therefore$ Pressure due to dry air alone $= 758{\cdot}6 - 13{\cdot}6 = 745$ mm.

Mass of 1 litre of moist air at 30°C. and 758·6 mm.

= mass of one litre of dry air at 30°C. and at pressure 745 mm.

+ mass of litre of water vapour at 30°C at 13·6 mm. pressure.

Vol of 1 litre of dry air at 30°C. and 745 mm. when reduced to N. T. P.

$$= 1 \times \frac{273}{273 + 30} \times \frac{745}{760} \text{ litres}$$

Vol. of 1 litre of aq. vapour at 30°C. and 13·6 mm. when reduced to N. T. P.

$$= 1 \times \frac{273}{273 + 30} \times \frac{13 \cdot 6}{760} \text{ litres.}$$

Mass of the volume of dry air $= 1 \cdot 293 \times \frac{273}{303} \times \frac{745}{760} = 1 \cdot 1419$ gm.

Mass of aq vapour contained $= 1 \cdot 293 \times \cdot 662 \times \frac{273}{303} \times \frac{13 \cdot 6}{760} = 0 \cdot 0129$

$\therefore$ Mass of 1 litre of moist air $= 1 \cdot 1419 + 0 \cdot 0129 = 1 \cdot 1548$ gm.

92. Condensation of Water Vapour in the Atmosphere.—The water vapour given off from various water surfaces rises up in the atmosphere and remains mixed up with air in gaseous form. The different conditions under which the condensation of water vapour takes place give rise to the formation of dew, fog, mist, cloud, rain, snow, frost or hail. When we breathe out in winter, the warm water vapour gets condensed in the colder atmosphere and presents a *misty* appearance.

Dew.—During day time the water vapour present in the atmosphere remains unsaturated due to higher temperature and hence the presence of water vapour is not directly noticeable. During the night, the warmth of the sun is no longer present and the surface of the earth as well as other objects fall in temperature. The layers of air *very near* the surface of the earth cool and ultimately become saturated with the water vapour contained in it. If the cooling proceeds further, the air can no longer hold the same amount of water vapour, a portion of which is, therefore, *deposited as dew* on the surface of the cold bodies.

The deposition of dew is more copious under the following conditions—(1) *A clear sky*, as this promotes radiation from the earth and objects lying on it, due to which the surfaces of bodies quickly cool and so dew is rapidly formed on them. (2) *Absence of wind*, as in this case a particular layer of air may remain in contact with the cold objects for sufficient time to be cooled below the dew point. (3) The bodies on which dew is formed are to be (*i*) *good radiators*, as they would cool rapidly ; (*ii*) *placed closed to the earth*. If the object is at a higher level, the air getting slightly cooled due to contact sinks down and a warmer layer from above comes to its place ; the result is that no layer of air which comes in contact with the body is sufficiently cooled to deposit its vapour as dew.

Fog : Mist.—If the temperature falls to such an extent that not only the objects on the surface of the earth but also the whole mass of air at or near its surface is cooled below the saturation point, condensation take place throughout a large mass of air in the *lower* regions on suspended dust and smoke particles, thus giving rise to *fog or mist*. In hot countries it is usually formed during a still, cloudless winter night after a calm, warm day. A very dense mist is ordinarily called *fog*. Water is transparent but a patch of fog appears opaque owing to diffused reflections from condensed globules of water.

Fog may be formed on land as well as on water surfaces. Land fog

is formed when a mass of warm air becomes sufficiently cooled on coming to contact with the cold surface of the earth during the evening, night or morning. In this case the fog gradually extends upwards and the condensation takes place on smoke and hygroscopic dust particles serving as nuclei for condensation. When the surface of the earth is cooled by radiation at night, the air in contact is cooled and some of the condensed vapour is deposited as dew and some remain hanging in the air. In autumn or winter, fog is often seen on water surfaces. Here the water and also the layer of air just in contact with its surface are warmer than the air above. Water vapour is then given off in larger quantities and when air in a nearly saturated condition cools owing to contact with the cold air above, the fog is formed.

The dense fog in large towns is due to the condensation of water vapour on smoke and dust particles, although smoke does not cause the fog. On some occasions the mist becomes very dense late at night and continues in the following morning. It is found that the day is warmer on these occasions. Owing to the warmth of the day greater evaporation takes place depositing large mass of water vapour in the atmosphere. When the temperature falls by radiation, condensation takes place readily and a dense fog is formed. As the air warms up by the heat of the sun, it rises above the dew point ; the condensed water particles therefore evaporate and the fog slowly disappears.

Clouds.—Clouds are mist formed *high up* in the air. Large masses of warm air charged with water vapour rise to higher levels and become cooled. The cooling may be brought about by the contact with cold air above or on account of the expansion due to lowering of pressure in the upper regions. If the temperature falls below the saturation point, the excess of moisture immediately condenses on floating dust particles as minute drops of water and becomes visible as clouds. Clouds may either remains stationary or may be moving with the wind. In any case, however, the constituent particles are in constant motion among themselves. A cloud remains suspended in air because the small particles while tending to fall down enter into a warmer region and quickly evaporate. Again as the mass of water vapour is in state of going higher up due to buoyancy, it expands due to lowering of pressure. The expansion lowers the temperature of vapour to the condensation point to form cloud again. Thus there is a constant vertical up and down motion of water vapour in the region of the clouds.

Forms of Clouds —According to the conditions under which they are formed clouds have a variety of forms and structure. The following classifications were made and names given by Luke Howard, a London Chemist, in 1803.

(1) Cirrus.—These are fibrous or feather like clouds of uniformly white colour occupying the highest regions of at least 25000 ft. in the atmosphere. They are composed of particles of ice formed by the freezing of the constituent water drops, owing to the very low temperature at the high altitudes.

(2) Cumulus.—These consists of extensive rounded masses convex above and flat below, resembling heaps of cotton wool. When columns of warm, moist air rise to cold higher regions, they get condensed at the upper extremities so that clouds are formed on the top of these columns at a height of about 5,000 to 10,000 ft. above the earth's surface.

(3) Stratus.—These consist of extended horizontal sheets of grey colour, hanging low in the atmosphere at a hight of about 2,000 ft. They are formed due to condensation, taking place at regions of contact of layers of air at different temperatures. They are characteristic of fine weather.

(4) Nimbus.—These are black rain clouds appearing like formless masses and with rugged edge at a height of about 1,000 ft.

Rain.—Whenever two fine droplets of water in a cloud come into contact, they coalesce to form a larger drop and fall down as *rain*. As a drop falls, some of the vapour in different layers of air condenses on the cold drop ; the drop thus gradually grows in size as it falls down. The rain drops vary in size and velocity of fall.

Rain Gauge.—The amount of rainfall in a locality is measured by a simple instrument called a *rain gauge*. The form now commonly in use and known as 'Simon's Rain-gauge' consists in its simplest form, of a funnel provided with a circular brass rim exactly five inches in diameter, fitted to a cylindrical body joined to the base. The base is kept fixed to the foundation and the height of the rim of the funnel is one foot above the ground. The rain falling into the funnel collects into a bottle placed within the cylinder and the quantity deposited during a period of observation is measured in a glass cylinder graduated to hundredths of an inch. *An inch of rain* means that the water collected would fill the depth of an inch within the cylinder with its base equal to the rim of the funnel. The annual rainfall at a locality is expressed in inches. A fall of one inch of rain on a square yard of surface represents 4·69 gallons. The average annual rain-fall in Bombay is nearly 45 inches and that at Cherrapunji is 100 inches, where we have the greatest amount of rainfall per annum.

Sleet.—If the falling rain freezes before it reaches the ground it is called *sleet*.

Hoar-frost.—If the temperature of the surface of the earth and of adjacent objects fall below 0°C. before the due point is reached, the water vapour in contact with the surfaces directly condenses into icy crystals which are deposited as fast as they are formed. This deposition is called *frost* or **hoar-frost.** Thus frost is not frozen dew but is formed directly by the freezing of the water vapour. Frost is injurious to plants.

Snow.—When condensation takes place at a height where the temperature is below 0°C. the water vapour directly condenses into crystals of snow around suspended nuclei. The crystals being aggregated form of *snow flakes* fall to the ground as *snow*. These crystals have a variety of forms and structure.

Hail.—Due to violent air current accompanying thunder-storms the condensed moisture in upper regions is carried up and down several times through regions of snow and rain, owing to which *half stones* are formed with alternate layers of white snow and clear ice. They may be of various shapes and sizes and are more or less transparent.

WILLIAM HENRY DINES (1865—1934)

Dines was born in 1865 at Horsham. He got his education at Windlesham and at Cambridge. He started his career as a tutor utilising his leisure in scientific studies. His main subject of interest was weather and its causes. Previous to Dines, Glaisher tried to ascertain the conditions of the upper atmosphere by ascending in balloons, reaching about 30,000 ft. from the ground. But to secure more stable and reliable reading at higher altitude, Dines constructed box-kites and pilot balloons with self-registering instruments and started investigation of the upper atmosphere in 1902. With these balloons he made some hundreds of observations upto a height of 9 miles.

He constructed an improved type of hygrometer for the measurement of humidity at any altitude. He also made valuable researches regarding the formation of 'air pockets' which aviators have occasionally to pass through. The meteorological charts of now-a-days which can forecast weather conditions of near future owe their origin due to his efforts. He died in 1934.

Summary

Hygrometry is that part of Physics which deals with the study and measurement of the amount of aqueous vapour present in the atmosphere.

The **dew point** is the temperature at which a given mass of air becomes just saturated with the quantity of vapour actually present in it.

The **relative humidity** denotes the degree of the saturation of the atmosphere as regards aqueous vapour. It is expressed as

$$\text{R. H.} = \frac{\text{Saturated vapour pressure at the dew point}}{\text{Saturated vapour pressure at the air temperature}}$$

Hygrometers are instruments for determining the relative humidity of air at any particular time and place. There are three types of the instruments:—(*i*) Dew point Hygrometer—(*a*) *Daniell's* (*b*) *Regnault's*; (*ii*) the Chemical *Hygrometer*; (*iii*) the Dry and Wet bulb Hygrometer; (*iv*) Hair Hygrometer.

Exercises on Chapter VII

Reference

Art. 84 1. Why does a glass tumbler 'cloud over' on the outside when ice cold water is poured into it?
(C. U.—1929, '30; Dac. U.—1929; P. U.—1923)

Arts. 84 & 85 2. Define 'Relative Humidity' and 'Dew Point' and explain how the determination of the dew point enables you to calculate the relative humidity at a particular place.
(C. U.—1953; *Cf.* Mad. U.—1950; R. P. B.—1941; *Cf.* Pat. U.—1953)

Reference

3. The average temperature of a summer day is much higher than that of a winter day, but still a damp cloth dries more quickly on a winter day. Explain why? (All. U.—1929) — Art. 85

4. The temperature of air in a closed space is observed to be 15°C. and the dew point is 8°C. If the temperature falls to 10°C, how will the dew point be affected? (Pressure of aqueous vapour in mm. of mercury at 7°C. = 7·47; at 8°C. = 8·02). (Pat. U.—1941; Gau. U.—1949) — Art. 85

A. Dew point = 7·75°C.

5. A hot day in Puri causes greater discomfort than an equally hot day in Delhi. Why? (C. U.—1948) — Art. 85

6. Define relative humidity of the air and state how it can be determined by knowing the dew point and using a table of saturated pressure of aqueous vapour. (Pat. U.—1932; U. P. B.—1938) — Art. 85

7. The temperature of the air on a certain day is 23°C. and the relative humidity is 55%. What fraction of the mass of the water vapour in the air would condense, if the temperature decrease to 10°C.? (Saturated vapour pressure at 23°C. = 21·1 mm.; at 10°C. = 9·2 mm.) — Art. 85

A. 0·21.

8. Describe a dew point hygrometer and explain the method of working it. What use is made of this instrument in weather forecasting? (C. U.—1933; M. U.—1933) — Arts. 85 & 86

9. Explain what you mean by humidity. Describe any method of determining it. — Art. 85

On a certain day the temperature of the air at a place is 17°C. and the dew point is 12°C. Find the relative humidity. (Maximum vapour pressure of water at 12°C. is 1·046 cm. of Hg. and at 17°C. is 1·442 cm.) (Dac. U.—1943)

A. 72%.

10. State what is meant by the term 'relative humidity' and describe how its value may be determined. (Nag. U.—1950; U. P. B.—1942; Andhra. U.—1950) — Art. 85

11. What is meant by Regnault's table of saturation pressure of aqueous vapour. Show how this table will enable you to determine the relative humidity of air, if dew point is known. (C. U.—1944) — Art. 87

12. The table below gives the maximum pressure of water vapour in mm. of mercury:—

4° .. 6·1	10° .. 9·1
6° .. 7·0	12° .. 10·4
8° .. 8·0	14° .. 11·9

Explain how these data have been obtained experimentally and find the actual pressure of water vapour present in a room at 14° when the dew point is found be 5°. What is the relative humidity of the air in the room? (Dac. U.—1941) — Arts. 79 & 90

A. 6·55 mm. 55%.

Reference

Art. 86 13. One day when the temperature of the air was 16·5°C., the dew-point was found to be 12°C. Find the relative humidity.

[Maximum vapour pressure of water at 12°C. = 1·046 cm. of Hg., at 16°C. = 1·364 cm. of Hg. and at 17°C. = 1·442 cm. of Hg.]

A. 74·5% approximately. (C. U.—1925)

Art. 87 14. Describe Regnault's hygrometer and explain how you can use it to determine relative humidity.

(U. P. B.—1942; *Cf.* R. P. B.—1952)

Arts. 87 & 89 15. Define 'Relative Humidity.' Describe Regnault's hygrometer and the Wet and Dry bulb Hygrometer. Point out their relative advantages. (Mad. U.—1948)

16. Define relative humidity.

Art. 89 Describe a wet and dry bulb hygrometer. How would you determine the relative humidity with its help?

(Del. U.—1943; C. U.—1948)

Art. 89 17. Explain the use of a wet and dry bulb thermometer.

(Mad. U.—1949)

Art. 91 18. A cubic metre of air, the relative humidity of which is 80% is dried at atmospheric pressure. Find the change in volume, if the atmospheric pressure is 755 mm. and the temperature is 30°C. (Max. pressure of water vapour at 30°C. = 32 mm.)

A. 31×10^6 c.c.

Art. 91 19. Calculate what fraction of the mass of the water vapour in the air would condense if the temperature of the air fell from 20°C. to 5°C and if originally humidity was 60%. (Saturation vapour pressure at 20°C = 17·5 mm. and at 5°C = 6·5 mm.)

A. ·381 (Mad. U.—1948)

Art. 92 20. How is dew formed and why it is more copious on some bodies than on others? (C. U.—1937; Pat. U.—1945)

Art. 92 21. Cloudless nights are better for the formation of dew than cloudy ones. Explain. (U. P. B.—1950; Dac. U.—1929)

Art 92 22. Why does a morning fog generally disappear before noon? (C. U.—1930)

Art. 92 23. Explain the formation of clouds.

(C. U.—1932; *Cf.* U. P. B.—1947)

24. Write a short essay on hygrometer.

(U. P. B.—1948; *cf.* B. II. U.—1951)

Art. 92 25. Write a brief illustrative account of the nature and mode of formation of clouds, fog and dew. Indicate the conditions that are favourable for their formation. (Utkal. U.—1951)

CHAPTER VIII

TRANSMISSION OF HEAT

93. Modes of Transmission.—There are three distinct processes by which heat may be transferred from one place to another. These are *Conduction, Convection* and *Radiation.*

(1) ***Conduction*** *is the process in which heat is transmitted from the hotter to the colder part of a body or from one body to a colder body in contact with it, without any transference of material particles* The agency of a material medium is necessary for this process. Heat is transmitted through solids by conduction. If one end of metal rod is held in a flame, the other end soon feels warm ; the end gradually gets hotter and ultimately becomes too hot to be held in hand Consider the rod to be composed of layers of material particles. Here heat is first communicated to the particles of the rod in contact with the fire ; these particles transfer heat to the adjacent particles and these to the next layer of particles and so on by way of *vibratory motion.* In this way heat passes on from one end of the rod to the other.

(2) ***Convection*** *is the process in which heat is transferred from one point to another by actual movement of heated material particles from a place of higher temperature to another place of lower temperature.* Liquids and gases are generally heated by the process of convection. Take a flask partly filled with water (Fig 65) and drop a few crystals of potassium permanganate into the water ; heat the flask at the bottom. A coloured stream of water is found to rise up from the bottom to the middle of the flask and then to diffuse back down the sides. The water at the bottom is heated first and being lighter it rises up the middle while colder water from the top comes down the sides to take its place. Two currents are thus set up in the mass of liquid,—an *upward* current through the middle and the *downward* current along the sides. These currents tend to heat the entire mass of the liquid indirectly and are, therefore, termed ***convection currents.*** Again if we hold our hand above a flame, we feel hot. The air in contact with the flame being heated becomes lighter and rises up, while colder air from the sides rushes in to occupy the space left by the heated air. The cycle of operation is repeated whereby we get a continuous current of heated air.

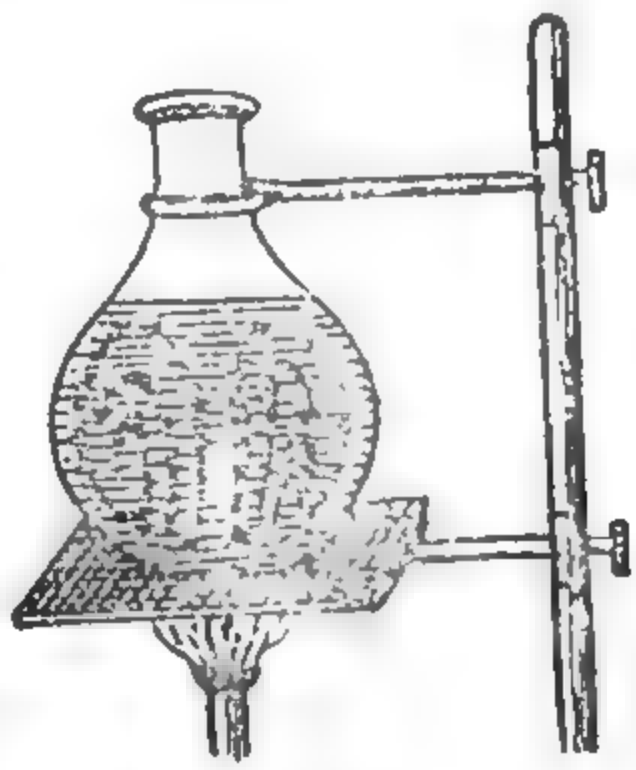


Fig. 65

(3) ***Radiation*** *is the process in which heat can pass from one point*

to another without the intervention of any material medium. By this process heat reaches us from the sun, although the vast space between the sun and the atmosphere of the earth is vacuous. Radiation is thus a distinct process. If we stand before fire, we feel hot. Heat comes to our body not by convection, for due to convection heated air rises upwards while cold air rushes from the sides *towards* the fire. Nor has the heat reached by the conduction, for air is a bad conductor and the effect of the cold stream of air moving inwards would neutralise the outward transference of heat by conduction. Heat here comes by *radiation*.

94. Distinction between Conduction, Convection and Radiation.—(1) In *conduction* and *convection*, heat is transmitted by means of *material particles* while in *radiation* heat can pass through vacuum. In *conduction* heat passes from layer to layer without any transference of the particles of the body, while in *convection*, the particles carrying heat actually move from one part to other. *Solids* are heated by *conduction* and *fluids* (liquids and gases) usually by *convection* while the process of *radiation* involves a wave-motion through empty space.

(2) *Conduction* and *convection* are *slow* processes, while *radiation* is an *extremely quick* process ; heat radiation travels with a velocity equal to that of light.

(3) In *conduction* and *convection*, heat is transmitted in *any* path while in *radiation* heat travels *in straight lines*. Thus the direct heat of the sun is cut off by an umbrella and the heat of a fire can be shut out by an intervening screen.

95. Comparison of Conductivities.—The property of a substance due to which it can transmit heat by conduction is known as **conductivity.** All substances do dot possess this property to the same degree Substances which readily conduct heat through them are called *good conductors*, *e.g* , metals, while those which do not conduct so readily are termed *bad conductors*, *e.g.*, glass, wool, felt, gases *etc.* The conductivity of different solids can be roughly compared by the following experiment.

Ingen-Hausz's Experiment.—The apparatus consists of a metal trough T (Fig 66) provided with a number of holes at its side. A number of rods of the same dimensions *a*, *b*, *c*, *etc.* but of different materials, such as brass, iron, copper, wood, glass are fixed exactly into these holes. The rods extend slightly to the same distance into the trough and are all coated outside with paraffin wax. The trough is filled with water, kept boiling by some heater. So one end of each rod is raised to the temperature of boiling water. While heat is conducted along the rods, the temperature rises gradually to the melting point of paraffin wax as indicated by the melting of wax over the rods.

When the steady state has been attained, it is found that wax has melted over different distances along different rods, the distance being large in case of copper and very small in the case of wood or glass. This shows that the conducting power is different for different substances. It can be proved from theoretical consideration that *the conductivity of a particular material is proportional to the square of the length over which wax has melted on the rod of the material, when the stationary temperature has attained.* This is a very ready method of comparing the thermal conductivities of different materials.

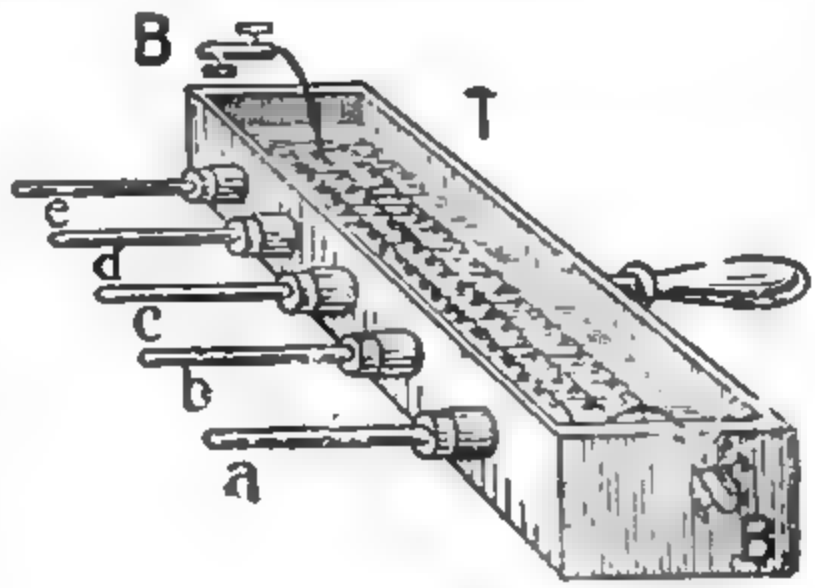


Fig. 66

96. Thermal Conductivity (*Coefficient of Conductivity*).—The *thermal conductivity* of a material is *the quantity of heat that flows in one second through unit area of a plate of that material of unit thickness, the difference of temperatures between the faces being unity and the flow of heat being normal to the faces.*

Let us take a plate of some material, with two opposite parallel faces at different temperatures. Heat will flow from the face at higher temperature θ_1 to the other face at θ_2. The total quantity of heat Q flowing through the plate is (*i*) directly proportional to A, the area of cross-section of the plate ; (*ii*) to the difference of temperatures $(\theta_1 - \theta_2)$, between the faces ; (*iii*) to the time t in seconds and (*iv*) inversely proportional to d, the thickness of the plate.

$$\text{Then} \qquad Q \propto \frac{(\theta_1 - \theta_2)At}{d}$$

$$\therefore \qquad Q = \frac{kA(\theta_1 - \theta_2)t}{d} \qquad (96.1)$$

where k is constant, depending solely on the nature of the material of the plate.

If $A = 1$ cm^2., $\theta_1 - \theta_2 = 1°$C, $t = 1$ sec and $d = 1$ cm., then Q becomes equal to k. The constant is called the **thermal conductivity** of the material of the plate.

The quantity $\frac{(\theta_1 - \theta_2)}{d}$ is called the *temperature gradient* or fall of temperature per unit length. Hence the *thermal conductivity* of a substance may also be defined as the quantity of heat flowing in unit time through unit area of any section of the substance, the temperature gradient normal to the section being 1° per unit length.

97. Determination of Thermal Conductivity.—The thermal conductivity of a *good conductor* can be readily determined by means of Searle's apparatus (Fig. 67). AB is a bar of the material of which the thermal conductivity is to be found. The bar is covered with felt so as to minimise loss of heat from its sides. The end A is placed in a

chamber through which steam is passed while round the other end a coil C of thin copper tubing is soldered, through which a steady flow of cold water is maintained. The temperatures θ_1 and θ_2 of the water at entrance and exit respectively are indicated by two thermometers. Two other thermometers are placed at a known distance apart into mercury contained in two grooves on AB.

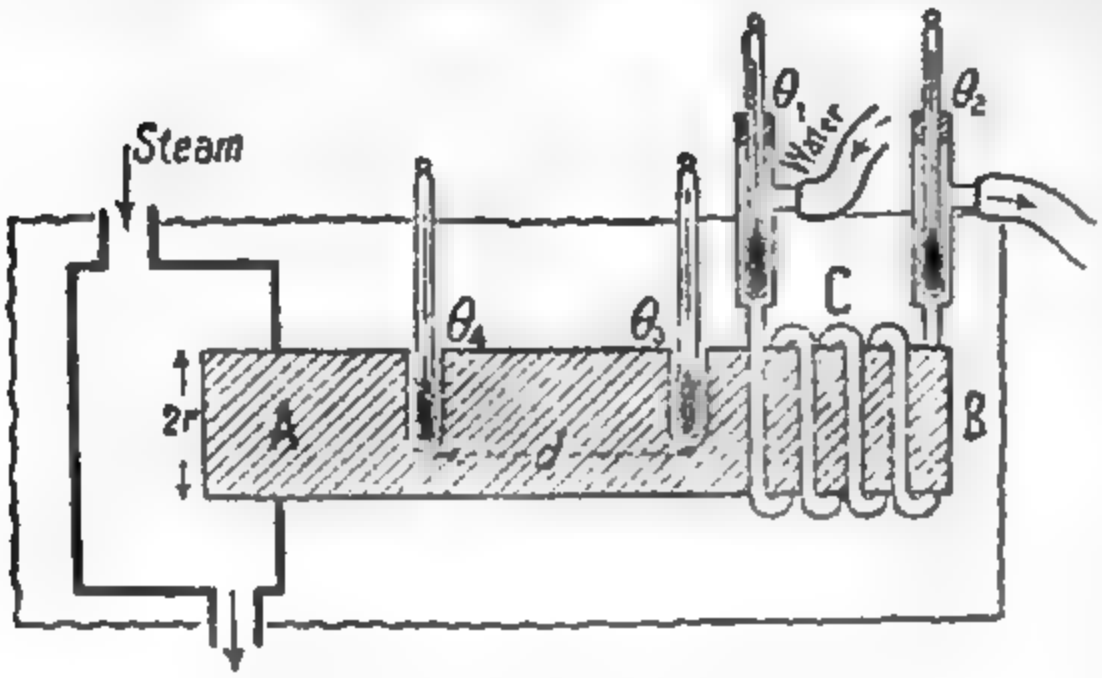


Fig. 67—Searle's Apparatus

By regulating the flow of water, a *steady state* of temperature is reached ; the quantity of heat now flowing through any section is the same, (any loss by radiation being neglected), so that the quantity of heat flowing into the bar at the hot end is equal to the quantity flowing out at the other. The quantity of water passing out in a given time is collected, temperatures at entrance and the exit being noted. Thus the quantity of heat absorbed by the water is found, which also gives the quantity of heat flowing through the bar during the time.

Let the cross-section of the bar $= S$ sq. cm.
length of the portion under observation $= d$ cm.
difference of temperature at the two points $= \theta_4 - \theta_3$
time during which experiment is continued $= t$ sec.
quantity of water collected $= m$ gm.
required thermal conductivity $= k$
Temperatures of water at the inlet and exit $= \theta_1$ and θ_2

Then the quantity of heat flowing through the bar during the time is

$$m(\theta_1 - \theta_2) = \frac{kS(\theta_4 - \theta_3)t}{d} \qquad (97,1)$$

whence k can be calculated. The unit of k in C. G. S. is calorie per cm. per deg. per sec

The following Table supplies thermal conductivities of various substances : —

Thermal Conductivities

Substance	Conductivity C. G. S.	Substance	Conductivity C. G. S.	Substance	Conductivity C. G. S.
Aluminium	·49	Silver	·97	Ebonite	4×10^{-4}
Copper	·92	Brass	·26	Felt	9×10^{-5}
Gold	·70	Glass	2×10^{-3}	Flannel	$2{\cdot}3 \times 10^{-4}$
Iron	·14	Fireclay	4×10^{-3}	Porcelain	$2{\cdot}5 \times 10^{-3}$
Mercury	·2	Asbestos	6×10^{-4}	Rubber	$4{\cdot}5 \times 10^{-4}$

98. Influence of Specific Heat on rate of rise of Temperature.—When one end of a road is heated, its temperature gradually rises. Each portion of the rod receives by conduction from the previous layer *more* heat than the quantity it *transmits* to the next layer and also it *loses* heat by radation : this excess being *absored* raises the temperature of the particular layer. The rod is now at the **variable state.** But after sometime the heat received by any layer becomes *equal* to the quantity transmitted to the next layer together with that lost by radiation from the particular, layer : there is now no further absorption of heat and so no further rise of temperature. This is called **the stationary state.**

In the *variable state, the rate of rise of temperature* at the point of the rod, that is to say, the quickness with which any point of the rod is raised in temperature, depends not only on the thermal conductivity but also on the specific heat of the material. In the *stationary state*, there is no further absorption of heat and so the flow of heat depends only on the thermal conductivity. Take a unit volume of the rod in the variable state of temperature.

Let the density (*i e* , mass per unit volume) of the material $= \rho$
sp. heat and thermal conductivity of the material $= s$ and k
rise of temperature of the substance per second $= t^{\circ}$C.
Then quantity of heat reaching unit volume per second $= \rho\ s\ t. = k$

$$\therefore \quad \text{Rate of rise of temp. } t = \frac{k}{s\,\rho} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity per unit volume}}$$

In other words, the rise of temperature of unit volume of the rod is directly proportional to the thermal conductivity (hence to the quantity of heat reaching the volume) and inversely proportional to the thermal capacity per unit volume (specific heat × density). The ratio $k\ s\ \rho$ is constant and determines the *rate of rise of temperature* of a given rod. This constant has been termed **diffusivity** by Kelvin and **thermometric conductivity** by Maxwell, the former term being more commonly used.

The rate of rise of temperature is slower for a body of higher thermal capacity. Hence during the *variable* state of a substance of low specific heat, the temperature may rise quickly to the steady point, although the conductivity is less. Thus, the thermal conductivity of iron is double that of lead while the specific heat of iron is about four times that of lead. If two rods of iron and lead, of the same dimension, be coated with paraffin wax and exposed at one end to the same temperature (as in Fig. 66) wax is found to melt first on the lead rod but ultimately when the steady state is reached, the wax would melt to a larger distance on the iron rod.

Examples :—

1. An iron boiler 1·25 cm. thick contains boiling water at atmospheric pressure. The heated surface is 2·5 sq. metres in area and the temperature of the underside is 120°C. If the thermal conductivity of iron be 0·2 and the latent heat of steam 536, find the mass of water evaporated per hour.

(R. P. B.—1946 ; Pat. U.—1930 ; Dac. U.—1933)

A. Evidently the water in the boiler is at 100°C.

$$2{\cdot}5 \text{ sq. metres} = 2{\cdot}5 \times 10^4 \text{ sq. cm.}.$$

Let the quantity of heat conducted per hour = Q calories.

$$\therefore \text{ From eqn. (96.1) } Q = \frac{0{\cdot}2 \times 2{\cdot}5 \times 10^4 \times (120 - 100) \times 60 \times 60}{1{\cdot}25}$$

$$= 288 \times 10^6 \text{ cal.}$$

$$\therefore \text{ Quantity of water evaporated} = \frac{288 \times 10^6}{536} = 537{\cdot}3 \text{ kgm.}$$

2. The effective wall area of a room is 100 square metres. Find the rate of flow of heat through the walls of thickness 60 cm. when the temperature of the interior is 30°C. and that of the exterior is 0°C. (Thermal conductivity of the material of the wall = 0·002 C. G. S.). (Mad. U.—1951)

A. Area of the walls = 100 sq. m. = 10^6 sq. cm.; thickness = 60 cm.; difference of temperatures = 30°C.; thermal conductviity = 0·002.

$$\text{Hence } Q = \frac{0{\cdot}002 \times 10^6 \times 30}{60} = 100 \text{ calories per sec.}$$

3. A hollow cube of metal with sides 10 cm. long 1 cm. thick is filled with melting ice and kept surrounded by water at 100°C. How much ice will melt in an hour? (The conductivity of metal = 0·5 and latent heat of fusion of ice = 80).

A. The cube has six faces, each of area 100 sq. cm.

$\therefore$ Total area through which heat entesr = 600 sq. cm.

Quantity of heat conducted per hour

$$= \frac{0{\cdot}5 \times 600 \times 100 \times 60 \times 60}{1} \text{ cal.} = 432 \times 10^5 \text{ cal.}$$

$$\therefore \text{ Amount of ice melting per hour} = \frac{432 \times 10^5}{80} = 54 \times 10^4 \text{ gm.}$$

4. Find the difference in temperatures between the two sides of a boiler plate 20 mm. thick and of conductivity 0·2 C. G. S. units when transmitting heat at the rate of 600 kilogram calories per square metre per minute. (Raj. U.—1953)

A Here Q = 600 × 1000 calories; conducting area A = 100 × 100 sq. cm. time t = 60 sec.; thickness d = 2 cm; conductivity k = 0·2 C. G. S.

$$\text{Hence } 600 \times 1000 = \frac{\cdot 2 \times 100 \times 100 \times (\theta_2 - \theta_1) \times 60}{2}$$

$\therefore$ $\theta_2 - \theta_1$ = difference of temperatures = 10°C.

5. A current of dry steam is maintained through a glass steam trap surrounded by melting ice on all sides. Calculate the thickness of the glass walls and the steam condensed per minute if the total area of the glass trap is 100 sq. cm.; ice melted per minute is 75 gm. thermal conductivity of glass is ·001. (Nag. U.—1953)

A. Here Q = heat absorbed by 75 gm. of ice in melting = 75×80 cal. taking latent heat of ice to be 80 cal/gm.; t = 60 sec.; conductive area A = 100 sq. cm.; decrease of temperatures $\theta_1 - \theta_2$ = 100°C.; conductivity k of glass = ·001. Hence according to eqn. (97.1)

$$75 \times 80 = \frac{\cdot 001 \times 100 \times 100 \times 60}{d} \text{ whence } d = \cdot 1 \text{ cm.}$$

99. Conductivity of Liquids—With the exception of mercury and metals in the molten state, liquids in general are bad conductors of heat. In measuring the conductivily of a liquid, difficulty arises on

account of (1) the convection current set up and (2) the conduction of heat through the walls of the containing vessel,

The thermal conductivity of water can be demonstrated in the following way. Take a thin walled test tube (Fig. 68) nearly filled with cold water ; sink a piece of ice, suitably weighed, into the water. Hold the test tube in an inclined way and heat the water near the top in small flame, It will be found that the water at the top may be boiling while the ice does not appreciably melt. By heating the tube at the top convection current is avoided and the low conductivity of water is clearly shown.

The conductivity of liquids can be *compared* by Despretz's method. The apparatus consists of a tall cylinder fitted with a number of equidistant holes arranged vertically. Each hole is provided with a water tight cork through which the bulb of a thermometer can be inserted within the cylinder. The cylinder is nearly filled with the liquid and cylindrical copper vessel is fitted on to the top. By circulating hot water at about 100°C, into the upper vessel the liquid in the cylinder is heated by conduction. Heat travels downwards very slowly, showing the low *conductivity* of the liquid. After a sufficiently long interval a steady state is attained when the temperature indicated by each thermometer is noted. The liquid is removed and the experiment is repeated with another liquid. If distances from the bottom of the copper vessel to points showing the same rise of temperature be l_1 and l_2 repectively in case of liquids of thermal conductivity k_1, k_2 then.

Fig. 68

$$\frac{k_1}{k_2} = \frac{{l_1}^3}{{l_2}^3} \text{ ruling}$$

100. Conductivity of Gases.—The conductivity of gases is extremely low, the gases (except hydrogen and helium) being worst conductors of heat. The measurement of conductivity of gases is rendered very difficult owing to effects of convection and radiation,

The conductivity of a gas was measured by Kundt and Warburg by observing the rate of cooling of a heated thermometer bulb of approximately known thermal capacity. The bulb was enclosed in a spherical vessel which contained the experimental gas and whose outer surface was kept at 0°C by a water bath. By reducing the pressure to less than 150 mm. the effect of convection was rendered negligibly small while conductivity was not affected. By observing the rate of cooling of the thermometer when the enclosure was made as perfect a vacuum as possible the effect due to radiation only was found. The difference between the two observations gives the quantity of heat transmitted by conduction only. Knowing the radius of the enclosure, the difference of temperatures and the time for the fall, the thermal conductivity k was calculated. The value of k for air as determined by more careful experiments, is 0·000056 C.G.S.

101. Good and Bad Conductors.—If a piece of wire gauge is placed upon the flame of a gas burner (Fig 69), the flame appears only *below* the gauge but does not get through it. The heat is conducted away from the flame by gauze so rapidly that the gas above the gauze does not get sufficient heat to reach the *ignition point* and so the fame does not extend above the gauze. If a lighted match-stick is now applied just over the gauze a flame is produced, showing that the gas has been all along passing through the meshes. If the gauze were held for a sufficiently long time so as to be heated above the ignition point of the gas, the flame would extend above the gauze.

Again a piece of wire gauze is held a little above the burner ; the gas is turned and lighted *above* the gauze ; flame is produced above but it does not extend below the gauze. As the heat of the flame is rapidly conducted by the gauge, the gas below is not sufficiently heated to reach the ignition point. If the gauze is moved up or down, the flame also moves. By applying a lighted match-stick below the gauze, the flame can be produced there.

If an wooden table and iron knife lying on it be touched, the knife feels colder than the table, although both are at the *same* room temperature. Iron being a better conductor takes away heat from the hand more rapidly than wood ; iron therefore cools the hand more rapidly and so appears colder. If the hand is colder than the common temperature of the two objects, the iron knife will appear warmer to the touch than the table ; for heat will pass into the hand more rapidly from the metal than the wood. Again if a piece of wood and piece of metal have been lying long in the sun, the latter will appear uncomfortably hotter to the touch. For the same reason, a marble floor appears colder than a cemented floor and a piece of oil cloth than a linen sheet.

Fig. 69

A glass rod can be melted in a flame by holding it at a point very close to the flame, while in metal rods, even a distant point becomes too hot to the touch. To prevent it from becoming inconveniently hot, handles of soldering "iron" and of metal kettles, tea-pots, cookers, etc., are provided with handles of non-conducting substances.

Water can be boiled in a vessel of *thin* paper. The temperature of the water does not rise above 100 C and as the paper is very thin, heat is conducted rapidly through it and so it is not sufficiently heated to be charred. Steam pipes exposed to air are covered with loosely packed asbestes, so that very little heat is lost. Fireproof safes have asbestos lining and are made of double sheets of iron with the intervening space filled with woodash, saw-dust, etc.

The *low conductivity of air* is observed in various cases. In winter we use woolen clothes, blankets, etc. to keep us warm. They prevent the escape of the heat of the body for two reasons ; firstly, wool is a

bad conductor of heat ; secondly, the air which is enclosed in the interstitches of the loosely woven fabrics is also a bad conductor. These clothes are generally designated as "warm" although they are not actually warmer than other objects in the room. It is also to be noted that the mass of air should be at rest, otherwise heat will be lost by convection. If this were not so, the air surrounding our bare body would have been enough to keep us warm and there would not have been any necessity of "warm" clothes.

Although the conductivity of wool and cotton does not differ much, the former is preferably used in the manufacture of the so called 'warm clothings' ; for wool being more *loose* holds a thicker layer of air for the same weight. A new quilt is 'warmer' than a old one, for in the latter much air has been pressed out. In cold countries rooms are often provided with 'double' windows to keep the room 'warm' owing to the enclosed layer of air. If ice is to be kept long, it is wrapped in a blanket or placed within saw dust, both of which being poor conductors prevent heat of the atmosphere from reaching the ice.

Davy's Safety Lamp.—The heat conductivity of a metal is utilised in the construction of the Davy's Safety lamp. This is an ordinary lamp the flame of which is completely surrounded by one or to layers of thick iron or copper wire gauze (Fig. 70) ; the air necessary for combustion passes in through gauze. If the lamp is taken inside a mine in which marsh gas is present, the gas enters the lamp and burns *inside* with a dull blue flame. The heat of the flame being quickly conducted by the gauze, the gas *outside* is not readily heated to the ignition point. The *appearance of the flame* indicates the presence of the explosive gas in the mine and acts as a warning to the miners. In the improved pattern, for stronger light the flame is surrounded with glass having wire gauze above and below.

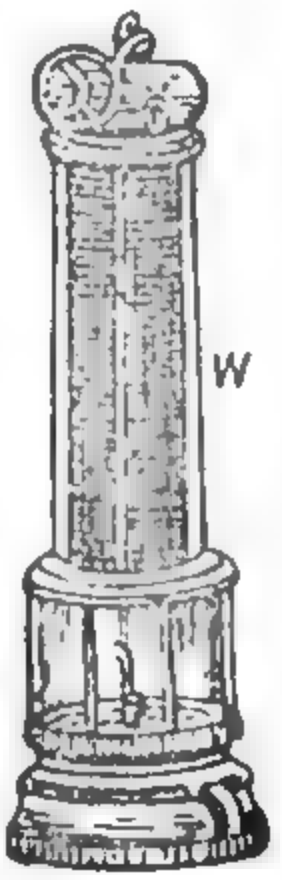


Fig. 70

102. Convection of Liquids.—The convection of heat in a liquid can be demonstrated in the following way. Arrange the apparatus as in Fig. 71. B is a flask fitted with a cork and C is a wide mouthed vessel open at the top and closed with a cork at the lower end. The glass tubes pass through both the corks. One of these D, which is straight, passes just into B and extends at the other end up into C ; while the other tube E, bent as shown in the figure, reaches almost down to the bottom of B and extends at the top just into C. Fill the whole apparatus with water, covering the upper end of tube D and taking care that no air is left in B. Colour the water in C with some dye and secure the apparatus with a suitable clamp. Heat B from below with a Bunsen burner. The water in B becomes heated and rises up along D to C while the coloured colder water from C flows down E into B to take the place of hot water. The circulation of water thus set up is clearly

indicated until ordinary and coloured water becomes mixed. The above experiment demonstrates the principle of the *hot water heating system* in buildings.

Fig. 71

103. Convection of Gases.—When a quantity of any gas is heated, it rises up and the colder gas from all sides rushes to occupy the space thus setting up convection currents. It is a fact of common experience that whenever a house is on fire, the wind blows from all sides towards the place and thus helps the fire. Stand a lighted candle on a basin and pour some water in it ; put an ordinary lamp chimney over the candle, so that the bottom of the chimney is under water. The flame gradually becomes weaker and finally goes out. At the ingress of air from below is cut off, the flame does not get fresh supply of oxygen which is essential to combustion and so it goes out The flame continues to burn if a slight opening is made for air by raising the chimney even slightly above the water.

Now take a piece of T shaped card-board and insert it at the top of the chimney so as to divide the upper part into two halves (Fig. 72). Light the candle again and observe that it continues to burn. Hold the piece of smouldering paper over the chimney near one side of the card-board ; the smoke goes down by one side and rises up by the other side of the card-board, thus indicating the direction of the air currents. So in one half of the tube there is an inward draught supplying fresh air to support the flame while throught the other half the heated air goes out, taking off the products of combustion. This experiment illustrates the principle of *ventilation* of rooms and in some coal mines.

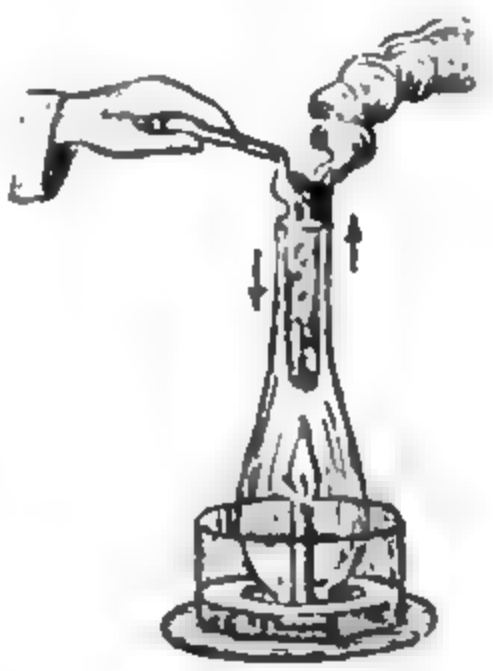


Fig. 72

Ventilation.—It is the process by which the warm and impure air in a room goes out while fresh air from outside takes its place. Ventilation is brought about by convection currents set up in the room. The air in a room is made warm due to the presence of the immates, and being lighter rises up, and escapes through a series of holes near the cornice, known as *ventilator*. Fresh air is drawn in from outside through opening below so that a continuous circulation of air is set up which keeps the room filled with fresh air. If there is a fire place in a room, the draught in the chimney also helps ventilation. The heated air and smoke go up the chimney while fresh air from the outside comes into the room.

Chimneys of lamps or furnaces are designed on the principle of *convection*. The heated air together with smoke and products of combustion go up the chimney while fresh air enters through the holes at the bottom of the burner or the fire place. A convection current is thus set up, which maintains the combustion of the flame or fire.

If the chimney be removed, then (*i*) the regular draught of air is no longer maintained ; the carbon dioxide is not therefore properly removed and a flame does not get a regular supply of oxygen ; (*ii*) also air coming from all sides lowers the temperature of the flame. In either case the combustion becomes incomplete and the flame becomes smoky. It can be shown that the intensity of the draught is greater, (*i*) the greater is the height of the chimney ; (*ii*) the larger is the difference of temperatures of air inside and outside the chimney. Hence to set up a draught, a chimney is sometimes to be warmed initially.

Gas-filled Lamps.—The gas filled electric lamps contain a small quantity of some inert gas such as argon. As the gas is heated, it rises up, carrying the heat away to the upper part of the bulb, so that the filament can be raised to a higher temperature without any risk of melting. Further due to the convection current the tiny particles thrown off by the filament are carried to the top of the bulb. So in the case of these gas-filled lamps the blackening caused by the deposit of these particles occurs mainly at the upper part and not over the entire inside surface ; these lamps therefore last for a longer period.

104. Natural Phenomena based on Convection.—The following natural phenomena depends upon the convection of air.—

Winds.—Winds are commonly due to convection currents set up in the atmosphere due to local variation of temperature. Air at a particular place gets heated somehow and rises upwards and colder air from surrounding places proceeds towards the region, producing winds. Again a region at higher temperature contains larger amount of water vapour due to the greater evaporation. As water vapour is lighter than air, the moisture would go up producing a convection current. Winds are therefore due to convection currents in the atmosphere produced due to a local decrease in density of air either due to rise of temperature or increased evaporation or both. When the fall of pressure at a place becomes very great, the cold air from all sides rushes to the place with a very high speed to produce *storms*.

Land and Sea Breezes.—Land absorbs heat from the sun more rapidly than water ; also owing to its lower specific heat, land is soon raised to higher temperature than water. Thus, during day time land becomes more heated than the sea. The air in contact with land becomes heated and being lighter rises up while the colder air over the sea blows *towards the land*, causing what is called a *sea breeze*. Again during night, the land cools more rapidly than the sea and so gradually attains a lower temperature than the latter ; the air in contact with sea being thus at a higher temperature rises up while the colder air over the land blows *towards the sea*, producing what is called a *land breeze*. The effect of such breeze seldom extends beyond twenty-five miles from the shore.

Trade winds.—The surface of the earth near the equator becomes greatly heated by the sun ; the air in contact being heated rises up and colder air from the north and the south is drawn towards the equator. If the earth were at rest this would produce a north wind in the northern hemisphere and a south wind in the southern hemisphere. But

owing to the rotation of the earth from west to east, there appear relative to the earth at the lower levels, a north easterly wind in the northern hemisphere and a south-easterly wind in the southern hemisphere. These are known respectivoly as the *North-East* and the *South-East Trade Winds.*

Monsoons.—These are *periodic* winds blowing from large continents towards the ocean and *vice-versa.* During the hot seasons the regions in Central Asia become more strongly heated than the waters of the Indian Ocean due to (1) the rays of the sun falling nearly vertical on the former and (2) the difference in the effects of the sun's heat on land and water. Currents of hot air rise upwards from the heated regions while stream of air charged with moisture blow from the Indian Ocean on the south towards these regions On the way the moist air becomes very warm and takes up plenty of moisture. When it meets a barrier of high mountains, it rushes upwards to great height and is thus cooled both by expansion and by elevation to colder regions. Hence the moisture condenses and falls down as rain. Thus the moist air from the Indian Ocean being obstructed by the Assam hills causes profuse rain in Assam and Bengal. In fact the world's greatest rain-fall occurs at Cherrapunji in Assam Owing to the rotation of the earth from west to east this current of moist air seems to blow from the south-west and is called the *south-west* or *rainy monsoon.* During winter the rays of the sun fall vertically over the waters of the Indian Ocean, which therefore becomes more heated. Hence the colder air from the regions of Central Asia appears to blow across India from the north-east and is called *north east* or *dry monsoon.*

Ocean currents.—They are caused partly by convection and partly by winds blowing along the surface. The cold waters in the polar regions sink down and flow towards the equator as a deep *under current* while the surface waters at the equator on being heated become lighter and flow towards the poles as a *surface current.* The **gulf stream** is a huge surface current which starting from the Gulf of Mexico flows along the east coast of the United States, crosses the Atlantic and runs along the west coast of Europe.

***105. Modern Air Conditioning.**—The subject of maintaining a closed room in a comfortable state of temperature, purity and ventilation of air with a congenial proportion of water vapour is called *air conditioning.* The importance of the subject is growing daily with rapid strides of material civilisation.

Necessity.—The modern civilisation has compelled groups of people to stay together in-door for hours and hours A boy has to spend a few hours each day in the company of other boys in the school classes. When he grows up older he attends College classes, meetings, libraries and participates in debating societies, social gatherings or cinema shows. He has often to move in over congested automobiles. In practical life he is employed in a firm or a factory till his age of retirement. In fact a good part of his life is spent in company with many people in some closed space of some form or other.

In a closed and crowdy space the foremost problem is that of

maintaining ventilation. The local atmosphere, due to insufficient supply of fresh air, becomes more and more polluted with carbon-dioxide and some organic matters given off by the people inside. Moreover the human body is a generator of heat which passes out from the body mainly in form of evaporation or perspiration and also by convection and radiation. If this heat leaves the body at a normal rate, the feeling is comfortable ; if it flows less the feeling is stuffy whereas if it flows more, the feeling is chilly.

Comfort Chart.—It is a natural observation that hot summer days become much more uncomfortable if the humidity of the air is high. On the other hand during winter in a drizzling atmosphere the feeling is much more chilly. Thus humidity plays an important part along with temperature in our feeling of comfort. In 1910, a series of experiments were undertaken at Chicago with a view to determine the most favourable temperature and humidity for the best comfort in school rooms. It was found that there was no fixed temperature or humidity required for the best comfort but a relative proportion of these two is always necessary for such a feeling. By varying the temperature and humidity of a closed room, a series of data for temperature and humidity was obtained, This is graphically represented in Fig. 73. The curve C and regions very near it shows the comfort line. The condition of a closed room corresponding to any point of this line would give best comfort Thus, a temperature of 71°F with 30% humidity is equally comfortable as a temperature of 61°F with 70% humidity. The curve D is a slightly warm limit. Any point on or above E appears very warm. Conversely, the curve B is cool and A very cold. This limiting condition of course slightly varies with persons and this is the average limit of the people of cold countries, For people of tropical countrties such as India, although no such statistical curve of comfort is yet available, but the 'warm line' D of the graph would most probably be the 'comfort line' for us.

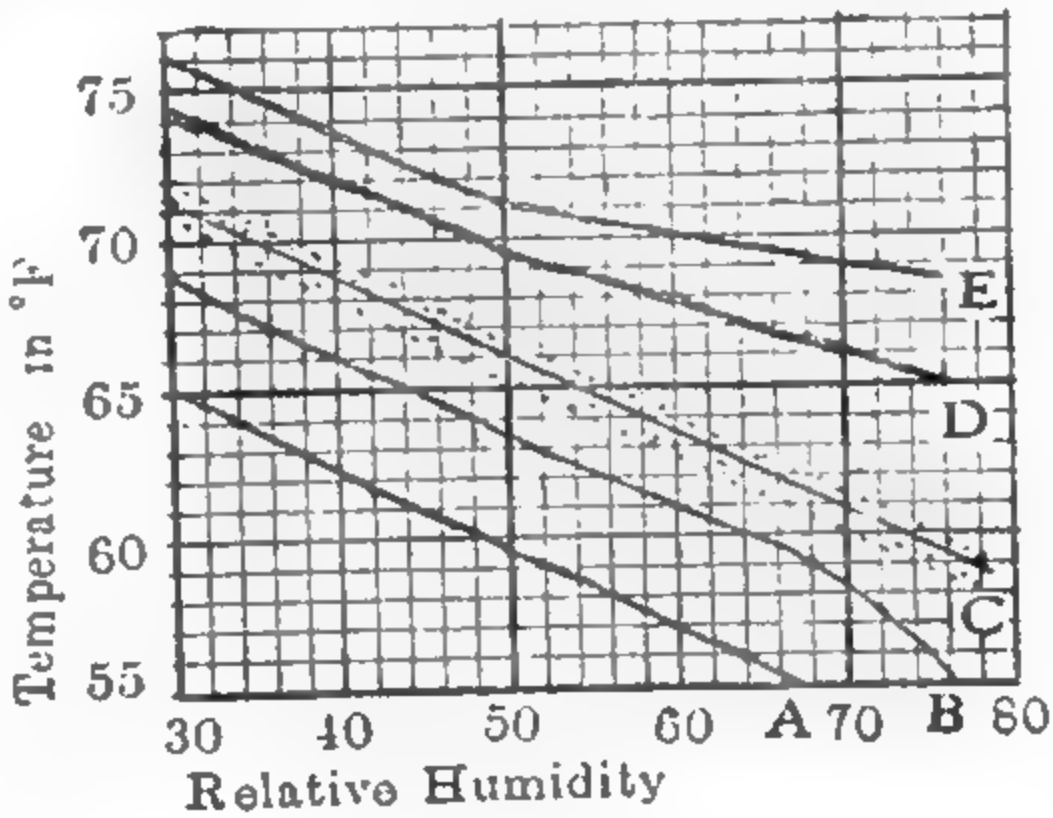


Fig. 73

Heating and Ventilation.—In cold countries the atmospheric temperature is much below 65°F for a major part of the year and for air conditioning some kind of heat is necessary. For warmth fireplace was constructed long ago in most houses. Just above the fireplace is fitted a chimney for heated air and smoke to escape and the warmth of the room is maintained by convection and radiation from the fire place. Pure air from outside is sucked in and the ventilation is maintained. But this sort of a fire side is a bad economy as only 10 to 15% of the heating value of the fuel is usefully utilised.

An improved form of heater by a circulation of hot water, employed in large buildings, is shown in Fig. 74. The water is heated in a boiler from a furnace below. The heated water rises up through a vertical pipe attached to the boiler and passes on to each floor through side tubes. Each room is provided with a radiator which is a series of highly polished bare copper tubes, through which the hot water is made to circulate. In the process of circulation the water gives up most of its heat to the air of the room through the copper tubes. The cooler water runs down through the return tube leading from the feed cistern and re-enters the boiler through its bottom part. A circulation is thus set up and is maintained so long as the boiler is heated. In order to drive away any air collecting in pipes, an air trap is usually fitted at the top of the out-flow tube. This type of hot water heating utilises a much greater proportion of heat of the furnace and there is some natural ventilation of air inside. Moreover as the circulation of heat takes place mostly by convection, the air of the room becomes warmer than objects inside. There is another system, called *panel heating*, in which the ceiling of the room contains concealed radiators. In this case heat comes from a region high up mostly by way of radiation but in this type of heating, the temperature of the room is lowest at the floor and highest at the ceiling which is not always comfortable. But such direct heating arrangements have a defect that there is nothing to regulate the humidity in the room which adds much to the comfort. Of late new types of indirectly heating units have been devised in which a blast of hot and pure air mixed with water vapour is circulated at a level 2 to 3 ft. above the head of the occupants of the room.

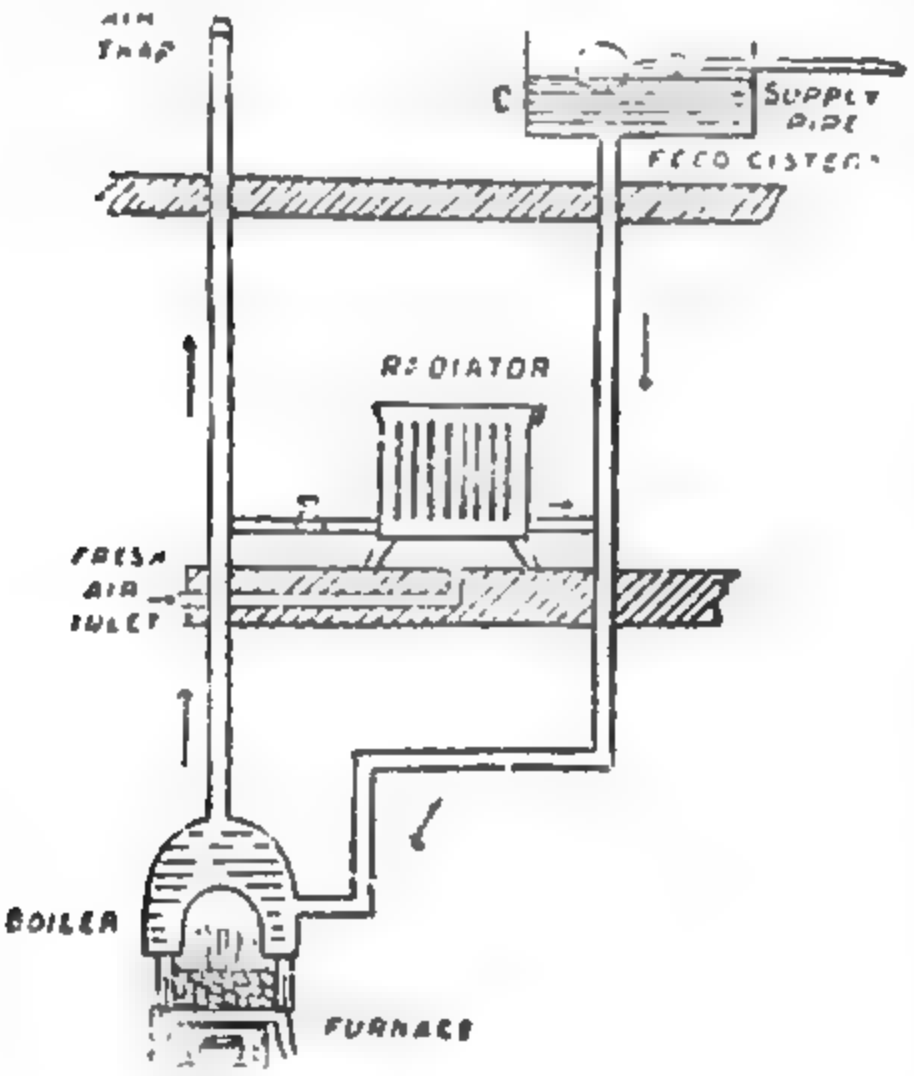


Fig. 74

Cooling and Ventilation.– In tropical and hot countries, the temperature for the most part of the year remains above 75°F and so for air conditioning artificial cooling is necessary. To cool a large closed space with a required degree of humidity, the complete installation may be broadly divided into three parts.—

(i) Refrigerator.—It is a machine to cool water to the required temperature. The water is kept in a reservoir within which there is the expansion coil of a refrigerant gas, such as ammonia, carbon-dioxide, sulphur dioxide or dichloromethane, the last mentioned substance being most frequently used. The principle of action of this part of the apparatus is described in detail in Art. 73. There are two pipes connected to the refrigerater,—one outlet pipe through which cold

water is sent to the *sprayer* or *air washer* and the other through which water returns from the sprayer after absorbing heat. The circulation of water is mintained by a pump.

(ii) Air Washer.—This is a chamber having three interconnected compartments. At the extreme end of the chamber there is a centrifugal air pump U which sucks in air from the other end of this unit through two ventilators O and A. Through one ventilator O outdoor pure air is drawn in and through other ventilator A partially heated and impure air from the closed room to be air-conditioned recirculates. The

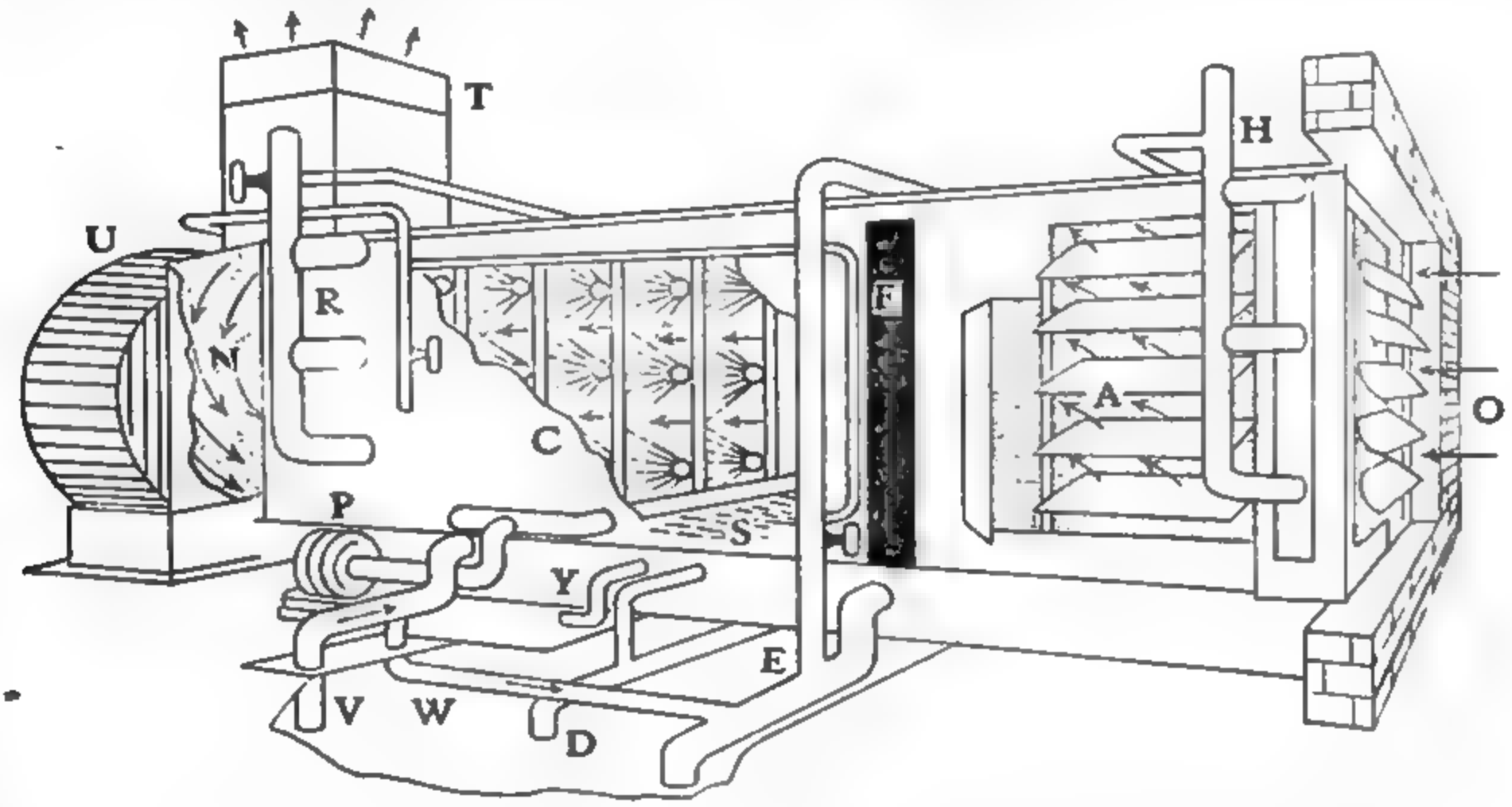


Fig. 75—Air washer system

mixture then passes through one automatic air filter F which absorbs carbon dioxide and dust particles. Then this current of air is led through a chamber C called the cooling chamber. This makes the temperature of the air much lower than the comfort line as also the air becomes saturated with water vapour. The cold water at the required temperature is brought from the refrigerator to the chamber through the pipe V and is sprayed through a number of jets by a compression pump P. The water from the jets in the process of evaporation absorbs the latent heat and thereby cools the air current. The excess water is deposited in a tank S wherefrom it is led back to the refrigerator through the pipe W to be cooled again and recirculated through V.

After passing through the cold sprays, the saturated air is heated to the comfort temperature by a heating unit R. This process makes the air again unsaturated, its temperature and humidity being kept on the comfort line. It is sometimes called dehumidifying air This air is finally sent through a large pipe T to the duct system.

Air washer system is also utilised in heating and ventilation. In this case the outer atmospheric air, supposed to be much colder than the room temperature, is admitted through the ventilator O where it is preheated by the heating coil H, to the temperature of the impure

air of the room to be resucked through the ventilator A. This makes the temperature of the mixture *a few degrees below* the desired temperature. The refrigerator is stopped and water at a temperature slightly higher than that of the mixture is sprayed through the jets of the chamber C by means the pipe E. Thus saturated air at a temperature of a few degrees below the desired temperature is again heated by the heater R before it is sent to the room. The temperature of saturated air is so adjusted that mere heating it gives the circulating air a temperture and humidity suited to the comfort chart.

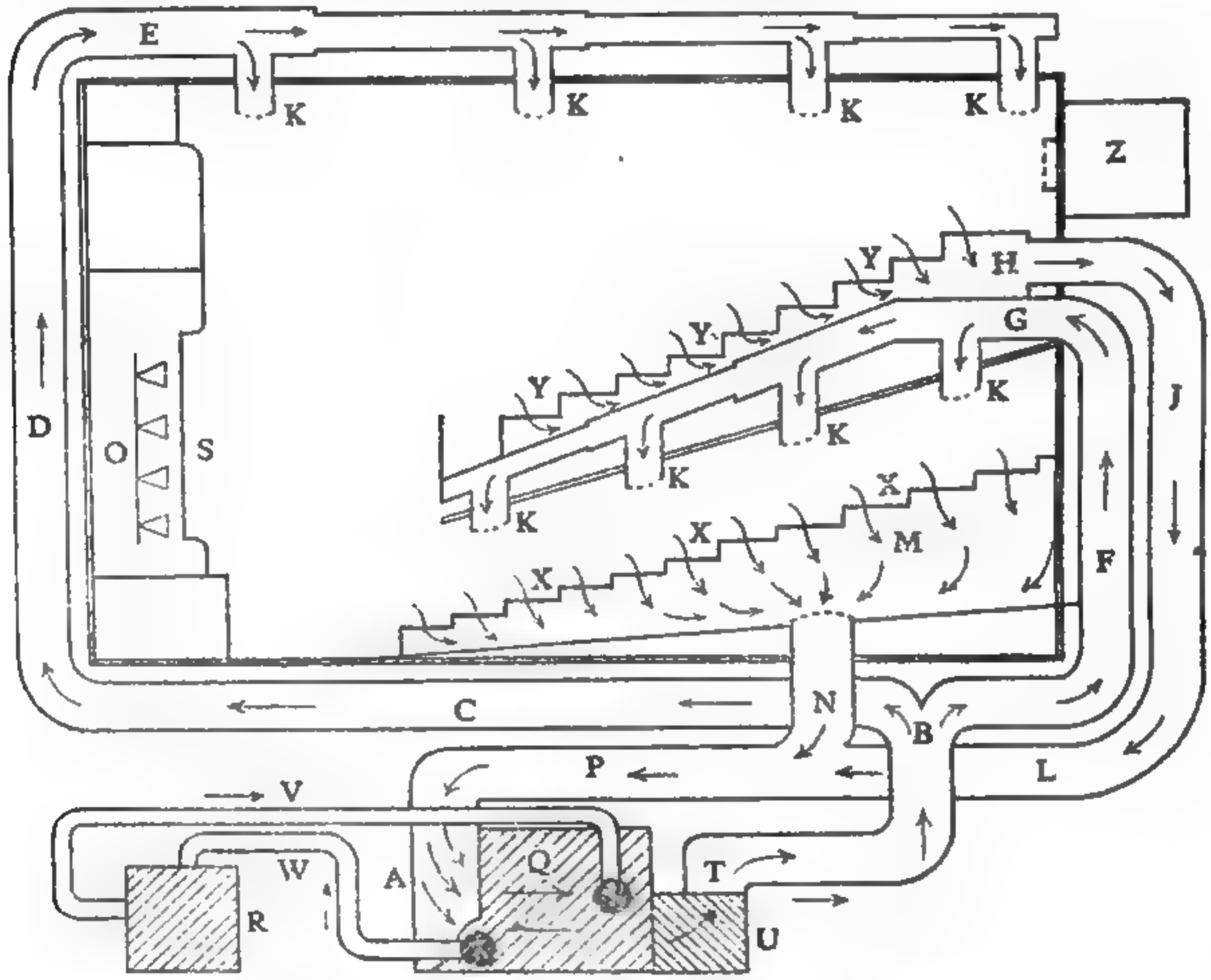


Fig. 76—Air conditioning system in an Auditorium.

(iii) Duct System.—The cold air is sent through a large pipe to a system of orifices evenly distributed on the ceiling of the room. The air circulates through the room and is sucked through a number of ducts on the floor and is returned to the air washer for a fresh purification and circulation. Fig. 76 shows an arrangement of air conditioning in an auditorium.

In the complete picture (Fig. 76), R represents the refrigerator from which cold water at the desired temperature is supplied to the air washer Q through the pipe V and is taken back through the pipe W. Air is sucked in by the pump through the air washer, which becomes first saturated at a cold temperature and is then properly warmed to attain the required degree of temperature and humidity for comfort. This air is sent through the pipe T to the auditorium. For a big hall

where there are upper and lower galleries, the inlet air pipe devides in two branches as shown at B. One branch BCDE leads cold air to the ceiling from which it is led into the hall through a number of ducts K and is led out through a number of orifices Y just below the seats on the upper gallery to the returning pipe HJLP to be reconditioned within the air washer. The other inlet branch BFG supplies air to the lower gallery through ducts K, which is led out through orifices X and is sent to the washer through MNP. There is thus a slow vertical cool air current in the hall Z and S represent the projection room and the screen.

106. Nature of Radiation —We know that heat transmitted by radiation can pass through a vacuum and also through a material medium It has been formerly supposed that the whole space of the universe, is filled with a continuous and elastic medium, called *ether* The vibrations of atoms and molecules of a heated body set *up transverse* waves in the surrounding ether, which travel out in all directions with an enormous velocity of about 186,000 miles per second and carry the heat energy from the source to the receiver. When these waves fall on a material body, they are transmitted with a reduced volocity and is partly absorbed in it producing a rise of temperature of the body. It is to be noted that at the source the energy manifests itself as heat and on reaching the receiver it becomes evident again is heat ; but when it passes through vacuum it exists not as heat but as *energy of wave motion in ether*. The latest view regarding the propagation of heat energy is that the radiation consists of transverse waves of electric and magnetic intensities moving through space.

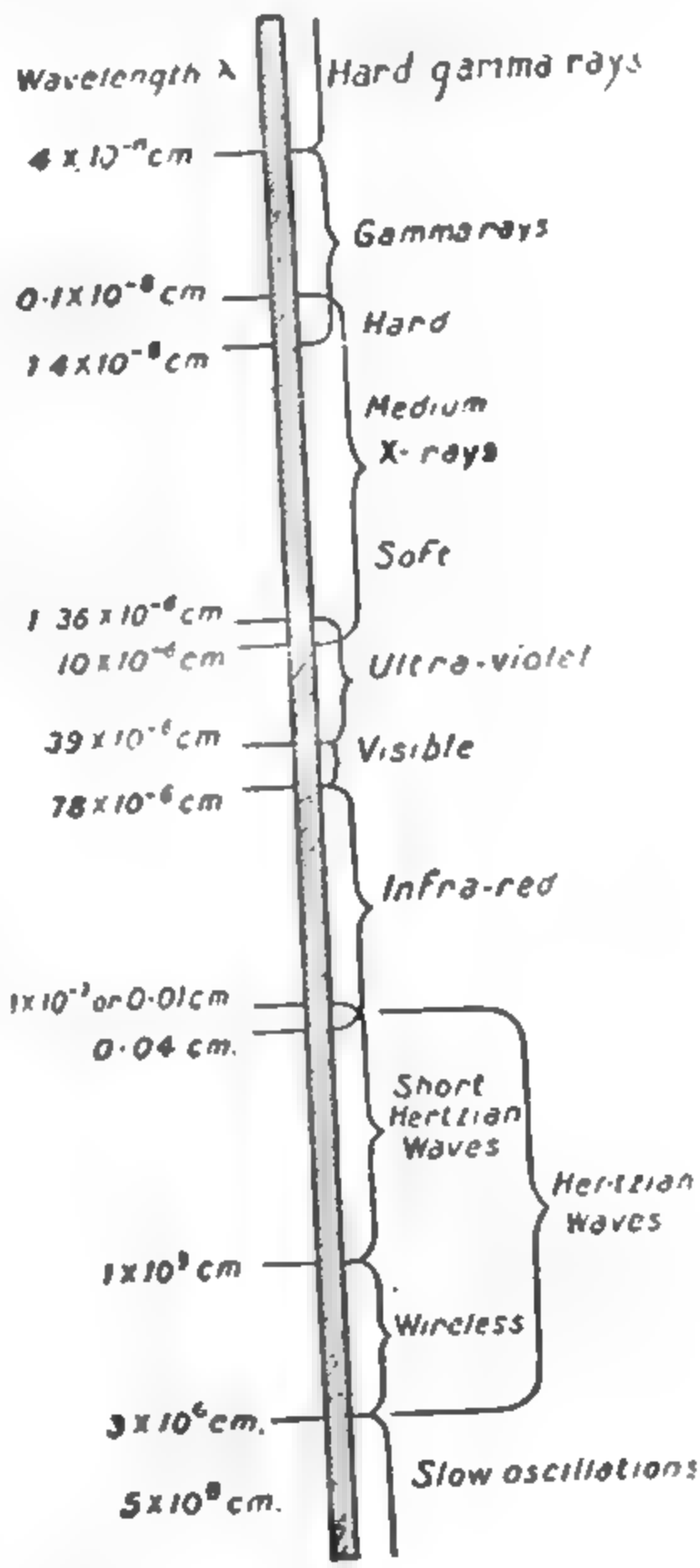


Fig. 77

The radiation from the sun on reaching us excites the sensations of heat and light, both of which are therefore supposed to be electro-magnetic waves It has been found that gamma rays, X-rays, visible light, thermal radiation and wireless waves are all due to the same physical agency, being transverse electromagnetic waves (vide Electricity Chapter XIV). The difference lies in the frequencey of the waves and consequently in the wave length. Thus waves whose

lengths are between 7800×10^{-8} cm. (red) and 3900×10^{-8} cm. (violet) produce of sensation of vision. Radiation caused by these waves is shown as **luminious radiation.** Again when the wave-length is greater than 7800×10^{-8} cm. the radiation does not affect the sensation of vision but produces only the sensation of heat : such radiation is termed infra-red or **thermal radiation**. Beyond thermal radiation, there is the region of Hertzian waves and the radio waves. At the extreme end there are slow oscilliation of alternators. The radiation can also produce chemical reactions, the effect being most marked when the visible length is less then 3900×10^{-8}cm. beyond the violet end of the visible light ; such radiation is termed **ultra-violet radiation.** Beyond ultra-violet, we get X-rays and radiation of still shorter wave-lengths constituting gamma rays. In recent days analysis of cosmic rays reveals that a part of this radiation consists of waves of very short wave lengths, called hard gamma rays. A complete radiation spectrum is given in Fig. 77.

Radiation from a hot body is not visible, as it emits thermal radiation only and no luminous radiation. But if the temperature of the body is gradually increased so as to make it red hot, it becomes visible as the body then emits red light along with thermal radiation, With a gradual rise of temperature, the other colours appear in an increasing proportion the thermal radiation being always present. Ultimately when the body becomes white hot, it emits all kinds of radiations—thermal, luminous and ultra-violet.

107. Instruments to Detect and Measure Thermal Radiation— 1, *Ether Thermoscope.* - It consists of two glass tubes (Fig. 78) connected by a bent glass tube and is held vertically in a suitable stand. The instrument contains only some quantity of coloured ether and ether vapour, the whole of air having been expelled before the instrument is sealed. One of the bulbs is coated with black paint. When thermal radiation falls on this bulb, it is absorbed : so the temperature of the bulb and consequently that of the contained vapour rises. The pressure of vapour on the ether within this bulb therefore increases and so the level of the ether in the other branch rises.

(2) *Differential Air Thermometer.*—This was first used by Leslie. It consists of two equal glass bulbs containing air and connected by a glass tube bent twice at right angles. The tube contains some non volatible liquid (coloured) such as sulphuric acid. The quantity of air in the bulbs is so adjusted that when the two bulbs are at the same temperature, the liquid stands at the same level in both the limbs. If one bulb is raised to a higher temperature than the other, the air in it expands and so the level of the liquid in this stem is depressed while that in the other rises. The instrument thus indicates a small difference of temperatures. In addition to these, there are a few electrical instrument *e. g.*, thermopile, bolometer, radio-micrometer etc. (vide Voltaic Electricty).

Fig 78

108. Thermal Radiation and Light Compared.— The similarity between thermal radiation and light is established from

the fact that they obey identical laws and have many properties in common. The main points of similarity are :—

(i) Transmission through Vacuum.—*Thermal radiation like light can travel through vacuum.* The heat as well as the light from the sun come to us through empty space. A thermometer with a blackened bulb is placed within an evacuated jar. If the whole of it is now placed in the sun or before a fire, the thermometer shows a rise of temperature.

(ii) Rectilinear Propagation.—*Thermal radiation travels very approximately in straight lines ;* so we may use the term *heat rays* in a similar sense as *light rays.* The heat coming from a flame can be cut off by a screen in much the same way as to stop the light coming from it. It will be seen subsequently that both heat and light waves have got the property of bending their paths by the edges of opaque bodies. This bending is more pronounced in the case of heat waves.

(iii) Uniform Emission.—*In a homogeneous medium thermal radiation like light, is emitted uniformly in all directions.* An incandecent metal sphere is placed in a dark room ; if a delicate thermometer with a blackened bulb is placed on any side at the same distance from the sphere, it gives the same reading. The sphere also appears equally bright from all sides.

(iv) Velocity of Propagation.—*Thermal radiation and light travel through space with a constant velocity* (186,000 miles per sec.) During a total solar eclipse of the sun, the stoppage of light is simultaneously accompanied by a fall of temperature.

(v) Inverse Square Law.—*Thermal radiation, like light, follows the law of inverse squares ;* hence the intensity of the radiation at a point varies inversely as the square or the distance of the point from the source. Take a large flat-sided tin vessel *S* (Fig. 79) with one face coated with lampblack and fill it with boiling water. Place a thermopile fitted with the cone collector at T is front of the surface ; observe the deflection of the galvanometer, Now remove the thermopile to T, *twice* the former distance ; the deflection of the galvanometer remains unchanged.

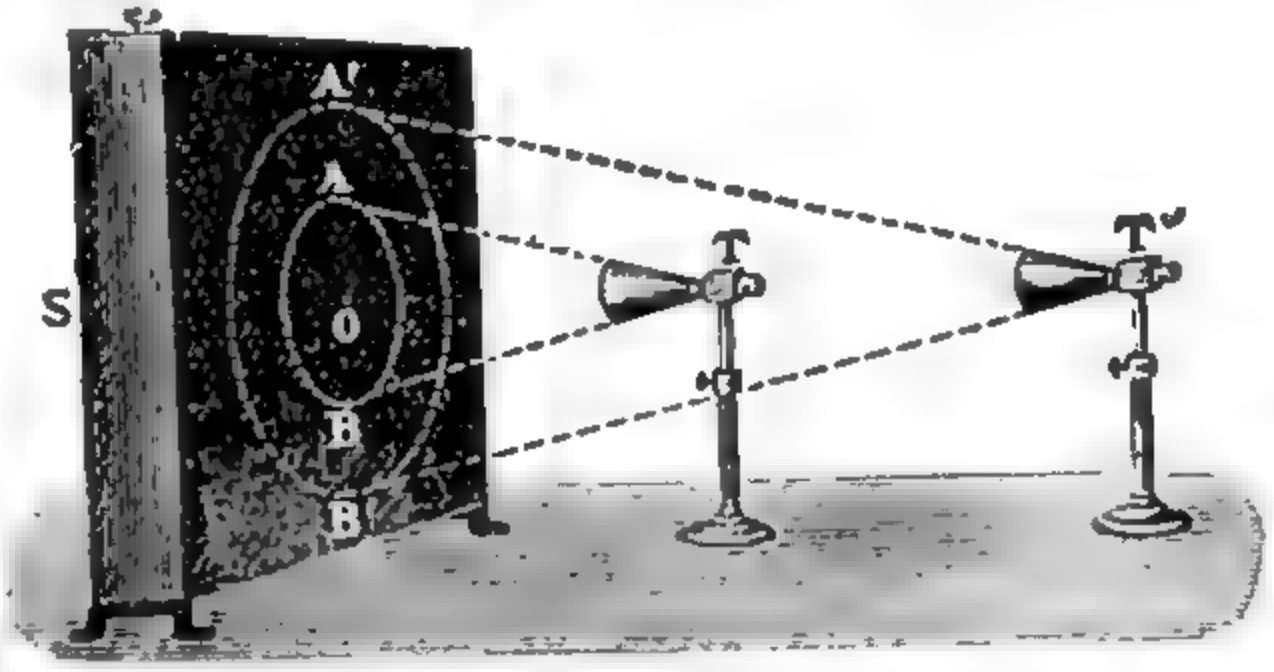


Fig. 79—Proof of the Law of Inverse Squares

The area of the blackened surface from which radiation now falls on the thermopile is found to have increased four times ; but as the deflection remains the same, the amount of radiation received on the pile must have to remain the same. Hence the intensity of radiation on the thermopile has been reduced to *one fourth*, in other words, the intensity of radiation varies inversely as the square of the distance.

(vi) Reflection.—*Thermal radiation like light, is reflected from polished surface, obeying the laws of reflection.*

(*a*) *Reflection at a plane surface.*—A polished tin plate is supported vertically. Two tubes with their axes in the same horizontal plane are placed in front of the plate. A heated ball is placed near the end of one of the tubes and a thermopile is placed near that of the other, direct radiation from the ball to the thermopile being prevented by suitable screens. By rotating the tube the position for maximum effect of the the thermopile is obtained. It will be found that the tubes are *equally* inclined on opposite sides of the normal to the reflector. If one of the tubes is slightly displaced, the effect at once grows less.

(*b*) *Reflection at a spherical surface.*—Two large concave metallic mirrors are placed co-axially facing each other, at some distance apart. If a luminous source is placed at the focus of one, an image is produced at the focus of the other. Again, if a red hot ball is placed at the focus of the former, a piece of phosphorus being placed at that of the latter, takes fire.

(vii) Refraction.—*Thermal radiation like light, is refracted obeying the law of refraction.* If solar rays be passed through a convex lens, preferably of rock-salt, the light rays converge to the principal focus of the lens ; with any detecting apparatus it can be shown that the heat rays are also concentrated almost at the same point. If the solar rays previously passed through a glass vessel containing a solution of iodine in carbon bi-sulphide, luminous radiation is cut off, but not the thermal radiation.

A burning glass is a convex lens of short focus used to converge the heat rays from a source to a focus. A large burning glass devised by Tobrinhausen was used to burn up diamonds for the first time.

(viii) Continuity of Spectrum.—Forming spectrum of the solar light with a rock-salt prism, the heating effect along the entire spectrum can be demonstrated with a thermopile. The heating effect is found to increase towards the red part of the spectrum and becomes maximum in the infra-red region at a little distance beyond the visible red. The presence of the dark lines in this region has also been demonstrated.

(ix) Polarisation.—*Thermal radiation, like light, can be polarised ;* that is, the vibrations are brought to a particular plane after being transmitted through certain substances *e.g.* tourmaline plate.

(x) Production of Light by Heating.—If electric current is passed through a piece of platinum wire, it first sends out thermal radiation and finally at about 1600°C. it gives out white light. Thus, at a rather low temperature the body gives out the long heat rays and at a sufficiently higher temperature it emits light rays along with the heat rays.

We thus find that the nature of the thermal radiation as well as of luminous radiation is the same ; the difference lies only in the wave length. *Radiant heat is then invisible light.*

109. Absorption, Transmission and Reflection of Radiation.—When thermal rediation falls on a body, it is generally split up into three parts. (1) A portion is *reflected* at the surface, obeying the laws of reflection. (2) A portion is *irregularly reflected* or diffused at the surface so as to be scattered in various direction, (3) A protion is *refracted* into the body. Of this protion, a part may be *transmitted* through the body and the remainder may be *absorbed* within the material. The portion absorbed raises the temperature of the body. Different bodies possess different powers of reflection, transmission or absorption.

Selective Absorption.—The power of absorption of a substance depends greatly on the nature of the incident radiation and also on the temperature of the source emitting the radiation. Thus, glass is transparent to light and also to thermal radiation coming from a source of high temperature, but is parctically opaque to thermal radiation from a source at a low temperature (below red heat). The light and heat from the sun can pass through moist atmosphere to a great extent. The surface of the earth absorbs the heat, gets thereby heated and sends out thermal radiation in its turn ; but this radiation is greatly obstructed by the moist air. Again rock-salt is very transparent to thermal radiation while quartz behaves similarly towards actinic rediation only. A solution of iodine in carbon bisulphide is opaque to luminous radiation but readily transmits thermal radiation.

110. Emission of Radiation.—The heat energy radiated by a body is transformed into wave-motion. As the transformation takes place at the surface, the *emissive* or *radiating power* of a body is likely to depend on the nature of the surface. It is found that the emissive power of different surfaces, although at the same temperature, is different.

Emissive Powers of different Surfaces.—Take a Leslie's cube on a suitable stand. It consists of a thin metal cube (Fig. 80) with a hole in the upper lid through which a thermometer can be inserted. The faces of cube are different. Thus, one of the faces may be coated with lamp black, another painted white, a third roughened and the fourth highly polished. Fill the cube with boiling water and keep each of the faces turned for the same time towards a thermopile is series with a galvanometer and placed at a fixed distant apart. Note the deflection of the galvanometer each time; evidently the deflection is proportional to the emissive power of the particular surface.

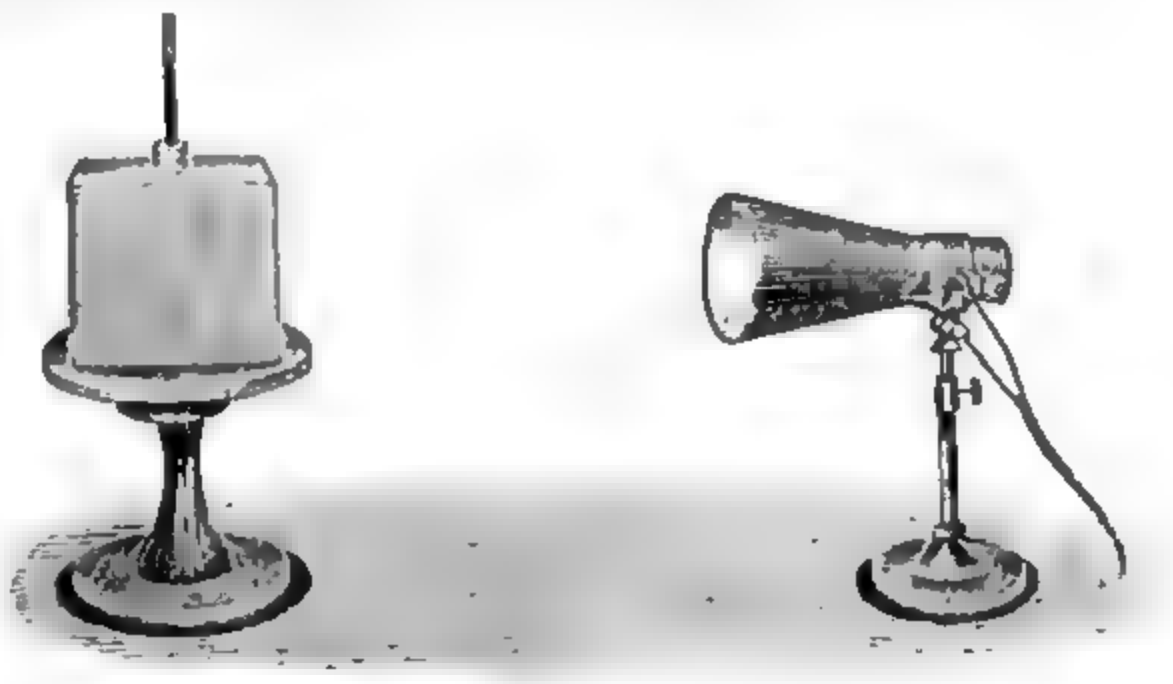


Fig. 80

It is found that for the polished surface the deflection is very small ; it becomes greater for the rough surface, increases further for the coated surface and maximum for the lamp black surface. At a temperature of 100°C. the radiating power of white lead is the same as that of lamp-black but at higher temperatures lamp-black surface is the best radiator. Hot liquids kept in a blackened vessel cool more rapidly than in a polished one. The emissive power of a lamp-black surface is, therefore, usually taken as the standard with which that of any other surface is compared. The radiating powers of different substances are different, even when coated with the same polish.

The ***emissive*** *(or radiation)* ***power*** or the *coefficient of emission of a surface of the ratio of the amount of radiation emitted in a given time by the surface to the amount emitted in the same time by an equal area of a perfectly black surface at the same temperature relative to that of the surrounding.*

Let the radiation from a surface in a certain time = e

radiation from an equal area of a lamp-black surface in the same time = E

Then the Coefficient of Emission, $\epsilon = e/E$.

111. Absorption of Radiation.—Absorption of radiation on the surface of a body varies with the nature of the surface.

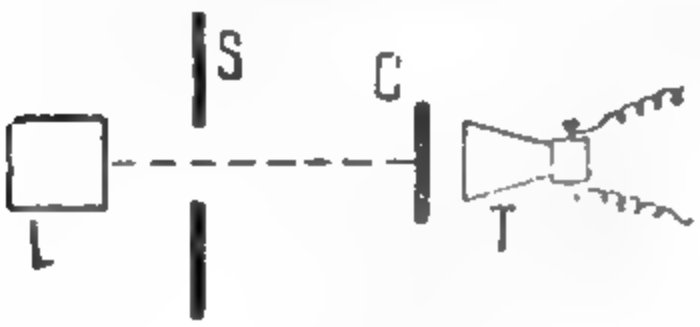


Fig. 81

Absorbing Powers of Different Surfaces.—Take a suitable source L (Fig. 81), say a Leslie's cube filled with boiling water. Take a thermopile T in series with a galvanometer and a perforated screen S, so that the aperture is on the same line as the source and the thermopile. Place a thin disc C of copper, with the face towards the pile coated with lamp-black and the other coated with the experimental substance. Different surfaces absorb different amounts, as is shown by the difference in the deflection of the galvanometer.

The ***absorbing power*** or the ***co-efficient of absorption*** *of a surface is the ratio of the amount of radiation absorbed in a given time by the surface to the total amount incident on the surface in the same time.* It is found that the absorbing and emissive power of a surface are proportional to tach other.

Relation between the Emissive and Absorbing powers of a Surface.—This is demonstrated by Ritche's experiment. The apparatus devised by Ritche consists of two equal cylindrical metal vessels B and C (Eig. 82) filled with air and connected by a glass tube which is bent twice at right angles and contains some coloured liquid. A large cylindrical vessel A, is supported between B and C. The face of A towards B is coated with lamp-black while the other face is polished. The faces of B and C which are turned towards A are respectively polished and coated with lamp-black.

A is filled with boiling water. The level of liquid in the stem on the two sides is found to remain the same, which shows that B and C are at the same temperature *i.e.*, the amounts of energy received by them are equal. Here the polished face B absorbs a fraction of the amount emitted by the black face of A, while the black face of C absorbs wholly the fraction radiated by the polished face of A. As the liquid column does not move, the amounts of heat energy received by B and C are equal, *i e.*, the *emissive power is equal to the absorbing power.*

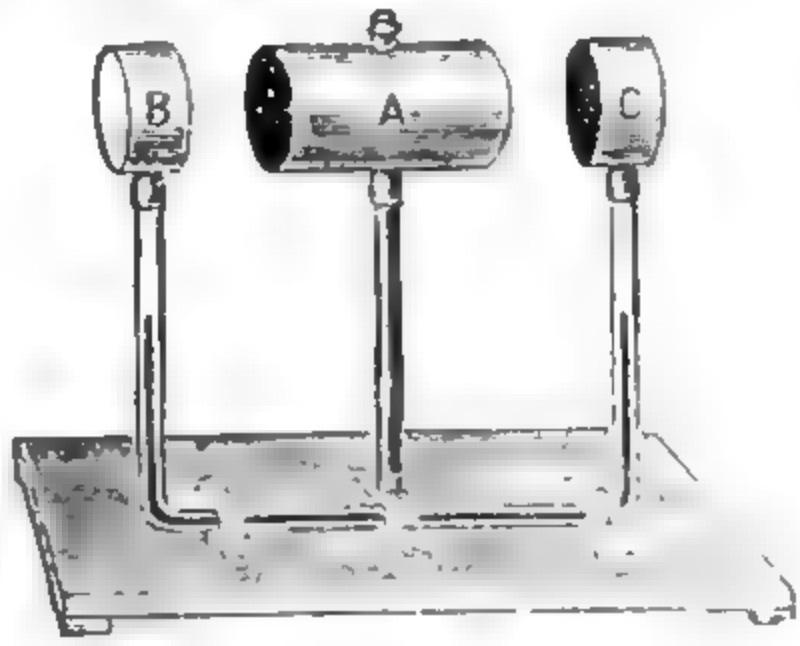


Fig. 82

Hence good radiators are good absorbers are poor radiators are poor absorbers. Thus lampblack surface absorbs the greatest portion of the incident radiation and emits more than any other surface. Further the absorbing power depends on the temperature of the surface and also on the wave-length of the incident radiation. A body which absorbs the *whole* of the incident radiation is termed a **perfectly black body.** The radiation emitted by such a body at any temperature is called the **full radiation** for the temperature A lamp-black surface is the nearest approach to a perfectly black body.

To Compare the Relative Heat Transparency of different materials.—The radiation from suitable source is allowed to fall directly on a thermopile and the deflection of the galvanometer in series is noted. A plate of a material is now interposed and the deflection is noted again. The ratio of the latter deflection to the former gives the ratio of the portion transmitted by the plate to the total radiation incident on it. By taking plates of different materials but all of the *same thickness*, the percentage transmitted by each is obtained and can be compared. In case of liquids, narrow cells of the glass containing the liquid may be similarly used.

The percentage transmitted is found to vary widely for different materials—solids, liquids and gases. Thus for a plate 0·25 cm. in thickness, rocksalt transmits 93 per cent of the incident radiation, glass about 28 per cent while a solution of alum allows only about 2 per cent. The transparency of rocksalt is the greatest and remains the same for radiation from a source at different temperatures. For other substances *the transparency increases with the temperature of the source.*

The ***transmitting power of a plate is the ratio of the amount of radiation transmitted to the amount directly incident on it.*** The fraction of the incident radiation transmitted by a plate 1 cm. thick, is called the coefficient of transmission of the material.

112. Reflection of Radiation.—The portion of the incident radiation which is reflected by a surface depends on the nature of the surface and also on the angle of incidence.

The ***reflecting power*** *of a surface is measured by the ratio of the amount of radiation regularly reflected by the surface to the amount of radiation directly incident on it.*

To compare the Reflecting Powers of different Surfaces.—Take a suitable source H (Fig. 83). Allow the radiation to pass through an aperture in the screen S and to fall directly on a thermopile T connected with a galvanometer and placed at a suitable distance apart; note the deflection of the galvanometer. Allow the radiation now to fall on the experimental plate P; shift the thermopile to T′ at the same distance from P as at T, to receive the reflected radiation, direct radiation to T being prevented by the screen S′; note the deflection again. The ratio of this deflection to the former gives the *reflecting power* of the plate for the particular angle of incidence. It is found that a lampblack surface is the poorest reflector while polished metals are the best reflectors. It is evident that *good absorbers are poor reflectors.*

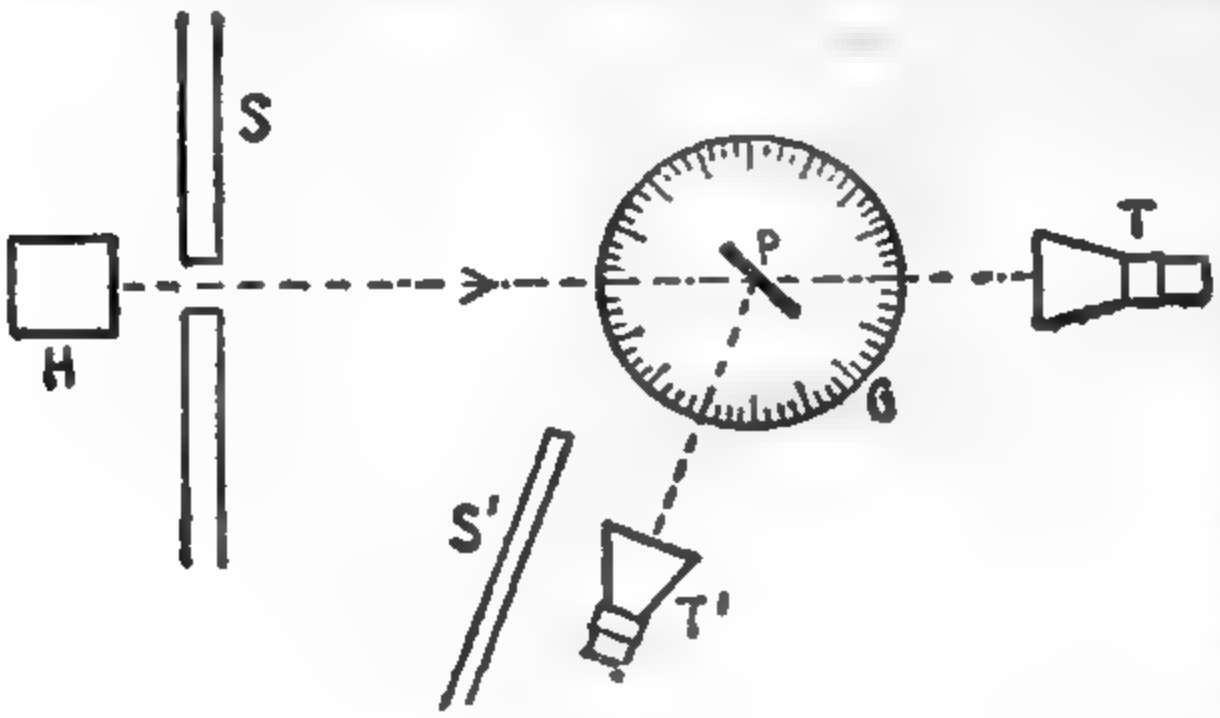


Fig. 83

*113. **Radiometer.**—This small apparatus was first designed by Sir William Crookes. It consists of a glass bulb B evacuated to have a low gas pressure inside (about 1 mm of mercury). Four thin aluminium or platinum vanes V are mounted on a light frame work and are capable of revolving with least possible friction. One surface of each vane is blackened while the other is polished, and all the black surfaces are turned in the same way. If the radiometer is exposed to heat on one side, such as burning of a match stick within a foot from it, the vanes continue revolving as if the dark sides of the vanes are pushed by heat.

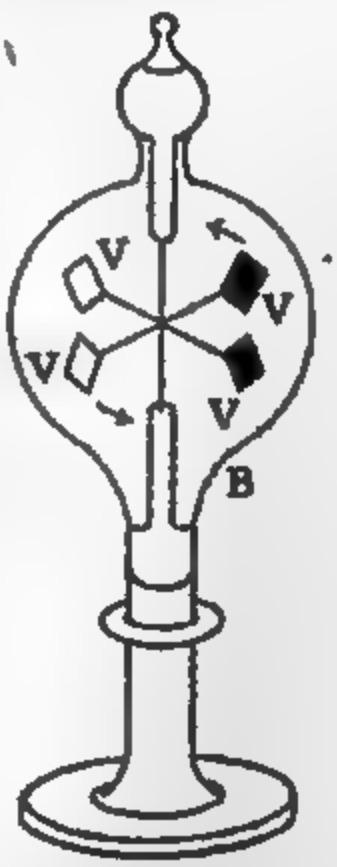


Fig. 88

The principle of action of the instrument is explained by considering the molecular motions within the bulb. When the pressure of gas inside is low, much lesser number of molecules are there within the bulb and the molecules get a better freedom of motion. When heat radiations fall on the vanes, the black surfaces absorb and radiate heat much more than the polished surfaces. Consequently the black surfaces are more heated and gas molecules which collide with a black surface have their kinetic energy much raised and they rebound from this surface with a higher velocity than the polished surface. Each rebound of a molecule gives a tiny tap on the vane. These taps take place on both sides of a vane but

the taps on the black side are more vigorous than the other side. The number of these taps per second on a vane is equivalent to the force contributed by the molecules. Since this force is greater on the black side, all these forces together constitute a couple which rotates the frame. If however the evacuation be proceeded to the extreme point so that molecular action is negligible, it may be that the vanes in presence of heat or light radiation might rotate in the opposite direction. This is due to the pressure of heat or light radiation acting more on the polished side than the dark side. This radiation pressure is extremely feeble and is outweighed by the radiometric action at low pressure.

114. Practical observations on Thermal Radiation.—We can now make a common observation that good radiators are good absorbers while good transmitters or good reflectors are poor absorbers and poor radiators. Polished metals are the best reflectors and are so used in electric radiators. Firemen's helmets are highly polished. Also any vessel meant to keep liquid hot for a long time is highly polished on the outside ; the outside of a calorimeter is polished and the steam pipes of a locomotive are kept brightly polished. On the other hand hot water pipes for heating a room may be blackened on the outside to get greater radiation of heat from them. Hot liquids cool down sooner in a black stone vessel than in a metal cup with polished surface. Well-polished shoes and hats are cooler than dull ones.

Again black bodies absorb more than white ones. Cooking vessels should preferably be black and rough on the outside. A liquid is heated rapidly in common unglazed earthen vessels. A thermometer with a blackened bulb, when kept in the sun, gives a higher reading than an ordinary one. White linen is more suitable in summer as it absorbs very little of the sun's ray ; while black clothing is preferred in winter for it absorbs almost the whole of the incident radiation and thus becomes warm. A white-painted building is warmer in winter and cooler in summer than one painted dark.

The selective absorption by glass of radiation from source at a temperature below the red heat is utilised in a **green-house**. The thermal radiation from the sun passes through the glass covering of the green-house. The ground and the plants inside absorb the radiation, get heated and send out thermal radiation. But glass being opaque to this radiation from a source at low temperature, the heat remains confined and keeps the inside warm. The action of the *glass fire-screen* depends on the same principal. The portion of the thermal radiation possessing the greatest heating effect is absorbed while only a part of the thermal radiation is transmitted along with the luminous radiation.

Since moist air greatly absorbs thermal radiation it prevents the earth from becoming too much heated. Again as the radiation escaping from the heated surface of the earth into space at night comes from a source of comparatively low temperature, it is absorbed by the moist air. Thus too much heating during the day and the cooling by

radiation at night are both diminished. Again in summer, the envelope over plants and valleys maintains them comparatively warm during night but deserts and mountain-tops over which there is little moisture cool down rapidly.

It feels rather warm on a cloudy night because the heat radiated by the earth is obstructed and sent back by the clouds. Dew is more copius on a clear night, as radiation takes place freely and the surface of the earth cools rapidly. As gases are poor radiator of heat, the hot gases in a furnace are made to play on the fire-bricks which are thus heated and radiate heat in their turn.

Earth absorbs the heat of the sun during day and radiates the heat at night. During hot seasons the day is longer than night, and so more heat is absorbed than emitted. This causes an accumulation of heat on the earth from day to day and so the weather feels hotter. During the cold seasons the night is longer than the day so the earth radiates during night not only all the heat it has absorbed during the day but also a portion of the internal heat. The earth is, therefore, cooled down from day to day, causing the weather to feel colder.

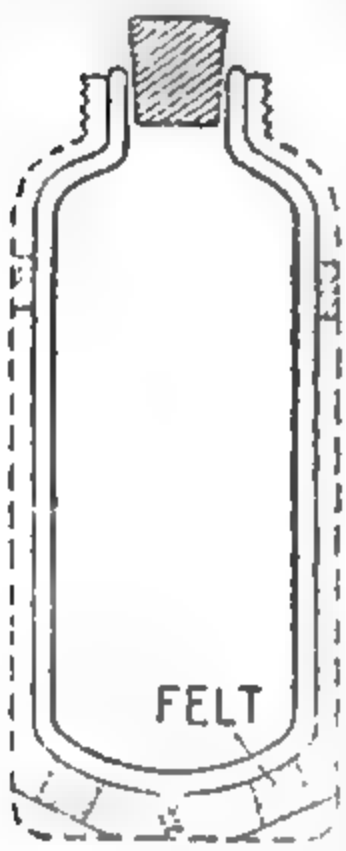


Fig. 85

Thermos flask.—That is used for keeping hot drinks hot and cold drinks cold for a long time. It consists of a double walled glass vessel (Fig. 85) the space between the walls is exhausted of air through a fine tube at the bottom of the outer wall, which is finally sealed off. The outer surface of the inner wall and the inner surface of the outer wall are silvered. The vessel is placed on a spring within a metallic or wooden case, its mouth being closed by a cork stopper, the space between the flask and the outer cover is sometimes packed with felt. Hence the vacuum prevents the passage of heat by conduction as well as by convection, while the silvering minimises radiation to a very great extent. Any transference of the heat by conduction through the wall of the vessel is minimised by the non-conducting packing. Thus in thermos flask passage of heat to and from the interior by conduction, convection as well as by radiation is prevented. It is sometimes called a *vacuum flask*. A **Dewar flask** first designed to store liquefied gases is of similar construction.

115. Loss of Heat by Radiation :—It is experimentally found that the rate at which a body loses heat by radiation depends on (i) the temperature of the body ; (ii) the nature and extent of its surface, and (iii) the temperature of the surrounding space into which radiation takes place.

Newton's Law of Cooling.—Newton first investigated the loss or gain of heat by a body placed in given surrounding and enunciated the following law. *The rate of loss of heat from a body is proportional to the difference between the temperatures of the body and its surroundings.* This law refers to bodies cooling in air ; it is appli-

cable only for small difference of temperatures and does not hold good when temperature difference is large (exceeding about 10°C.) For further details vide J. Chatterjee's Intermediate Practical Physics. All India Edition.

The determination of specific heat of liquid by the method of cooling, based on Newton's law, has been already described in Calorimetry. The laws of cooling have been stated by Dulong and Petit and then by Stefan and Boltzmann. Finally Plank established the law from a different stand-point. According to Stefan-Boltzmann Law, the rate at which the energy is radiated by a hot black body is proportional to the fourth power of its absolute temperature :—

$$E = RT^4 \quad \ldots \quad \ldots \quad \ldots \quad (115,1)$$

where E = energy emitted per sq. cm. per sec. by a full radiator and R = a constant ($5{\cdot}72 \times 10^{-5}$ ergs per sec. per sq cm. per degree absolute^4) and T the absolute temperature. A full radiator absorbs all the the radiant energy falling on it ; a lamp black surface approaches this condition. The total energy radiated is equal to the continued product of E and surface area and time.

According to Wien's Law, the product of the absolute temperature of a body and the wave length at which highest energy is radiated in a given time is constant. Thus

$$T\lambda_{max} = B \quad \ldots \quad \ldots \quad \ldots \quad (115,2)$$

when λ_{max} = the wave length at which maximum energy is radiated and B = a constant (0·2890 cm. deg. absolute) Thus as the temperature of the body rises, wave length at which maximum energy is radiated decreases. Thus as a body is more and more heated till it glows, the first glow appears as red and on further increasing the temperature, the colour extends towards the yellow and green parts, until it becomes white hot

116. Prevost's Theory of Exchanges.—If a red hot metallic ball is placed in front of a thermopile, it indicates that the hot ball is sending out thermal radiation. If the hot ball is now replaced by a lump of ice, it now shows that ice is also sending out some radiation. Again if a hot body is suspended in an evacuated chamber the walls of which are impervious to heat, so there is no loss of heat by conduction or convection, the body will gradually cool by radiating heat to the enclosure. After some time the temperature of the body and that of the enclosure become the same and there is then no further change of temperature. It cannot be supposed, however, that the body suddenly loses its power of radiation ; for if the temperature of the enclosure is now lowered, that of the hot body will at once begin to fall again, showing that radiation has been taking place all the time. Hence it must be concluded that *every body sends out radiant energy at all time and at all temperatures*, the amount of which depends only one its own temperature, and nature of its own surface but is not effected by the presence of surrounding bodies. Thus a red hot ball radiates the same amount of heat whether it is placed near a very hot or a very cold body.

Hence we may conclude that there is always a *mutual exchange* of radiation between any body and the surrounding objects and the rise or fall of temperature observed in a body depends upon the difference between the amount of radiation emitted by the body and absorbed by it from surrounding objects. This is known as **Prevost's Theory** of Exchanges, as it was first formulated by Prevost of Geneva in 1792. Thus when we stand before a fire we feel warm, becase our body is receiving per unit of time more heat energy from the fire than it is losing by its own radiation. Again when we stand before a block of ice we feel cold ; for our body loses more heat by radiation than it receives from the ice which is at lower temperature. When a body absorbs more radiant energy than it itself emits, the body rises in temperature and when less, the body cools. When the quantity attached is equal to the quantity emitted the temperature of the body remains constant. A body will cease to emit any radiation only when it is at the temperature of absolute zero. It follows from this theory that the absorbing power of a surface is equal to its emissive power.

HUMPHRY DAVY (1778—1829)

Davy was born at Penzance, Cornwall on December, 17, 1778. Ate arly age, he did not show much prominence. At his age of sixteen, his father died and he became an apprentice to an apothecary's shop with an intention of becoming a medical practitioner. At a little corner of this shop he set up his first laboratory with most ordinary chemicals and apparatus. He secured an old air pump and began experimenting with pneumatic chemistry.

Fig. 86—Sir Humphry Davy

At the age of twenty he was employed as a research assistant under Dr. Thomas Beddoes. Here in the laboratory he experimented with nitrous oxide and wrote a paper on the usefulness of this gas in producing local insensibility. Afterwards his attention was turned towards electrical apparatus such as Voltaic pile, Leyden jar and

electrical batteries. It was during such investigations that Michael Faraday was introduced with him and was employed as his assistant. With his worthy student, they carried out an elaborate work on "electrolysis" which resulted in the discovery of alkaline elements such as potassium, sodium, magnesium etc. He was knighted in 1812. During those days there were frequent explosions in coal mines due to sudden ignition of combustible gases, Davy devised a kind of lamp which could be used safely in mines without any chance of explosion. He died at Geneva, on May, 29, 1829.

Summary

Modes of Transmission of heat are (*i*) *Conduction*, (*ii*) *Convection* (*iii*) *Radiation*.

Conduction is the process in which heat is transmitted, from layer to layer of a material medium, without any transference of the particles involved.

Convection is the process in which heat is transmitted from one point to another in a medium by the actual movement of material particles.

Radiation is the process in which it can pass from one point to another without the intervention of any material medium. Heat from the sun comes by radiation.

Thermal Conductivity of a material is the quantity of heat that flows in one second through unit area of a plate of that material of unit thickness, the difference of temperatures between the faces being unity and the flow of heat being normal to the faces.

Nature of Radiation.—The vibration of the molecules of a body set up transverse electro-magnetic waves in space. These waves according to the difference in frequency constitute different kinds of radiation.

Thermal radiations possess properties similar to luminous radiations, from which it may be said that *radiant heat is but invisible light*.

The **absorbing power** as well as the **emissive power** of different surfaces are different, those of the same surface being equal to each other.

The **loss of heat by radiation** has been expressed by (*i*) Newton's Law of cooling, (*ii*) Dulong and Petit's Law and (*iii*) Stefan's Law.

Exercises on Chapter VIII

	Reference
1. Mention the ways in which a body loses heat. Give a brief explanation of each way with examples. Which will give you better protection against cold, one thick shirt or two of half the thickness, the material being the same. Explain your answer. (C. U.—1943)	Art. 93
2. What are the different modes of transference of heat? Carefully distinguish between them, giving examples. (And. U.—1951; C. U.—1941, '50; Dac.—1933, '38; Pat. U.—1952; Del. H. S.—1951; *Cf.* Gau. U.—1953)	Arts. 93 & 94
3. Distinguish between conduction and convection of heat. Illustrate their difference by examples. (C. U.—1939; Dac. U.—1943; Utkal U.—1948)	Art. 94

Reference.

Art. 95 4. Describe an experiment to compare the thermal conductivities of two metals in the form of rods. (And. U.—1950; Anna. U.—1950; Mys. U.—1951; *Cf.* Mad. U.—1951)

Art. 96 5. Define thermal conductivity. (R. P. B.—1944, '46; E. P. U.—1951; C. U.—1932; Pat. U.—1926, '36; All. U.—1926, '30; Nag. U.—1952; Raj. U.—1953; Mad. U.—1950, '51)

Art. 97 6. Describe Searle's method of finding the thermal conductivity of metals. (Nag. U.—1952; *Cf.* B. H. U.—1951)

Art. 96 7. The co-efficient of conductivity of copper is 0·96. How many heat units will pass per minute across a plate of copper, one metre long, one metre broad and one centimetre thick, when its opposite faces are kept at temperatures differing by 10°C. (C. U.—1939)

A. 576×10^4 calories.

Art. 96 8. Define thermal conductivity. Explain the statement that the thermal conductivity of glass is ·002 C. G. S. unit. (P. U.—1951; U. P. B.—1950)

Art. 96 8a. The inside of a glass window 0·3 cm. thick is at a temperature of 30°C and the outside at 40°C. Estimate the rate at which heat is entering the room through the window, if its area is 2 sq. metres. (U. P. B.—1944)

A. 1333·3 cals/sec.

Art. 96 9. Taking the conductivity of iron to be ·017 C. G. S. unit, what difference of temperatures should exist between the surface of an iron wall 3 cm. thick, through every square metre of which heat is streaming from a furnace to the boiling water at the rate of 30,000 units per minute. (R. P. B.—1944)

A. 8·8°C.

Art. 96 10. Water is kept boiling in an iron boiler at atmospheric pressure. The boiler is 2 cm. thick and the heated surface is 3 square metres in area. If temperature of the fire be 330°C. and the thermal conductivity of iron be ·2, and the latent heat of steam 536, find the mass of water evaporated in half an hour. (Dac. U.—1934)

A. 2317·16 kgm.

Art. 96 11. The opposite faces of a cubical block of iron of cross-section 4 sq. cm. are kept in contact with steam and melting ice. Determine the quantity of ice melted at the end of 10 minutes, the conductivity of iron being 0·2 (Latent heat of ice = 80 calories). (Pat. U.—1935; E. P. U.—1951)

A. 300 gm.

Art. 96 12. Heat is conducted through a slab of two layers of different materials of thermal conductivities of 0·2 and 0·3 respectively. The thickness of each layer is 1 cm. If the temperatures of two outer surfaces are 100°C. and 0°C., find the temperature of the interface. (Nag. U.—1952)

A. 40°C.

12a. One end of a metal rod 40 cm. long and 1·6 cm. diameter, is placed in boiling water (100°C.) and the other end in ice. If the thermal conductivity of the metal is 0·9 C. G. S. unit, calculate the mass of ice which melts per minute. (Latent heat of water = 80 cal. per gm.). (Mys. U.—1951)

A. 3·39 gm.

Art. 96 13. Find how much steam per minute is generated in a boiler made of boiler plate 0·5 cm. thick, if the area of the walls of the fire chamber is 2 sq. metres, the mean temperatures of the plate faces

Reference

200°C. and 120°C. respectively, the latent heat of steam 522, and the conductivity of steel plate 0·164.

A. 60·321 kgm.

13a. A large copper disc one cm. thick has one face maintained at 100°C. The other face is maintained at 10°C. by running water. Calculate the heat conducted to the cooling water per sec. per sq.. cm. of disc. (Thermal conductivity of copper = 0·91 C. G. S. unit). (Anna. U.—1950) Art. 96

A. 8·19 cal.

14. The glass window of a room have a total area of 8 sq. metres and the glass is 5 mm. thick. Calculate the rate of flow of heat through glass when the inside temperature is 20°C and the outside temperature is — 10°C. (U. P. B.—1950) Art. 96

A. 9600 cals/sec.

15. A metal vessel 1 sq. metre in area and whose sides are 0·5 cm. thick is filled with melting ice and is kept surrounded by water at 100°C. How much ice will melt in an hour? (Thermal conductivity of metal is 0·02 C. G. S. centigrade unit; Latent heat of ice is 80 calories/gm.) (Utkal. U.—1948) Art. 96

A. 1800 kgm.

16. Two metal bars A and B, of the same size but of different materials are coated with equal thickness of wax and placed each with one end in a hot bath. It is noted that the wax on A melts to a greater rate than that on B, but when a steady state has been reached a greater length of wax has been melted on B than on A. Explain this. (C. U.—1941) Art. 98

17. Water is being boiled in an iron vessel over an oven. Find the rate in gm./sec. at which water is evaporating, assuming that no heat is lost otherwise. The area and thickness of the bottom in contact with the oven are 250 sq. cm. and 5 mm. respectively. Latent heat of vaporation of water = 536 cal./gm., temperature of the oven = 400°C., conductivity of iron = ·2 e.g.s. units, boiling point of water = 100°C. (Pat. U.—1952) Art. 96

A. 56 gm./sec. nearly.

18. The effective heating surface of a boiler is 2 sq. metres and the temperatures of the two sides of the steel boiler plates 8 mm. thick are 120°C and 108°C respectively. Find the mass of water evaporated per minute after boiling has commenced. (Latent heat of steam 525 cals/gm.; conductivity of steel 0·14). (Mad. U.—1950) Art. 96

A. 10·8 kgm.

19. Explain thermal conductivity and describe a method of determining it for a metal. Arts. 96 & 97

A spherical glass of 14 cm. radius is filled with water at 100°C. and immersed in melting ice. At what rate must heat be generated inside the globe, if the temperature of water in it is to be maintained constant? (Thickness, of glass = 1 mm.; its conductivity = 0·0025) (R. P. B.—1951)

A. 6154·4 cals/sec.

20. One end of a metal bar is heated. Indicate clearly the factors on which the rate of rice of temperature at any point on it depends. Art. 98

(Pat. U.—1925; Utkal. U.—1951; R. P. B.—1942, '49 Raj. U.—1953)

21. Explain the working of Davy's safety lamp. (C. U.—1928) Art. 101

Reference

Art. 105 22. Explain giving diagrams how a building is heated by hot water pipes. (P. U.—1931)

23. Explain how you would prove experimentally that heat rays like light rays obey laws of reflection and refraction (Pat. U.—1951; *Cf.* P. U.—1952)

Art. 109 24. Can opaque substance transmit radiant heat and transparent ones absorb it? Give examples. (P. U.—1932)

Art. 110 25. Describe and indicate the use of a Leslie's cube. (C. U.—1948)

Arts. 110 & 114 26. Define emissive power and absorption co-efficient. How can you prove that the emissive power of a surface is proportional to its absorption co-efficient. (R. P. B.—1945)

Art. 110 27. A room heater is made up of 10 polished thin walled tubes of copper, each one metre long and 5 cm. in diameter. If hot water at 70°C. circulates constantly through the tubes, calculate the amount of heat in calories radiated in an hour in a room where average temperature is 15°C. (Emissivity of Copper $= 4 \times 10^{-6}$ cal./second per degree per sq. cm.) (R. P. B.—1948)

A. 12434·4 cals.

Art. 111 28. Give an account of experiments to show that good absorbers of heat are good radiators. (C. U.—1925; All. U.—1927)

28a. How will you show that absorbing power of different substances is difference? What relation does the absorbing power of a surface bear to its emissive power at the same temperature? (Utk. U.—1952)

Art. 109 29. Describe Ritchie's apparatus and its working to show that emissive power of a surface is equal to its absorptive power. (Nag. U.—1951)

Art. 111 30. What do you understand by the absorbing and emissive powers of a surface? (R. P. B.—1952)

Art. 111 31. Describe an experiment to compare either the emissive powers or the absorptive powers of two surfaces. (Nag. U.—1951)

Art. 111 32. You are given two ordinary clear glass thermometers and the bulb of one of them is coated with lamp-black. Compare their readings when exposed (1) on a damp cloudy night (2) on a clear dry night in a cold weather. (3) in the sun. (P. U.—1932)

Art. 113 33. Write a note on Radiometer. (R. P. B.—1952)

Art. 114 34. Describe the principle and construction of 'Dewar vacuum flask' or a "Thermoflask"?

(C. U.—1930; P. U.—1951; Gau. U.—1953; Del U.—1941; And. U.—1952)

Art. 115 35. State Newton's law of cooling and its limitations. How will you find the specific heat of a liquid by the method of cooling?

(E. P. U.—1953; U. P. B.—1951; R. P. B.—1943)

Art. 116 36. Give an outline of Prevost's theory of exchanges. The bulbs of two identical thermometers are coated one with lampblack and the other with silver. Compare their readings (*a*) when in a water bath in a dark room (*b*) when in the sun, (*c*) when exposed in a clear night. (Pat. U.—1927)

Arts. 115 & 116 37. Write notes on the Newton's law of cooling. (Raj. U.—1953; *Cf.* R. P. B.—1950; Dac. U.—1941; Del. U.—1943; And. U.—1952)

38. Explain Prevost's theory of exchanges. (R. P. B.—1948; E. P. U.—1950, '53)

CHAPTER IX

MECHANICAL EQUIVALENT OF HEAT

117. Heat and Work.—In earlier days, fire was produced by rapidly spinning the blunt end of a piece of hard wood in a shallow hole made in a block of softer wood. Sparks are produced when a piece of hard metal is pressed against the rim of a revolving grindstone or when a flintstone is struck against a piece of iron. Match sticks may be ignited by rubbing. When we work tools like saws, files, drills, etc., they are found to be considerably heated. Such illustrations show that heat is produced by friction.

Again when the motion of a body is arrested mechanically, the kinetic energy possessed by a body is converted into heat energy. Thus when a body falls from a height and strikes the ground, it is found to rise in temperature. If a piece of iron is struck several times with a hammer, is found to be heated. Sparks are produced when the iron shoe put to the horse's hoofs strikes hard against a metalled road. The action of a tinder box is a very suitable illustration of production of fire by impact. At the time of pumping a cycle, the barrel of the pump gets hot mainly due to the compression of air. All these facts go to prove that heat is produced by mechanial work Conversely heat energy can be converted into mechanial energy, as is illustrated by steam engines and other kind of heat engines. Such examples naturally suggest that there is a relation between the work done and the consequent heat produced.

118. Nature of Heat.—Various theories have been advanced to explain the nature of heat.

Caloric Theory.—According to this theory, which was prevalent before the nineteenth century heat was supposed to be an invisible and indestructible fluid, called *caloric*. The fluid was assumed to fill material bodies and to flow from a body at a higher temperature to one at a lower temperature. As caloric could neither be created nor destroyed, the total quantity of heat in a system was supposed to remain the same. So the caloric lost by one body of the system must be gained by other. The production of heat by friction or compression was attributed to the squeezing out of the caloric, like water from a sponze. The caloric was at once absorbed by the bodies in operation and produced a rise of temperature.

Rumford's Experiments.—Count Rumford, while supervising the boring of cannon at Munich in 1798, observed that the temperature of the gun as well as of the cutting tools rose considerably during the process. It appeared impossible that the generation of such a large quantity of heat was due to a change in the thermal capacity of so small quantity of the substance owing to a mere change in form. Rumford

also found experimentally that the specific heat of the metal was the same whether taken in a large mass or as chips. He further observed at there was no limit to the amount of heat that could be produced by friction. He therefore argued that heat, *considered as a material substance, could not thus be produced without limit* and so he suggested as an alternative that *heat must be a kind of motion.* The same conclusion was also arrived at by Sir Humphry Davy from independent experiments.

Dynamic Energy Theory.—It has been assumed that heat is due to a kind of motion. The question that arises is—motion of what ? According to *Dynamic Theory*, the molecules of a body are in a state of rapid vibratory motion. The total quantity of heat possessed by a body consists in the total amount of kinetic energy possessed by the molecules in virtue of their agitation. In solids, a part of the energy may be potential energy due to the attractive forces between the molecules. When a body is heated the rapidity of the movement of the molecules increases and so there is a gain in the kinetic energy. When the body is cooled, the reverse is the case. Hence *heat consists in the kinetic energy of agitation of the constituent molecules of a body.*

119. Mechanical Equivalent of Heat.—The generation of heat by mechanical work and the production of work by heat indicate an intimate relation between heat and work. Careful experiments show that there is a definite quantitative relation between the amount of work done and the amount of heat produced ; in other words, a definite amount of work must be performed to produce a definite quantity of heat. The relation is expressed in the **First Law of Thermodynamics** which may be stated as follows, after Maxwell,—

"*When work is transformed into heat or heat into work, the quantity of work is mechanically equivalent to the quantity of heat.*" If an amount of work W produces a quantity of heat H,

$$\text{Then} \qquad \frac{W}{H} = J \qquad \text{or} \qquad W = JH \qquad \ldots(119,1)$$

where J is a constant, provided that all the work done is utilised in generating heat and no part of it is wasted. *This constant is the energy equivalent of the unit of heat and is termed the mechanical equivalent of heat or Joule's equivalent. Hence* ***mechanical equivalent of heat*** *is the amount of work done to produce unit quantity (i.e.,* ***one calorie****) of heat.* It is denoted by 'J' after the name of Dr. Joule.

120. Methods for determining J.—Various methods have been devised to determine the value of the mechanical equivalent of heat (J) of which some are given below.

(1) **Vertical Fall Method.**—A cylindrical glass tube, about 50 cm. in length and closed at each end by padded procelain stoppers, contains a number of lead shots. The tube is held vertical when the shots collect on the bottom stopper. On now suddenly inverting the tube,

the shots fall down the entire length of the tube and strike against the lower stopper. If the tube is quickly inverted a number of times, the shots are found to rise in temperature slightly. This is because of the fact that the work done due to the fall of the shots is converted into heat energy which mainly goes to raise their temperature. It is here assumed that very little portion of heat energy is absorbed by the glass tube and the stoppers. So this method is an *approximate* one.

If m be the mass of the lead shots and h the height through which the shots fall down each time, then the loss of potential energy due to each fall is mgh. If there are n such falls, then the total loss of potential energy is $mngh$. So this is the amount of work done by the shots. Again if s be ths specific heat of lead and θ the rise of temperature, heat absorbed by the system is $ms\theta$. Then,

$$J = \frac{\text{Work done}}{\text{Heat produced}} = \frac{mngh}{ms\theta} = \frac{ngh}{s\theta} \quad \ldots (120,1)$$

We also know that loss of potential energy of the shots manifests as the gain in kinetic energy, which is converted into heat each time by impact with the stopper. If v be the velocity of impact, then the gain in kinetic energy is $\frac{1}{2}mv^2 = mgh$. Thus for n impacts, total kinetic energy is $\frac{1}{2}mnv^2$. Then also,

$$J = \frac{\text{Work done}}{\text{Heat produced}} = \frac{\frac{1}{2}mnv^2}{ms\theta} = \frac{nv^2}{2s\theta} \quad \ldots (120, 2)$$

Example :—

1. A closed card board tube 1 metre long contains some lead shots at one end and the tube is quickly inverted so that the shots fall through the whole length of the tube. This operation is repeated 100 times. Find the rise of temperature, if the sp. ht. of lead is ·03. (E. P. U.—1950)

A. If m = mass of lead shots in gm. then loss of potential energy for one inversion of the tube = $mg \times 100$ ergs. For 100 inversions, the work done = $m \times 980 \times 100^2$ ergs, taking $g = 980$ cm/sec.^{2}

Again heat developed = $m \times \cdot 03 \times \theta$ where θ = rise of temperature.

$\therefore \quad m \times 980 \times 100^2 = 4 \cdot 2 \times 10^7 \times m \times \cdot 03 \times \theta$

Whence $\theta = 7 \cdot 8°C$.

(2) Joule's Method.—The apparatus used by Dr. Joule consists of copper calorimeter C (Fig. 88), having four sets of lateral vanes V, V soldered to its sides. The calorimeter is provided with a water tight lid. Through a central opening in the lid a spindle S is introduced, which carries a number of paddles P, P. The paddles can just rotate through corresponding spaces within the fixed vanes. The position of the fixed vanes and the paddles attached to the spindle is shown separately in Fig. 87. The spindle S is attached by a removable pin G to a vertical wooden cylinder D. Two cords are wound round D in the same direction in such a way that they leave the cylinder at the opposite extremeties of a diameter. The other ends of the cords are stretched over pulleys P, P carrying at their ends two equal lead weights W and W. There are vertical scales L, L to indicate the height through which the

weights are allowed to fall. A measured quantity of water is put into the calorimeter and its temperature is noted by a sensitive centigrade

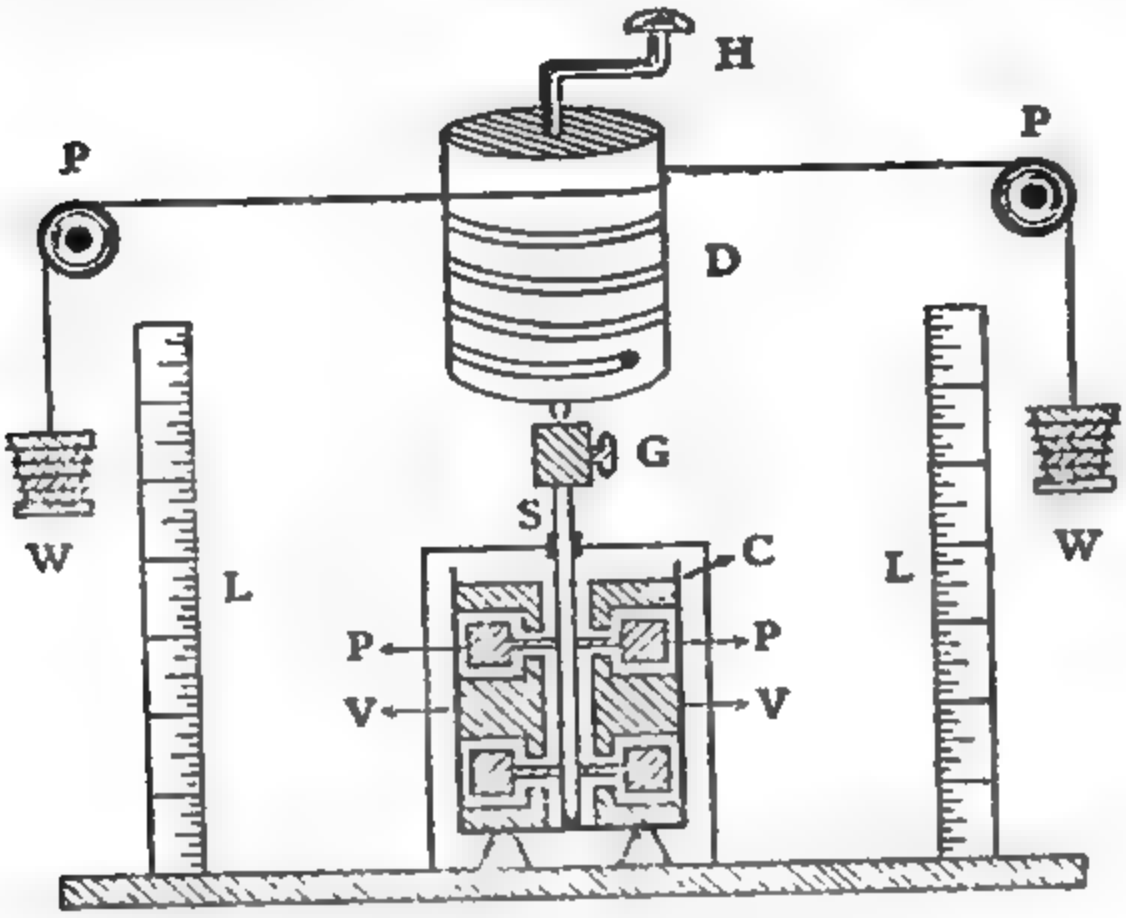


Fig. 87—Joule's Calorimeter

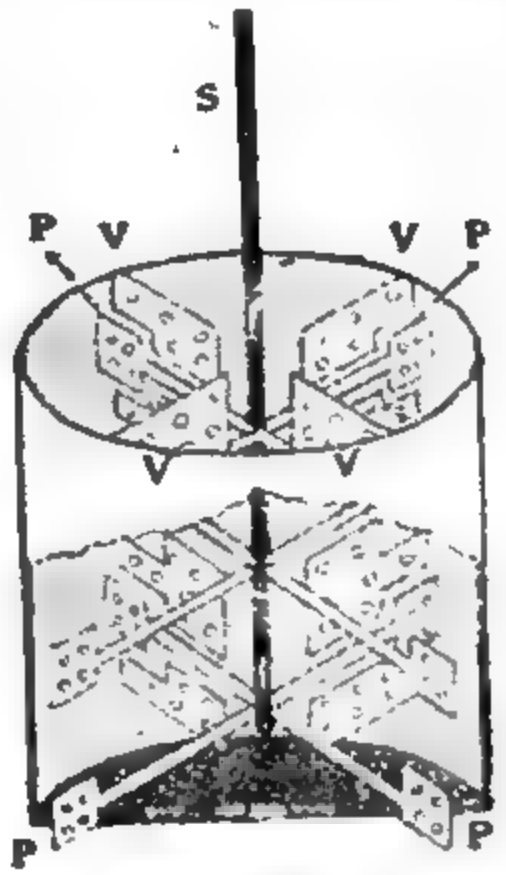


Fig. 88

thermometer, which is introduced into the calorimeter through an aperture in the lid.

By removing the pin, the cylinder D is detached from the spindle and is rotated by the handle H, till the weights are wound up to a definite height. The pin is replaced and the weights are released. As the weights descend, the paddles turn and tend to communicate a churning motion to the water but as this is partly prevented by the fixed vanes, the kinetic energy of the moving water is converted into heat. Just as the weights are about to reach the ground, the cylinder is quickly detached by taking away the pin and the weights are again wound up to the previous height ; the pin is replaced and the weights are allowed to fall as before. The operation is repeated several times and the final temperature of the water in the calorimeter is carefully noted. The amount of heat necessary to raise the temperature of the calorimeter and its contents must then have been supplied by the work done by the falling weights.

Let the mass of each weight	= M gm.
height through which each weight falls	= h cm.
number of times of fall	= n
water equivalent of calorimeter + contents	= m gm.
initial and final temperatures of water	= t_1°C. and t_2°C.
velocity of each weight just before reaching the ground	= v cm. per sec.
∴ Potential energy possessed initially by the two weights	= 2 Mgh ergs.
Kinetic energy retained by the weights just before striking the ground	= $\frac{1}{2}$Mv^2 × 2 ergs

Hence energy utilised in rotating the paddles in each fall
$= (2Mgh - Mv^2)$ ergs.

$\therefore$ Total work done (*i e.*, total energy utilised) in n falls
$= W = (2Mgh - Mv^2) \times n$ ergs.

Heat produced $= H = m(t_2 - t_1)$ calories.

Hence $$J = \frac{W}{H} = \frac{(2Mgh - Mv^2)n}{m(t_2 - t_1)}$$ ergs per calorie. ... (120,3)

Precautions were taken against and corrections were made for loss of heat due to *conduction* and *radiation* respectively. The loss of energy due to *friction* at the pulleys and the energy lost as sound waves due to the slight humming produced by the rotating paddles were also taken into account. Joule, however, on the authority of Regnault, assumes the sp. ht. of water to be the same at all temperatures. The mercury thermometer, he used, was not calibrated in comparison to a gas thermometer and the rise of temperature of water in the calorimeter was also very small.

From a large number of experiments, Joule found the mean value of J to be 773 ft. lbs. of work per B. Th. U. ; that is in order to produce the amount of heat required to raise 1 lb. of water 1°F, 773 ft. lbs. of work are to be performed. By using a modified form of the apparatus Joule finally obtained the mean value at 778 ft. lbs. per B. Th. U. or $4{\cdot}16 \times 10^7$ ergs per calorie.

(3) Searle's method. (*Friction Cone method*).—This is a very simple and convenient laboratory method for determining the mechanical equivalent of heat. The apparatus consists of two hollow truncated cones, C and C_1 of brass, one fitting smoothly in the other, and it is

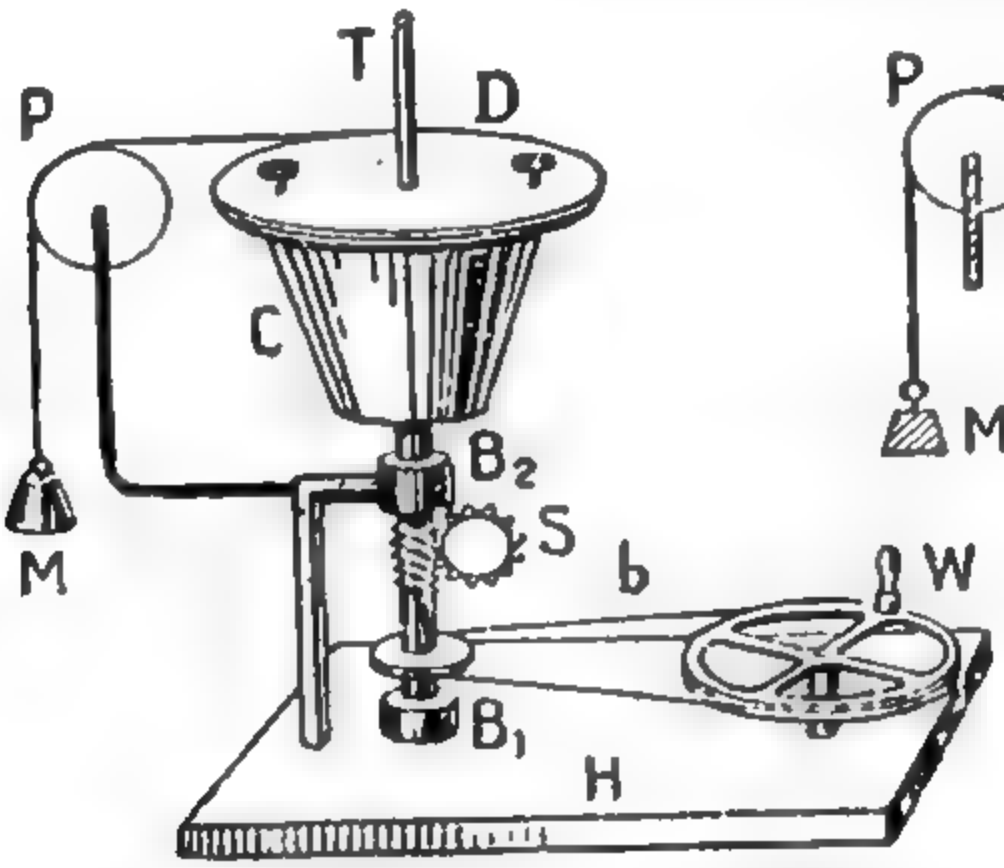


Fig. 89—Friction Cone Method

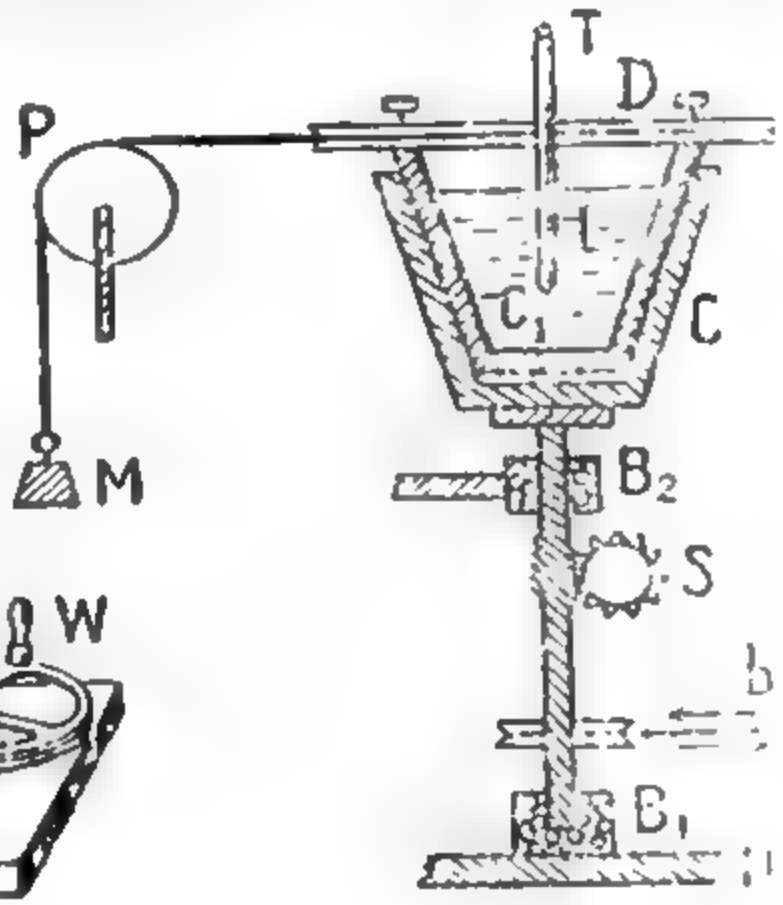


Fig. 90

attached at its upper end to a circular wooden disc. D (Fig. 89). The outer cone C is fixed through ebonite to a brass cylinder, attached to a shaft, which can be rotated rapidly by means of belting b going to a hand-wheel W or a motor. The inner cone C_1 is kept in position by an ebonite ring near the top (Fig. 90). The inner cone C_1 is partly

filled with a known mass of water ; the bulb of a delicate thermometer T passing through a hole in the disc is immersed in the water. A cord is wound round the circumference of the disc D, the end of which passes over the pulley P and carries a weight M at its end. The cylinder supporting C is held by sockets B_1 and B_2, in which there is ball bearing arrangement. The actual apparatus is shown in Fig. 89, and that in section in Fig. 90.

As the outer cone C is rotated, inner cone C_1 tends to follow the same direction but is prevented and held in position due to the opposing couple caused by the weight M. By properly adjusting the rotation of the outer cone and the weight M, the latter can be kept stationary in suspension. In this case the torque due to the frictional force between the cones is equal to the moment of the force due to the weight M. The rate of revolution of the cone is given by the speed counter S.

Let the radius of the wooden disc $= r$ cm.
the frictional force between the cones $= F$ dynes.
mean radius of the cones $= a$ cm.
number of revolutions of the disc per sec. $= n$
water equivalent of the cones and their contents $= m$ gm.
rise of temperature (corrected for radiation) $= t^{\circ}C$.

The moment of frictional force about the axis of rotation $= Fa$
Moment of the pull due to M „ „ „ „ $= Mgr$.
∴ For steady suspension $= Fa = Mgr$
Now work done in n revolutions $= 2\pi na\ F = 2\pi nMgr$ ergs.
Also heat developed by the rotations $= mt$ calories

Hence mechanical equivalent, $J = \frac{2\pi n\ Mgr}{mt}$ ergs per cal. ... (120,4)

(4) Electrical Method.—described in Voltaic Electricity.

121. The value of J in different units.—The correct value of J is 778 ft. lbs. per B. Th. U. of heat. This means that to produce one pound-fahrenheit degree of heat 778 foot-pounds of work have to be done. It can be converted into C. G. S. units in the following way.—

Now 1 ft. lb. $= 30{\cdot}48 \times 453{\cdot}6 \times 981$ ergs.
1 B. Th. U. = 1 lb. degree Fah. $= 453{\cdot}6 \times \frac{5}{9}$ calories.

$$\text{Hence } J = \frac{778 \text{ ft. lbs.}}{1 \text{ lb. deg F.}}$$

$$= \frac{778 \times 30{\cdot}48 \times 453{\cdot}6 \times 981 \times 9 \text{ ergs.}}{453{\cdot}6 \times 5 \text{ calories}}$$

$= 4{\cdot}186 \times 10^7$ ergs per calorie
$= 4{\cdot}186$ joules per calorie.

The value of J is approximately taken to be $4{\cdot}2 \times 10^7$ ergs per calorie.

Examples :—

1. With what velocity must a lead bullet at 50°C. strike against an obstacle in order that the heat produced by the arrest of the motion, if all the heat produced within the bullet is utilised, might be just sufficient to melt ? (sp. ht. of lead $= 0{\cdot}031$; melting point of lead $= 335°C$; latent heat of fusion of lead $= 5{\cdot}37$; $J = 4{\cdot}18 \times 10^7$ ergs per calorie).

A. Let the mass of the bullet be m gm. and let the required velocity be v cm. per sec.

Then the kinetic energy $= \frac{1}{2}mv^2$ ergs.

This energy is converted into heat which raises the temperature of the bullet from 50°C. to 335°C. and then melts it.

$$\therefore \text{ The heat produced} = \{ m \times 0{\cdot}031 \times (335 - 50) + m \times 5{\cdot}37 \}$$
$$= m \times 14{\cdot}205 \text{ calories.}$$

$$\text{As } W = JH, \quad \tfrac{1}{2}mv^2 = 4{\cdot}18 \times 10^7 \times m \times 14{\cdot}205 \text{ ergs}$$
$$v = \sqrt{2 \times 4{\cdot}18 \times 14{\cdot}205 \times 10^7} = 34{\cdot}46 \times 10^3 \text{ cm. per sec.}$$

2. A piece of steel of mass 1200 gm. is struck by a hammer weighing 960 kgm. dropped from a height of 1·6 metres. Assuming 75% of the heat generated to be retained by the steel piece whose specific heat is 0·12, calculate its rise in temperature. (Mad. U.—1950)

A. Mass of hammer $= 960$ kgm. $= 96 \times 10^4$ gm.; Height whence it falls $= 1{\cdot}6$ metres $= 160$ cm ; Assuming $g = 980$ C. G. S., the potential energy of the hammer $= 96 \times 10^4 \times 980 \times 160$ ergs and this amount is to be wholly converted into kinetic energy just before striking the piece of steel.

Now heat produced $= W/J$; assuming $J = 4{\cdot}2 \times 10^7$ ergs/calorie

$$H = \frac{96 \times 10^4 \times 980 \times 160}{9{\cdot}2 \times 10^7} \text{ calories} = 3584 \text{ cals.}$$

Of this amount of heat, only 75% is retained by the steel piece. Thus heat retained by the steel piece is $\frac{3}{4} \times 3584 = 2688$ cals. If rise of temperature of steel piece be t°C, then,

$$t°C = \frac{2688}{1200 \times 0{\cdot}12} = 18{\cdot}6°C \text{ nearly.}$$

3. If the mechanical equivalent of heat be $4{\cdot}2 \times 10^7$ ergs/cal., from what height must a mass of water be dropped in order to raise its temperature by 0·1°C, assuming that all the kinetic energy gained by the mass of water is used up in raising the temperature ? ($g = 980$ cm./sec.^{2}) (Pat. U.—1952)

A. Let h be the height in cm. Since the kinetic energy of fall is equal to the potential energy at the height,

$$\tfrac{1}{2}mv^2 = mgh \text{ where } m = \text{mass of water in gm.}$$

$$\text{Now, } W = JH \text{ or, } mgh = JH = J \times m \times 0{\cdot}1$$

$$\text{or, } 980 \times h = 4{\cdot}2 \times 10^7 \times 0{\cdot}1 \quad \text{whence} \quad h = 4286 \text{ cm. nearly.}$$

4. A copper calorimeter weighing 120 gm. (sp. heat 0·1) and containing 3901 gm. of oil (sp. heat 0·5) is stirred by a rotating paddle. If the couple driving the paddle is 10^5 dynes-cm. and if the rise of temperature is 8° after 1000 revolution, calculate J. (And. U.—1952)

A. Work done by a couple = moment of couple × angle turned through. Therefore 10^9 dynes-cm. in turning through 1000 revolution does an amount of work $= 10^9 \times 2\pi \times 1000 = 2\pi \times 10^{12}$ ergs. = W, say.

Now heat absorbed by the calorimeter and contents = $(120 \times 0{\cdot}1 + 3901 \times 0{\cdot}5) \times 8 = 15700$ cals. = H, say.

Then. $$J = \frac{W}{H} = \frac{2 \times 3{\cdot}14 \times 10^{11}}{15700} = 4{\cdot}0 \times 10^7 \text{ ergs. per caloric.}$$

5. The radius of the top disc of a friction cone apparatus is 20 cm., and a weight of 100 gms. remains suspended when the speed of rotation is 300 revolutions per minute. Calculate (*i*) the moment of the couple due to the frictional forces about the central axis and (*ii*) rate of development of energy in watts. (J = 4·2 joules per calorie; $g = 980$ C. G. S.) (R. P. B.—1951)

A. Moment of the couple about the central axis = $Mgr = 100 \times 980 \times 20$ $= 196 \times 10^4$ dyne. cm. 300 revolutions per minute = 5 revolutions per second.

So work done per second = $2\pi nMgr = 2 \times 3{\cdot}14 \times 5 \times 100 \times 980 \times 20$ ergs.

$$\text{Rate of energy} = \frac{2 \times 3{\cdot}14 \times 5 \times 100 \times 980 \times 20}{10^7} = 6{\cdot}154 \text{ joules per sec.}$$

$= 6{\cdot}154$ watts.

***122. Difference between two Specific Heats of a Gas.**— If a quantity of a gas is heated at constant volume, no external work is done by the gas and so the heat supplied is wholly utilised in raising the temperature of the gas, while the pressure increases. But if the volume is allowed to expand so as to keep the pressure of the gas constant, an additional amount of heat is necessary to supply the energy equivalent to the external work done. Hence the specific heat of a gas at constant pressure (C_p) *exceeds* the specific heat at constant volume (C_v) by the amount of heat equivalent to the external work done when one gramme of the gas is heated through 1°C. at constant pressure.

Let us take one gramme of a gas at pressure P_o dynes per sq. cm. in a cylinder fitted with a piston of cross-sectional area A sq. cm. Then the pressure on the piston is P_oA dynes.

Let the gas be now heated at constant pressure through 1°C. and let the piston move outwards through a distance of *d* cm.

Then external work done during expansion = P_oAd ergs.

Now the increase in the volume for a rise of 1°C = Ad

Let the volume of 1 gm. of the gas at 0°C, and under a pressure P_o be V_o. Then $Ad = V_o/273$.

$$\therefore \text{ External work done for a rise of } 1°\text{C.} = P_oAd = \frac{P_oV_o}{273} \text{ ergs.}$$

From Charles' Law, $P_oV_o = RT_o$, where T_o is the absolute temperature corresponding to 0°C. and is therefore equal to 273.

$$\therefore \quad \frac{P_oV_o}{273} = \frac{P_oV_o}{T_o} = R$$

Hence, the external work done for a rise 1°C = R ergs.

This is equivalent to the difference between the specific heats of a gas. In other words.

$$C_p - C_v = R./J. \qquad (122,1)$$

where J is Joule's equivalent.

Mayer's method of determining J.—Mayer used the above expression in the case of hydrogen in calculating the value of J, before it was experimentally determined by Dr. Joule.

For hydrogen $C_p = 3{\cdot}409$ calories per gm. per. °C.

$C_v = 2{\cdot}411$ " " " " "

Now the volume V_o of 1 gm. of hydrogen at N. T. P. = 11160 c c.

Also the normal pressure $P_o = 76 \times 13{\cdot}6 \times 981$

$= 1{\cdot}014 \times 10^6$ dynes per sq. cm.

Hence for 1 gm of hydrogen, $R = \dfrac{1{\cdot}014 \times 10^6 \times 11160}{273}$

$= 4{\cdot}145 \times 10^7$ ergs per °C

From eqn. (122,1), $3{\cdot}409 - 2{\cdot}411 = \dfrac{4{\cdot}145 \times 10^7}{J}$

whence $J = 4{\cdot}152 \times 10^7$ ergs per calorie

Examples :—

1. How much work is done against atmospheric pressure when 1 gramme of water at 0°C. is converted into steam? Express your result in calories.

A. The atmospheric pressure at which water is converted into steam at 100°C. is a pressure of 76 cm. of mercury.

∴ The pressure at the time $= 76 \times 13{\cdot}6 \times 981$ dynes per sq. cm.

When 1 gm. of water is changed into steam at 100°C. the increase in volume is 1672 c.c.

Hence the required work done $= 76 \times 13{\cdot}6 \times 981 \times 1672$ ergs.

This is equivalent to $\dfrac{76 \times 13{\cdot}6 \times 981 \times 1672}{4{\cdot}2 \times 10^7} = 40{\cdot}37$ calories

2. The specific heat of air at constant volume is 0·1614 and at constant pressure is 0·2375. Calculate the value of J. Density of air at N. T. P. = ·001293 gm. per c.c.

A. The normal atmospheric pressure $P_o = 76 \times 13{\cdot}6 \times 981$ dynes per sq. cm.

Volume of 1 gm. of air at 0°C. is $V_o = 773{\cdot}3$ c.c.

Thus for 1 gm. of air, $R = \dfrac{P_o V}{273} = \dfrac{76 \times 13{\cdot}6 \times 981 \times 773{\cdot}3}{273}$

From eqn. (122,1) $0{\cdot}2375 - 0{\cdot}1648 = \dfrac{76 \times 13{\cdot}6 \times 981 \times 773{\cdot}3}{273 \times J}$

whence $J = 4{\cdot}16 \times 10^7$ ergs per calorie.

JAMES DEWAR (1842—1923)

Dewar was born at Kincardine-on-Forth on September 20, 1842. He received his education at the Dollar Academy and later at Edinburgh University. At the age of twenty one he was appointed assistant to Lord Playfair, then Professor of Chemistry at Edinburgh and in 1868, he worked for some months under Prof. Kekule. One of his most important researches is the discovery of the explosive 'cordite' in collaboration with Sir Frederick Abel.

Dewar then turned his attention towards production of low temperature. He was then appointed Professor of Experimental Philosophy at Cambridge. In 1891 he was successful in liquefying oxygen and to preserve the liquid he designed a double-walled flask known as Dewar's flask. Later on he could liquefy and even solidify hydrogen bringing down the temperature to $-260^{\circ}C$. Once Dewar remarked "As we approach the zero point of the absolute temperature, we seem to be nearing what I can only call the death of matter". He was knighted in 1904. His last attempt was to tackle the helium gas, but he could not finish his experiments owing to an accident. He died on March 27, 1923.

JAMES PRESCOTT JOULE (1818—1889)

Joule was born at Salford near Manchester on December, 24, 1818. His father was the proprietor of a large brewery where the boy got opportunities of carrying on with his experiments. He had a very delicate health and had to be educated at home. Fortunately John Dalton was employed as his private tutor from whom the boy got the inspiration for Science.

Fig. 1—James Prescott Joule

The first few years of his student life were spent in electrical researches due to the influence of Faraday. He designed an electromagnetic engine and then his attention was concentrated upon the manner in which work was done by an engine. He found that in all engines work is done at the disappearance of certain quantity of heat. Then he wanted to show the relation between work done and

heat necessary to do the required amount of work. To complete his investigations on the mechanical equivalent of heat he devoted thirty-five years from 1843 to 1878. Joule also invented various apparatus including electric meters and speed counter of hydrogen molecules. In 1872, Joule's health gave away and he was forced to retire. He died on October 11, 1889.

Summary

Nature of heat.—Heat is due to the agitation of the constituent molecules of a body.

Mechanical Equivalent of Heat.—When work is transformed into heat or heat into work, the quantity of work is mechanically equivalent to the quantity of heat.

If by an amount of work W, the quantity of heat produced is H, then

$$J = W \div H \text{ or } W = JH$$

where J is a constant and is known as the **mechanical equivalent of heat.**

Difference between Specific Heats of Gas $C_p - C_v = R/J$.

Exercises on Chapter IX

Reference

1. Describe an experiment to establish the connection between heat and work and deduce from them the idea of mechanical equivalent of heat. (R. P. B.—1942; Pat. U.—1930; Dac. U.—1941) — Art. 116

2. Write an essay on 'Heat as a mode of motion'. (Dac. U.—1940) — Art. 118

3. Explain what is meant by saying that heat is a form of energy. (Dac. U.—1928, '30) — Art. 118

4. From what height must a block of ice drop in order that it may be melted completely? It is assumed that 50% of the energy of fall is retained by ice. (Assume $g = 980$; $L = 80$ and $J = 4{\cdot}2$ Joules/calorie). (Del U.—1942) — Art. 119

A. $6{\cdot}80 \times 10^5$ cm.

4*a*. How long will it take for on electrical heating rod of 42 watts to heat 100 c.c. of water by 10°C., if no heat is lost? ($J = 4{\cdot}2 \times 10^7$ ergs/calorie). (B. H. U.—1953)

A. 10 sec.

5. Calculate the velocity of a lead bullet whose temperature on striking an unyielding target, rises by 200°C. assuming that the whole of the heat generated by the impact remains in the lead. Specific heat of lead is 0·03. (C. U.—1937, '41) — Art. 119

A. $22{\cdot}45 \times 10^2$ cm. per sec.

6. What is meant by Mechanical Equivalent of heat? How can it be determined experimentally? (C. U.—1947, '49, '50; Utk. U.—1953; Mys. U.—1952; Gau. U.—1951, '53; Dac. U.—1942; Del. H. S.—1951; Del. U.—1942, '51; R. P. B.—1940,'49; Pat. U.—1944, '52) — Arts. 119 & 120

Reference

Art. 120 7. A meteorite weighing 2000 gg. falls into the sun with a velocity of 1000 metres per sec. Assuming that all its energy is converted into heat, how many calories of heat will be produced? $J = 4{\cdot}19 \times 10^7$

A. $2{\cdot}4 \times 10^8$ calories.

Art. 120 8. A lead ball, dropped from an aeroplane at a temperature of 15°C just melts on striking the ground. Supposing the whole of its kinetic energy is converted into heat, find out the height of the aeroplane at the moment the ball was dropped. Sp. ht. of lead = ·03, melting point of lead = 350°C; latent heat of lead = 35 cal/gm. (Gau. U.—1951)

A. 19286 metres.

Art. 120 8a. One kilogram of coal generates $1{\cdot}7 \times 10^4$ calories of heat during combustion. Assuming that only $\frac{1}{50}$th of the energy so generated is utilised for mechanical purposes, find the mass of coal required per hour for running a 10-horse power engine. [1 h. p. = 746×10^7 ergs per second; $J = 4{\cdot}2 \times 10^7$ erg/calorie]. (Utk. U.—1953)

A. 188·05 kg./hour.

Art. 120 9. What is meant by mechanical equivalent of heat? In an experiment to determine it 800 gm. of lead shot were placed in a long card board tube held vertically, the length of which was such that on reversing the tube the shot fell through 100 cm. The original temperature of the shot was 25°C. and after 50 inversions of the tube the temperature was found to have risen to 28·84°C. Find J in ergs per calorie. (Sp. ht. of lead = ·031, $g = 980$). (C. U.—1950)

A. $4{\cdot}12 \times 10^7$ ergs/calorie.

Art. 120 10. Mercury falls from a height of 50 metres in vacuo upon a perfectly non-conducting surface. What will be the rise in temperature after the fall, if the sp. ht. of mercury be ·02 and $J = 4{\cdot}2 \times 10^7$ ergs per calorie. (R. P. B.—1946)

A. 0·01°C.

Art. 120 11. Calculate the difference in temperatures between the water at the top and that at the bottom of a waterfall which is 50 metres high, given $J = 4{\cdot}2 \times 10^7$ ergs/calorie. (Gau. U.—1953)

A. 0·12°C.

Art. 120 12. A cannon ball of mass 100 kgm. is projected with a velocity of 400 metres per sec. Find the amount of heat that would be produced if the ball were suddenly stopped. ($J = 4{\cdot}2 \times 10^7$ ergs per calorie.) (And. U.—1951)

A. $10{\cdot}05 \times 10^6$ calories.

Art. 120 13. A mass weighing 2000 grammes falls from a height of 300 cm. If all the energy is converted into heat, find the amount of heat developed. Mechanical equivalent of heat = $4{\cdot}2 \times 10^7$ C. G. S. (Del. H. S.—1951; C. U.—1920; Dac. U.—1942)

A. 14 calories.

Art. 120 14. A block of metal slides down a rough inclined plane, starting from rest at a point 10 metres above the ground. Just before reaching the ground the velocity of the block is 10 metres per second. If half of the heat produced by friction remains in the block, by what amount

Reference

has its temperature been raised while sliding down? (Sp. heat of the metal = 0·1).

A. 0·057°C.

Art. 120

15. A copper calorimeter weighing 200 gm. contains 280 gm. of water. A mechanical stirrer is able to raise the temperature of water at the rate of 0·3°C. per minute. Calculate the energy in watts used by the stirrer. (Sp. ht. of copper = 0·1 and J = 4·2 joules per calorie). (Mad. U.—1950)

A. 6·3 watts.

Art. 120

16. A bullet is suddenly stopped and all its energy is converted into heat causing a rise of temperature of the bullet by 100°C. If the specific heat of the bullet is 0·21, calculate the velocity with which the bullet was moving. (J = 4·2 × 10^7 erg./cal.) (Raj. U.—1953)

A. 42 × 10^3 cm./sec.

Art. 120

17. A lead bullet strikes a target with a velocity of 200 metres per sec. Calculate its rise in temperature, assuming that all heat developed remains in the bullet. (Sp. ht. of lead = ·03). (P. U.—1952)

A. 158·7°C.

Art. 120

18. A lead bullet strikes against a steel armour plate with a velocity of 480 metres per sec If the bullet falls dead after the impact find the rise in temperature assuming that the heat produced is equally shared between it and the target. (Sp. ht. of lead = ·03, J = 4·2 × 10^7 C. G. S.)

A. 457·1°C.

Art. 120

19. A lead bullet travelling with a speed of 200 metre per second is brought to rest after hitting a target. Assuming that 80% of the kinetic energy is converted into heat in the bullet itself find the rise in temperature of the bullet after impact. (Sp. ht. of lead = ·03, J = 4·2 × 10^7 ergs./cal.) (Nag. U.—1952)

A. 126·9°C.

Art. 120

20. Describe a method for the determination of the mechanical equivalent of heat; and mention clearly the units in which it is measured. (U. P. B.—1953; P. U.—1926, '30; *cf.* Nag. U.—1950; B. H. U.—1953 C. U.—1939, '41)

Art. 120

21. Describe how the mechanical equivalent of heat is determined by frictional cone method. (Raj. U.—1953; R. P. B.—1951; U. P. B.—1951; Mys. U.—1952)

Art. 120

22. A copper calorimeter of mass 122 gm. contains 1680 gm. of aniline oil, which is stirred by a rotating paddle requiring couple of 10^7 dynes cm. to drive it. The temperature of liquid is raised 8°C. after 450 revolutions. Calculate the value of J. (Sp. ht. of aniline = 0·5 and that of copper = 0·095).

A. 4·15 × 10^7 ergs./calorie.

Art. 120

23. Describe Joule's method for determining the mechanical equivalent of heat. (Nag. U.—1951, '53; C. U.—1934; Del. U.—1953; Del. H. S.—1954)

Art. 120

24. Mention different methods to determine the mechanical equivalent of heat and describe any one of them. (Del. U.—1940)

Reference

24*a*. What is meant by mechanical equivalent of heat? Describe Searle's method of determining the same, giving precautions to be taken. (B. H. U.—1950)

Art. 119 25. How much work must be done for supplying heat necessary to convert 10 gm. of ice at $-5°$C. into steam at 100°C.? (Sp. ht. of ice 0·5 and mechanical equivalent of heat $= 4·2 \times 10^7$). (R. P. B.—1942)

A. 30177×10^7 ergs.

Art. 122 26. Explain how the difference of specific heats of a gas enable you to evaluate the mechanical equivalent of heat. (R. P. B.—1949; U. P. B.—1952)

27. Prove that $C_p - C_v = R$ ergs. (R. P. B.—1941, '48)

Art. 122 28. Calculate the value of the mechanical equivalent of heat from the following data for air:— (U. P. B.—1949)

Specific heat at constant pressure $= 0·2375$ cal.

Specific heat at constant volume $= 0·1690$ cal.

Atmospheric pressure in dynes per sq. cm. $= 1·013 \times 10^6$

Volume of 1 gm. of air at N. T. P. $= \frac{1}{0·00129}$ c.c.

A. $4·19 \times 10^7$ ergs per calorie.

Art. 122 29. Specific heat of argon at constant pressure is 0·125 calories per gm. and at constant volume 0·075 calories per gm. Calculate the density of argon at N. T. P. ($J = 4·18 \times 10^7$ ergs/cal.; normal pressure $= 1·01 \times 10^6$ dynes/sq. cm. (R. P. B.—1949)

A. $1·8 \times 10^{-3}$ gm./c.c.

CHAPTER X

HEAT ENGINES

123. Heat Engines.—These are machines by means of which heat energy is transformed into mechanical work. There are two principal types of engines, namely, (*i*) External combustion engines, (*ii*) Internal combustion engines. In the former type, water is converted into steam in a chamber, outside the working portion of the engine while in the latter type, the fuel used to generate heat is either a gas or liquid and the combustion of the fuel takes place *inside* the working portion of the engine.

Conversion of Linear to Rotatory Motion.—If we can obtain a *continuous motion* from a machine, either linear or rotatory, it can be converted to any desirable type of motion. Almost in every class of engines, the primary object is to secure a rotatory motion of a massive wheel, called the fly-wheel. Every other motion is derived from this fly-wheel dy direct or indirect coupling processes.

Consider a heavy wheel W (Fig. 92) rigidly attached to a shaft and capable of rotation. One end of a stout metal rod R, called the crank rod, is hinged at one spoke of the wheel and its other end is also hinged at the end B to another rod capable of moving to-and fro within a

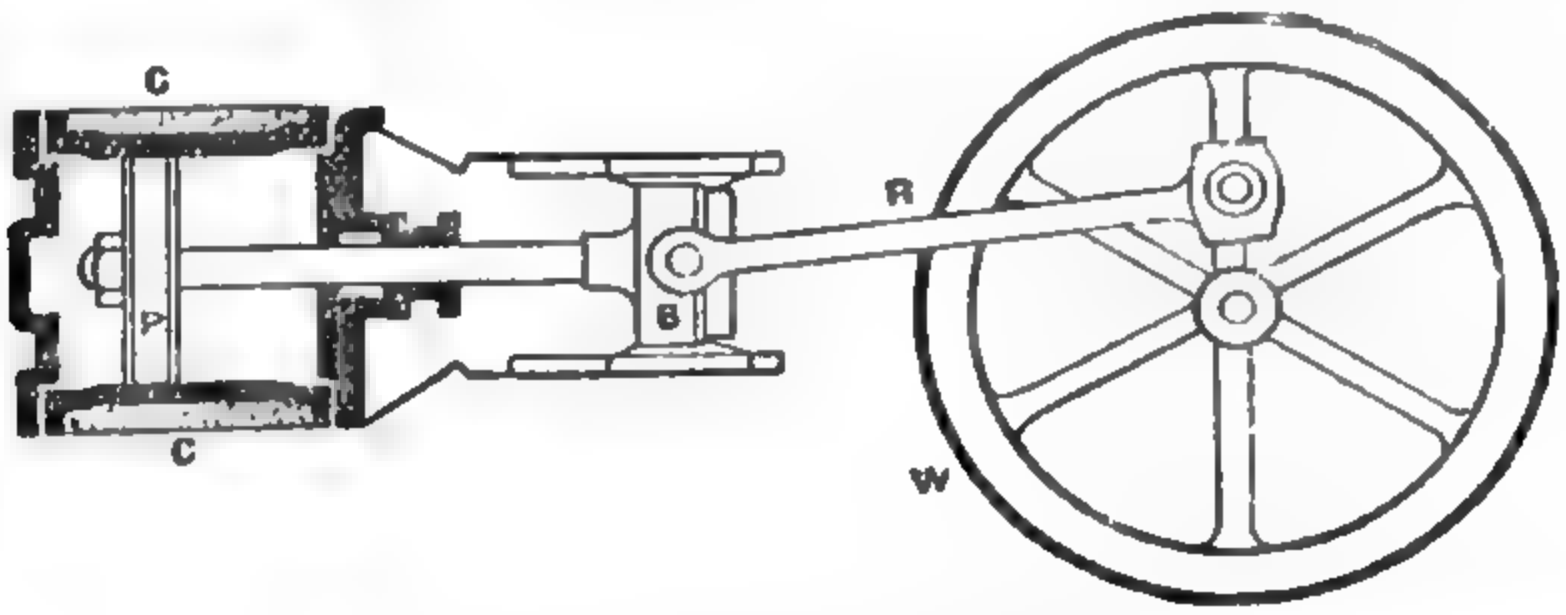


Fig. 92—Piston rod and Fly-wheel

cylinder. The hinged combination of the crank rod R and the piston rod at B is known as a *stuffing box*. This second rod is provided with an air-tight piston P moving within the cylinder C. If now some force acting on the piston causes it to move to-and-fro within the cylinder, the link rod would revolve the fly-wheel continuously. All classes of engines are provided with such appliances.

124. Steam Engine.—The principle of the steam engine was discovered by James Watt in 1768. Although many important improvements have since been introduced by Trevithick, Stephenson and others, the fundamental features remain the same.

The essential *parts* of a steam engine are :—(Fig. 93)

(1) *Boiler* L.—Here steam is generated by boiling water in a strong metal vessel over the furnace F in which coal is burnt. The steam is led away by a metal tube U into a chamber S, called the steam chest. There is a regulating key M with the tube, by means of which the supply of steam into the chest can be controlled.

(2) *Steam chest* or *Valve chest.*—This is a stout box S into which steam is injected from the boiler through the pipe. It has three

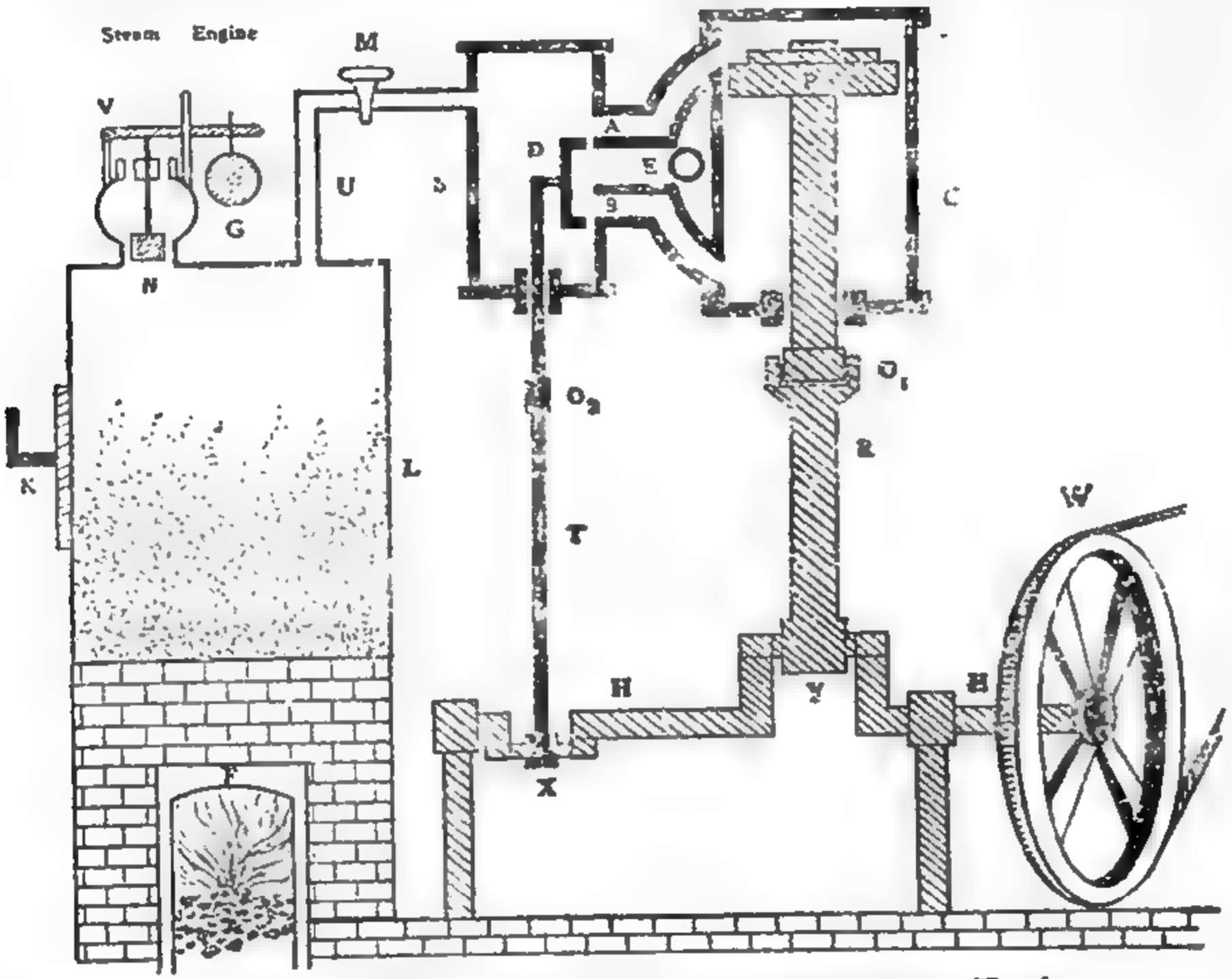


Fig. 93—Sectional diagram of a complete Steam Engine

openings or *ports* ; the middle one, E is connected with the *exhaust pipe*, while the other two A and B lead into the cylinder C.

(3) *Slide Valve* or the D-*valve.*—This is a D-shaped valve moving within the steam chest and regulating the supply of steam through the ports. The slide valve is connected to the rod T through a stuffing box O_2. The rod is finally connected to *eccentric disc* X of the main shaft H.

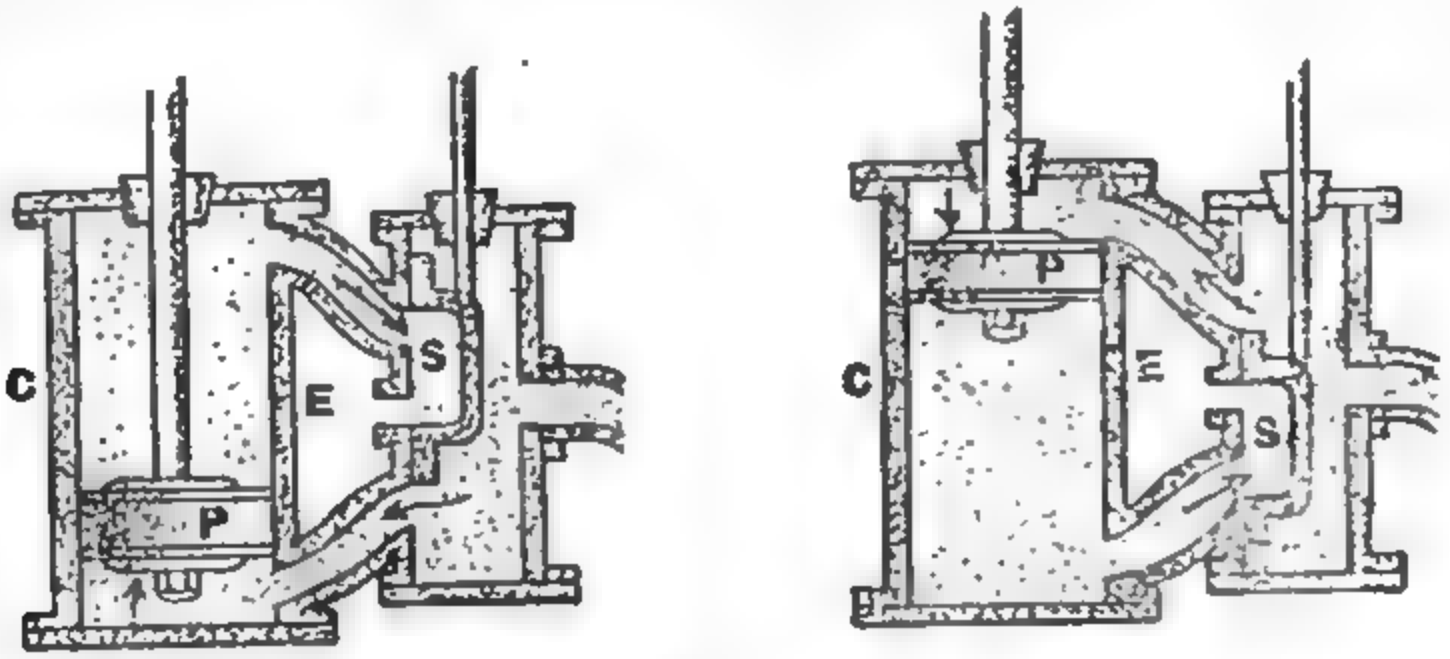
(4) *Cylinder*, C.—It is a strong cylindrical vessel communicating with the steam chest through the two tubes. A steam-tight piston P of cast steel just fits and works within the cylinder. The *piston* works through a *stuffing box* O_1 on the *driving rod* R which is hinged at Y to the *main shaft* H.

(5) *Safety valve*, G.—It is a mechanism for safety of the boiler against a prohibitive rise of steam pressure within it. There is an

outlet pipe fitted with a valve N, being worked by a lever V. If the steam pressure goes above the safe limit, the valve automatically opens and a part of the steam escapes, lowering the pressure.

(6) *Fly-wheel*, W.—It is a large wheel connected with the main shaft.

Action.—Super-heated steam at a very high pressure (20 atmosphere or even more) from the boiler is led through the pipe, U into the steam chest and thence into the cylinder, C through any one of the ports say, lower one (Fig. 94,*a*). When the steam enters the cylinder, the slide valve covers the exhaust and the other port so that these two are put into communication. The pressure of the steam due to its expansive action inside the cylinder pushes the piston upward.



(*a*) Fig. 94 (*b*)

The movement of the piston rotates the crank-shaft, whereby the motion is communicated through the eccentric X to the valve-rod T. The valve, therefore, *automatically* moves in the opposite way. When the piston has moved nearly to the end of the cylinder, the valve covers the lower port and the exhaust, but opens the upper port (Fig 94, *b*). The steam now enters the cylinder through the other passage and pushes the piston in the opposite way until it almost strikes the wall; at the same time the exhaust steam goes out through the exhaust pipe. This operation is repeated successively; the slide valve opens and closes one of the ports alternately just at the proper moment, so that the piston moves to-and-fro within the cylinder. The to-and-fro motion of this piston is converted into a rotatory motion of the main shaft through the crank. Twice during each revolution *i.e.*, at the top and the bottom of each stroke, the crank, and the connecting rod are in the same line, when the piston momentarily comes to rest and has no turning effect on the shaft : these are called the '*idle centres*' or 'dead points'. Again at two points during each revolution the crank is at right angles to the connecting rod, when the turning effect on the shaft becomes maximum. The shaft carries a large and heavy *fly-wheel* W which due to its large moment of inertia smoothes down any variation of motion of the crank shaft.

The supply of steam from the boiler into the cylinder is regulated

by a valve, known as the *throttle valve*, which is worked upon by a piece of self acting machinery known as the *governor*. If the engine works too fast, the balls of the governor move upwards and pull down the collar and in so doing operate the throttle valve so as to reduce the supply of steam. The motion of the shaft may be communicated to any machinery by means of a belt passing over a pulley attached to the main shaft. The type of engine described here is called a *Double-acting Engine*, for the steam acts alternately on two faces of the piston thus producing a to-and-fro motion.

Condensing and Non-Condensing Engines.—The type of the engine in which the waste steam escapes through the exhaust-pipe into the open atmosphere is called a *non-condensing engine*. Such an engine can be easily recognised by the puffs of exhaust steam, escaping into atmosphere at each stroke of the piston. While in a *condensing engine*, the exhaust steam is led into a chamber, called the *condenser* where the steam is condensed due to low pressure. The condensed steam is again used in the boiler to produce steam. The efficiency of the condensing type of engine is greater.

In a railway **locomotive** the principle of working is similar, but there are two pistons placed below the boiler, one on either side. These pistons are connected by crank shafts to the large diving wheels and they move backwards and forwards *alternately*. The boiler has a large *fire box* at the rear and a *smoke-box* at the front end. The boiler itself is composed of a large number of tubes through which the heated gas from the furnace goes out into the smoke-box to escape finally through the chimney.

125. Efficiency of an Engine.—It is measured by the ratio of the heat utilised or transformed into work in a given time to the total heat supplied by the fuel in the same time. This is more often called *Thermal Efficiency*.

If Q be the heat produced by the fuel consumed per unit of time, and W, the power developed *i.e.*, the amount of mechanical work done per sec, then the thermal efficiency is given by

$$\gamma = \frac{W}{JQ} \qquad \ldots (125,1)$$

Thus γ gives the fraction of the total heat converted into work and is always less than unity. The efficiency of the best type of steam engine is not more than 17%. The output energy is measured in terms of **horse-power** (H. P.) An engine is said to have one horse-power when it performs 550 ft-lbs. of work per second or 33,000 ft. lbs. of work per minute. Heat, taken in, may be measured by the quantity of steam used in a given interval and the initial and final temperatures and pressures of the steam.

The power developed by steam pressure within the cylinder of the engine is known as *Indicated Horse Power*, which depends upon the mean effective pressure of the steam on the piston during a stroke, the cross-sectional area of the pistion, length of the stroke and the number

of strokes per minute. This power cannot be fully utilised by the fly-wheel as a certain part of this power is used up in driving the engine itself by way of friction, weight of moving parts etc. Thus the available power, known as the *Brake Horse Power*, is always less than the Indicated Horse Power. The ratio of the latter to former is called the Mechanical Efficiency. Thus

$$\text{Mechanical efficiency} = 100 \times \frac{\text{Brake Horse Power}}{\text{Indicated Horse Power}}$$

126. Internal Combustion Engines —In this type, the combustion of the fuel is carried out *inside* the engine cylinder. These are, therefore, termed *internal combustion engines*. According to the nature of the fuel used, an internal combustion engine may be (1) **petrol engine**, (2) **an oil engine** or **gas engine.**

Petrol engine.—This is a type of *internal combustion engine* in which combustible liquids such as petrol, benzene, *etc* , are used as fuel. It is commonly employed in motor cars and in air ships.

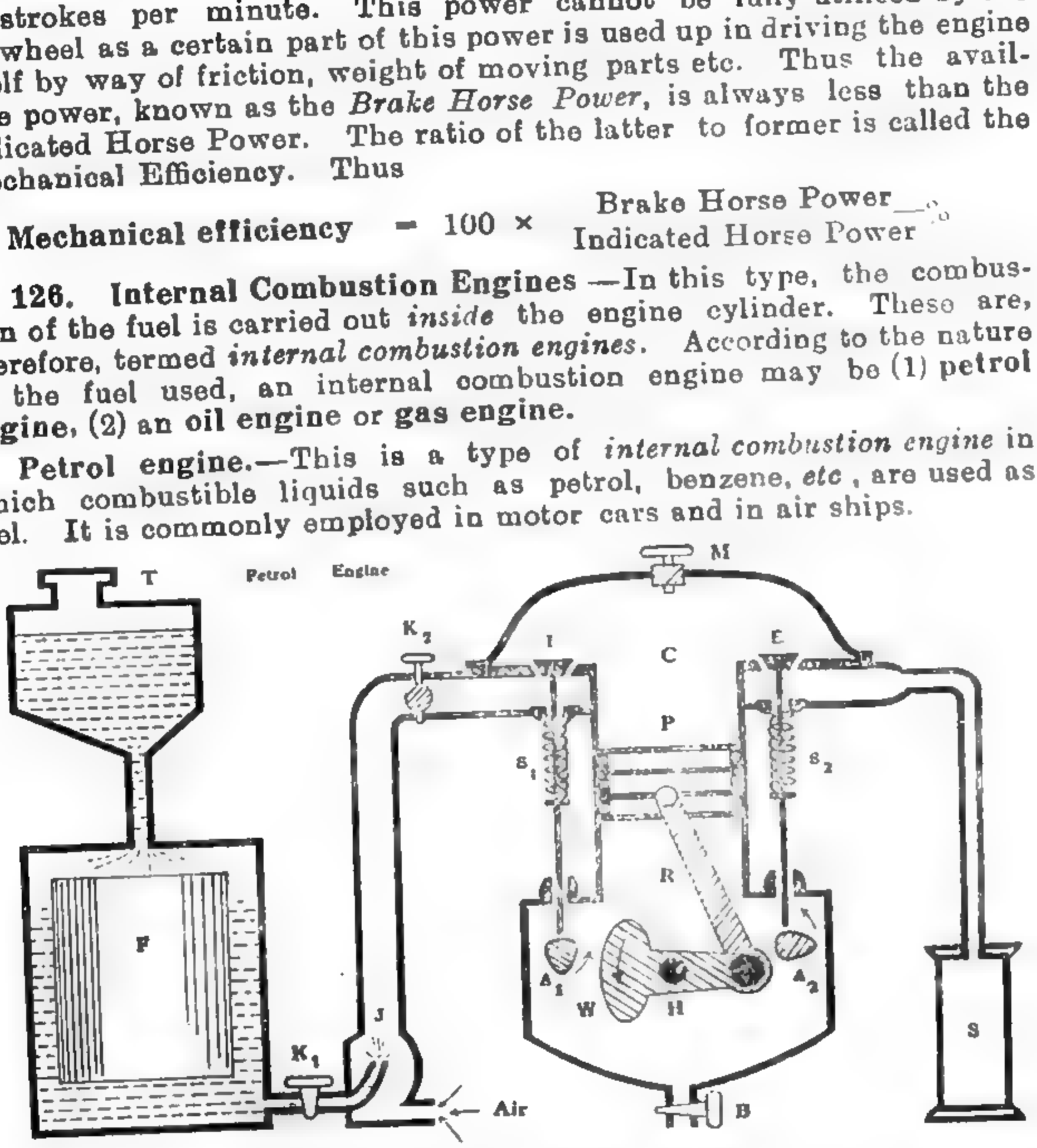


Fig. 95—A complete Petrol Engine

A complete picture of a Petrol Engine is shown in fig. 95. The petrol is stored in the *petrol tank T* whence it passes into a chamber containing a float F, and by pressure, petrol is forced into the *carburettor*. The quantity of fuel admitted into the carburettor may be adjusted with a key K_1. Within the curburettor the petrol is vaporised and is mixed with proper proportion of air. A regulated supply of the mixture which forms the fuel, passes through an adjustable valve, called the *throttle valve* K_2, into the *combustion chamber*, placed just above the cylinder C. In this chamber the ignition is caused by electric sparks from a *plug* M, the current being supplied by a small dynamo, called the *magneto*, which is driven by the engine itself.

Within the cylinder C an air tight piston P works. The cylinder is provided with an inlet and an exhaust *mushroom valves* I and E with projecting rods and operated respectively by rotating *cams* A_1, A_2, which are driven by the main shaft. Each can has a protruding end and is designed to rotate at half the speed of the main shaft H. In course of rotation when this end points upward, it gives the bottom of the valve rod an upward push and thereby the corresponding valve opens. For other position of the cams the valves are kept pressed on the holes by powerful springs S_1 and S_2. Hence the opening and the closing of these valves are controlled by the motion of the piston. In order to prevent the cylinder from becoming too hot during the operation, it is surrounded by water-jacket through which there is a continuous circulation of water (not shown).

The engine is called a **four-stroke engine,** as the supply of the fuel is made only *once* during the time the piston makes *four* motions, two forward and two backward. As the first successful gas engine was designed by Dr. Otto in 1876, the operation of the four consecutive strokes of the cycle is known as the Otto cycle. In a complete cycle, the four strokes occur in the following order,—

(*i*) *The Charging Stroke.*—The piston moves downwards, the inlet valve I, opens and a quantity of the curburetted mixture is sucked into the cylinder (Fig. 96,1).

(*ii*) *The Compression Stroke.*—The piston moves upward and compresses the combustible mixture to about $\frac{1}{5}$th of the volume. Both the valves E and I, remain closed during the stroke and the temperature of the mixture rises to about 600°C. (Fig. 96,2). At the end of this stroke, the mixture is fired by a series of sparks from the sparking plug.

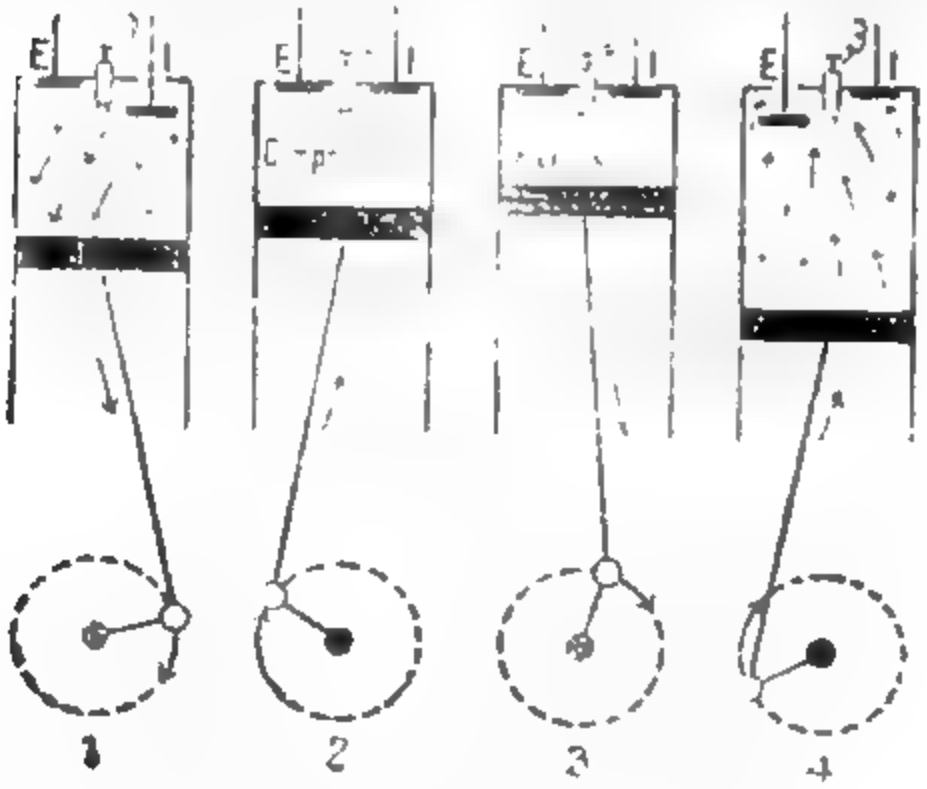


Fig. 96—Four Cycles of Operation

(*iii*) *The Working Stroke.*—Due to the combustion much heat is developed, whereby the temperature inside rises to about 2000°C. and the pressure rises to about 15 atmospheres. Owing to the expansive action of the gas, the piston is driven outward with a great force, both the valves still remaining closed (Fig. 96, 3). This is the only stroke during which the engine does work.

As the explosive mixture required a little time for full combustion, the maximum force of the explosion developes a short time after the sparking of the mixture.

(*iv*) *The Exhaust Stroke.*—The piston moves upward and the exhaust valve, E then opens and the spent production of combustion is forced out, the inlet valve I remaining closed (Fig. 96,4).

After the exhaust is complete, the initial condition is attained and a fresh cycle begins again. As the engine receives energy only during the working stroke, *the fly wheel must be very heavy* so that inertia gained by it during this stroke will maintain the speed without any appreciable loss during the next three strokes. In fig. 95, the flywheel is replaced by a heavy counter weight W attached to the opposite end of the shaft.

A petrol engine in a motor car or aeroplane is provided with a bank of cylinders, the number being a multiple of two, generally four, six or eight or even more. The pistons of all the cylinders work on the same shaft thereby increasing the working capacity or the horse-power of the engine. The arrangement of the different parts in a motor car engine is represented in fig. 97 in which A and B represent two cylinders, M and N piston rods, KSLQ the main shaft, W the fly-wheel and R the radiator of the water circulation system. Since such a type of the engine requires to be started by external agency ; H is the handle by the rotation of which the engine is started. There is also a mechanical arrangement to give the first start of the engine. This called the self-starting lever.

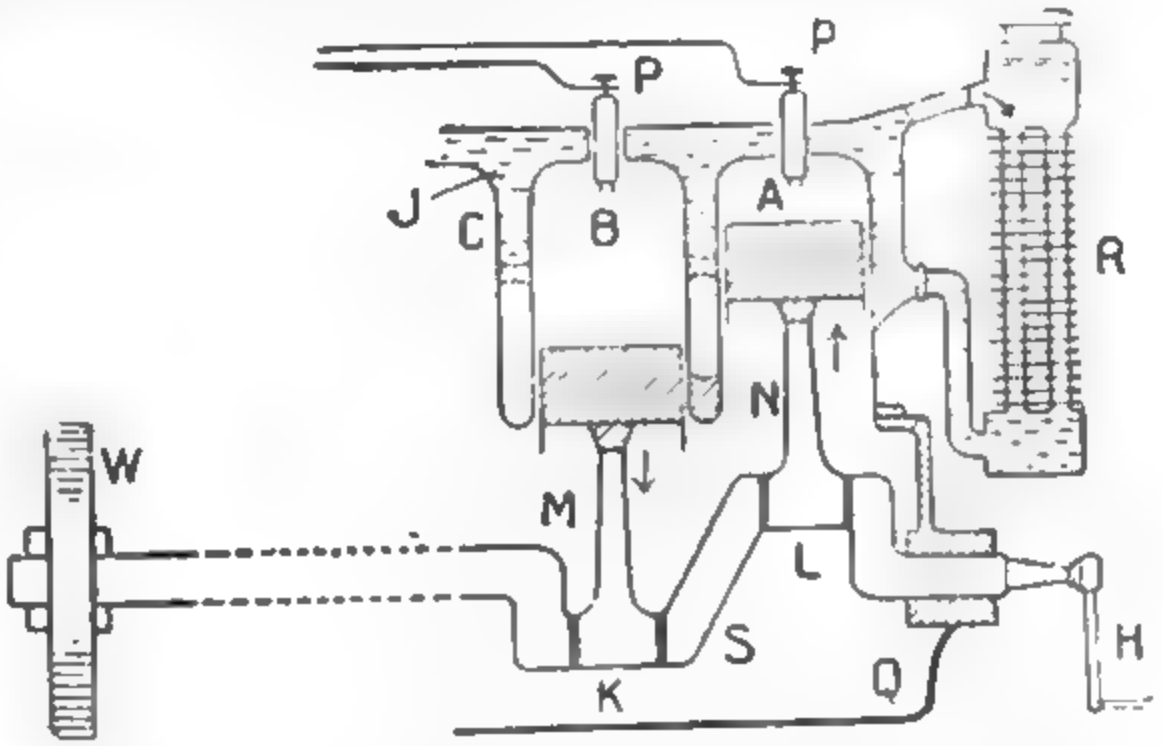


Fig. 97—View of a Motor Car Engine

127. Oil Engine.—It is a type of *internal combustion engine*, using as fuel an explosive mixture of air, and some oil. The oil is admitted in the form of spray into the *vaporiser* which is a red-hot metal tube and into which air is also admitted. The oil evaporates, mixes with the air and is exploded at suitable intervals. A four-stroke cycle also works in this case. In a special type of machine called the **Diesel Oil Engine** after the name of inventor, the cycle is modified in the following way.

During the *first stroke*, a requisite quantity of air is sucked within the cylinder. During the *second stroke* the air is very strongly compressed, so that great heat is developed within the cylinder and the temperature rises to about 540°C. At the beginning of the *third stroke* oil spray is injected at a very high pressure into the cylinder containing the highly compressed and intensely heated air ; the mixture takes fire and burns non-explosively, maintaining parctically a constant pressure. During the *fourth stroke*, as usual, the burnt gas is forced out of the cylinder.

The *efficiency* of the engine is very high, being about 70 per cent. It is very economical, being adapted to a wide range of fuels from

kerosine to tar spray. They are used in submarines, many merchant vessels and for various purposes on land.

Gas Engine.—It works similarly as a petrol engine and employs as fuel a mixture of about 1 part by volume of coal gas and 8 parts of air. The same four-stroke cycle works and the valves are operated in the same way by cams and spring which regulate the inlet of the mixture and the exhaust. The firing of the gaseous mixture is effected within the cylinder by contact with the walls of a red-hot porcelain tube or by electric sparks occuring at properly timed intervals. The engine is also cooled by water, flowing through a jacket round the cylinder.

Comparison of a Gas engine and a Steam engine.—The efficiency of a gas engine is higher than that of a steam engine being often as high as 70 per cent, while that in a steam engine the highest value is about 17 per cent. The former is free from smoke, occupies smaller space and may be instantly started. But the fuel (gas or gasoline) used in the former is comparatively expensive.

Examples :—

1. A petrol engine uses every hour 1 lb. of petrol which produces 22,000 B. Th. U. of heat, and has an efficiency of 30%. What is its H. P. (Gau. U.—1952)

A. Efficiency $= \frac{\text{output energy}}{\text{input energy}}$ $\therefore$ Output energy $= \frac{80}{100} \times 22000$ B. Th. U. per hour $= 6600$ B. Th. U. per hour $= 6600 \times 778$ ft. lbs. of work per hour.

Now, 6600×778 ft. lbs. per hour $= \frac{6600 \times 778}{60 \times 60}$ ft. lbs. per sec.

Since 1 H. P. $= 550$ ft. lbs. per sec., the output horse power of the engine

$= \frac{6600 \times 778}{60 \times 60 \times 550} = 2{\cdot}59$ H. P.

2. An engine consumes 80 lb. of coal in drawing a train of weight 240 tons over a distance of 1 mile, the resisting force being equal to 35 lbs. per ton. The calorific value of the particular sample of coal is such that 1 lb. is capable of converting 15 lb. of water at 100°C. into steam at the same temperature. Find the efficiency of the engine (Latent heat of steam $= 536$; $J = 1400$ ft. lb. deg. Centigrade).

A. The total resisting force $= 35 \times 240$ lb.

$\therefore$ Work done in moving the train 1 mile $= 35 \times 240 \times 1760 \times 3$ ft. lb.

Heat developed by the consumption of coal, $Q = 15 \times 536 \times 80$ calories.

Hence the efficiency, $\gamma = \frac{W}{JQ} = \frac{35 \times 240 \times 1760 \times 3}{1400 \times 15 \times 536 \times 80} = 0{\cdot}049$ or $4{\cdot}9\%$.

3. A motor car uses petrol whose calorific value is 11×10^4 B. Th. U. per gallon. The car covers on average 20 miles to the gallon running at 24 miles an hour during which the average output is 10 H. P. Find the overall efficiency on the car.

A. Work obtained $= 10 \times 550$ ft. per sec.

Petrol consumed per hour $= 24 \div 20 = \frac{6}{5}$ gallons.

$\therefore$ Heat energy given per sec. $= \frac{6}{5} \times \frac{1}{3600} \times 11 \times 10^4$ B. Th. U.

$$\therefore \text{ Mechanical energy given per sec.} = \frac{6}{5} \times \frac{1}{8600} \times 11 \times 10^4 \times 778 \text{ ft. lbs.}$$

$$\text{Hence the required efficiency} = \frac{\text{work obtained}}{\text{energy given}}$$

$$= \frac{5500 \times 5 \times 3600}{11 \times 10^4 \times 6 \times 778} = 0{\cdot}193 = 19{\cdot}3\%$$

JAMES WATT (1736—1819)

Watt was born at Greenock, Scotland, on January 19, 1736 In his boyhood, he had a delicate health but he showed an outstanding skill with tools and drawing At school he was very proficient in geometry. After his school career he went to London to learn business but he could not stand the foggy climate there and had to return to Glasgow. He was then appointed as an instrument maker.

In his workshop in collaboration with Boulton he carried out the work of utilising the steam pressure in receiving continuous motion. He would very often communicate with Prof. Black regarding the theoretical background of the properties of super-heated steam. Ultimately he was able to build up a steam engine from which continuous work could be derived. Soon after his invention, steam engines to run on rail roads were constructed by Trevithick, Stephenson and others. But Watt was the real inventor of steam engine.

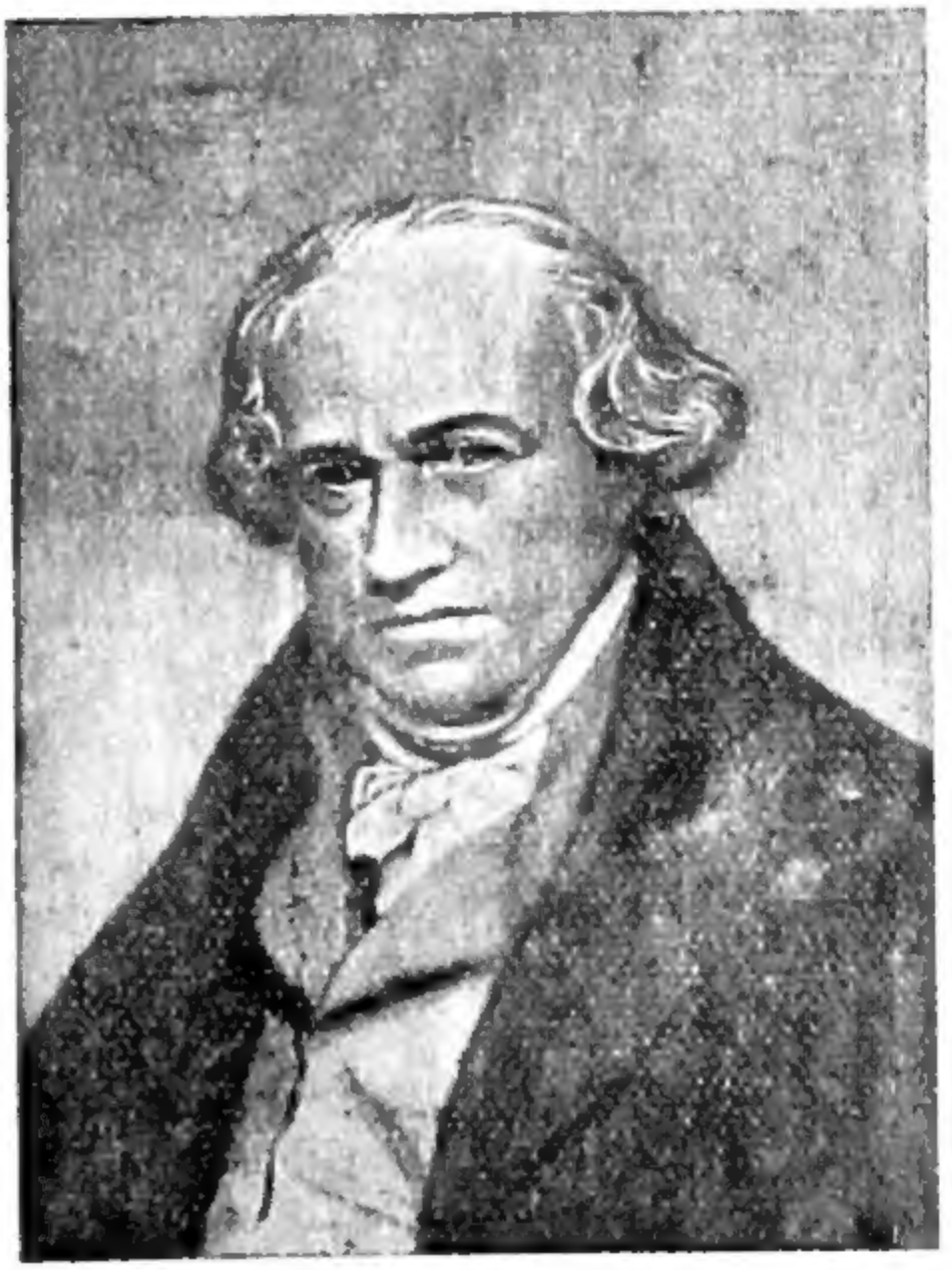


Fig. 98—James Watt

Apart from this, he designed a musical organ, letter-copying press and various moulds. He designed apparatus for the manufacture of gases and determising specific gravities of liquids. He died near Brimingham, on August 19, 1819.

Summary

Heat Engines are appliances by means of which heat energy is transformed into mechanical work. There are two types of engines, namely. (*i*) *Steam Engines* (*ii*) *Internal Combustion Engines.*

Steam Engines.—Highly heated steam from the boiler is led into the steam-chest from which it passes into the cylinder, alternately above and below the piston

through two ports. The expansive action of the steam moves the piston inwards and outwards alternately and this to-and-fro motion is converted by proper device into a rotatory motion of the shaft.

Internal Combustion Engines may be *(i) Petrol engines* **(ii) Oil and (iii)** *Gas engines.*

The **efficiency** of an engine is measured by the ratio of the heat utilised or transformed into work W to the total quantity Q supplied by the fuel.

The **capacity** of an engine is its rate of performing work. This is called the **horse-power (H. P.)** which is equivalent to 550 ft. lbs. of work per second.

EXERCISES ON CHAPTER X

Reference

Art. 124 1. Describe the principle and action of a Steam Engine, giving a sectional diagram. (Del. U.—1952; U. P. B.—1950; C. U.—1947; Del. H. S.—1950; E. P. U.—1952; Dac. U.—1941; P. U.—1931)

Art. 124 2. Describe with sketch the principle underlying the construction of a Steam Engine, and point out the transformation of energy which occur. (Utkal U.—1947, '49, '52)

Art. 126 3. Describe with a neat diagram any form of a modern Petrol engine. How does it act?
(C. U.—1940, '42, '48, '53; Gau. U.—1950, '52; E. P. U.—1953)

Art. 126 4. Describe the working of an internal combustion engine with diagram. (Dac. U.—1942; U. P. B.—1947; B. H. U.—1950)

Art. 125 5. A 33-H. P. engine consumes 22500 gm. of coal per hour. What proportion of the coal is wasted? The calorific value of coal is such that 1 gm. of coal can convert 16 gm. of water at 100°C. into steam at 100°C. (1 H. P. $= 746\times 10^7$ ergs. per sec. and $J = 4{\cdot}2\times 10^7$ ergs. per calorie; latent heat of steam = 536).
A. 0·9.

Art. 125 6. A gas engine having an overall efficiency of 20 per cent. burns 1500 cu. ft. of gas per hour. If the calorific value of the fuel is 800 B. Th. U. per c. ft., find the H. P. of the engine.
A. 94 H. P.

Art. 125 7. A steam engine burns 100 gm. of coal per minute, the calorific value of the coal being 6000 calories per gm. The engine is used to turn a wheel around which passes a rope which lifts a weight of 140 kgm. The wheel makes 30 revolutions per min. and its radius is 50 cm. Find the efficiency of the machine.
A. 5·1%.

Arts. 124 & 126 8. What is a Heat engine? Describe the construction and working of an external combustion engine and derive an expression for its horse-power. (Raj. U.—1953)

9. What is the essential difference between a steam engine and an oil engine? (*Cf.* E. P. U.—1950; C. U.—1948)

Art. 125 10. The combustion of 1 pound of coal results in the evolution of 14,000 British Thermal Units. What is the efficiency of a steam engine pumping plant which consumes 20 pounds of coal per hour and raises each minute 20,000 pounds of water to a height of 12 feet?
A. 6·6%.